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On neighbourhood system and convergence of soft nets in redefined soft topological spaces

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Abstract

The incentive of this paper is to enhance different notions in redefined soft topological spaces. We introduce a concept of strong soft neighbourhood operator which fully characterizes the redefined soft topology. A notion of weak soft base is introduced and its characterizations are investigated. Finally soft nets and their convergence are defined in a redefined soft topological space and the characterizations of continuity of a soft function and soft $p-T_2$ property of redefined soft topological spaces are investigated in terms of soft nets.

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1 Introduction

Due to the inadequacy of parameterization in fuzzy set theory Molodtsov proposed the theory of soft sets for first time in 1999. Later in 2002 some operations are defined on soft sets. Since then, many researchers have developed different algebraic structures on soft sets and shown several applications (for references please see [1, 13, 6, 4, 7]). Soft set theory has potential applications in fields like the smoothness of functions, Perron integration, measure theory, game theory, operation researches, Riemann integration, etc. Shabir and Naz [14], in 2011 introduced a definition of soft topological spaces on the collection of soft sets. In 2015 Shi and Pang[15] has commented that soft topology in the sense of Shabir and Naz [14] can be interpreted as a crisp topology. Very recently in 2017, Chiney and Samanta [2] redefined soft topology using elementary union and elementary intersection and elementary complement, though these operations are not distributive and do not obey the excluded middle law.

In the present paper we introduce notions of strong soft neighbourhood operator, weak soft base, soft nets and their convergence in Chiney and Samanta[2] - type soft topology and study some of their properties.

The organization of the paper is as follows :

Section 2 contains the preliminaries where definition of soft set and some of the basic properties of redefined soft topology are discussed. In section 3, we introduce strong soft neighbourhood operator and soft topology induced by strong soft neighbourhood operator. In section 4, we introduce weak soft base a soft topology and study some of its basic properties. In section 5, we define soft net and its convergence in a redefined soft topological space and studied the characterizations of continuity of a soft function and soft $p-T_2$ property of a soft topological spaces with the help of convergence of soft net. Section 6 concludes the paper.

2 Preliminaries

Definition 2.1. *[11] Let X be a universal set and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\alpha \in A$, $F(\alpha)$ may be considered as the set of α approximate elements of the soft set (F,A).*

In [8] the soft sets are redefined as follows:

Let E be the set of parameters and $A \subseteq E$. Then for each soft set (F,A)

over X, a soft set (H,E) is constructed over X, $\forall\alpha \in E$,

$$H(\alpha) = \begin{cases} F(\alpha) & if\ \alpha \in A \\ \phi & if \alpha \in E\backslash A. \end{cases}$$

Thus the soft set (F,A) and (H,E) are equivalent to each other and the usual set operations of the soft sets (F_i,A), $i \in \Delta$ is the same as those of the soft sets (H_i,E), $i \in \Delta$. For this reason, in this paper, we have considered our soft sets over same parameter set A.

Following Molodtsov and Maji et al. [9, 10, 11] definition of soft subset, absolute soft set, null soft set, arbitrary union and arbitrary intersection of soft sets etc. are presented in [12] considering the same parameter set.

Definition 2.2. *[12] For two soft sets (F,A) and (G,A) over a common universe X.*

(a) (F,A) is said to be a soft subset of (G,A) if $F(\alpha)$ is a subset of $G(\alpha) \forall\alpha \in A$.

(b) Two soft sets (F,A) and (G,A) over a common universe X are said to be soft equal if (F,A) is a soft subset of (G,A) and (G,A) is a soft subset of (F,A).

(c) The complement or relative complement of a soft set (F,A) is denoted by $(F,A)^C$ and is defined by $(F,A)^C = (F^C,A)$, where, $F^C(\alpha) = X\backslash F(\alpha), \forall\alpha \in A$.

(d) (Null soft set) (F,A) over X is said to be a null soft set if $F(\alpha) = \phi, \forall\alpha \in A$ and it is denoted by $(\tilde{\Phi},A)$.

(e) (Absolute soft set) (F,A) over X is said to be an absolute soft set if $F(\alpha) = X, \forall\alpha \in A$.

(f) Union of two soft set (F,A) and (G,A) is denoted by $(F,A)\tilde{\cup}(G,A)$ and defined by
$[(F,A)\tilde{\cup}(G,A)](\alpha) = F(\alpha) \cup G(\alpha), \forall\alpha \in A$.

(g) Intersection of two soft sets (F,A) and (G,A) is denoted by $(F,A)\tilde{\cap}(G,A)$ and is defined by $[(F,A)\tilde{\cap}(G,A)](\alpha) = F(\alpha) \cap G(\alpha), \forall\alpha \in A$.

Definition 2.3. *[3] Let X be a non-empty set and A be a non-empty parameter set. Then a function, $\tilde{x} : A \to X$ is said to be a soft element of X. A soft element $\tilde{x}$ of X is said to belong to a soft set (F,A) over X, which is denoted by $\tilde{x}\tilde{\in}(F,A)$, if $\tilde{x}(\lambda) \in F(\lambda), \forall\lambda \in A$. Thus for a soft set (F,A) over X with respect to the index set A with $F(\lambda) \neq \phi, \forall\lambda \in A$, we have $F(\lambda) = \{\tilde{x}(\lambda) : \tilde{x}\tilde{\in}(F,A)\}$, for all $\lambda \in A$.*

Let X be an initial universal set and A be a non-empty parameter set. Throughout the paper we consider the null soft set $(\tilde{\Phi},A)$ and those soft sets (F,A) over X for which $F(\alpha) \neq \phi, \forall\alpha \in A$. We denote this collection by $S(\tilde{X})$. Thus for all $(F,A)[\neq (\tilde{\Phi},A)] \in S(\tilde{X}), F(\alpha) \neq \phi, \forall\alpha \in A$. The soft set constructed from a collection $\mathscr{B}$ will be denoted by $SS(\mathscr{B})$. For any soft set $(F,A) \in S(\tilde{X})$, the collection of all soft element of (F,A) is denoted by $SE(F,A)$.

Definition 2.4. *[3] Let $\mathbb{R}$ be the set of real numbers and $\mathfrak{B}(\mathbb{R})$ be the collection of all non-empty bounded subset of $\mathbb{R}$ and A be a set of parameters. Then a mapping $F : A \to \mathfrak{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F,A). If specifically (F,A) is a singleton set, then after identifying (F,A) with the corresponding soft element, it will be called a soft real number.*

$\tilde{r}, \tilde{s}, \tilde{t}$ denotes soft real numbers whereas $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r, \forall\lambda \in A$.

Proposition 2.5. *[5] For any soft sets $(F,A), (G,A) \in S(\tilde{X})$, $(F,A)\tilde{\subseteq}(G,A)$ iff every soft element of (F,A) is also a soft element of (G,A).*

Definition 2.6. *[5] For any two soft sets $(F,A), (G,A) \in S(\tilde{X})$,*

(a) elementary union of (F,A) and (G,A) is denoted by $(F,A)\Cup(G,A)$ and is defined by $(F,A)\Cup(G,A) = SS(\mathscr{B})$, where, $\mathscr{B} = \{\tilde{x}\tilde{\in}(\tilde{X},A) : \tilde{x}\tilde{\in}(F,A)$ or $\tilde{x}\tilde{\in}(G,A)\}$; i.e. $(F,A)\Cup(G,A) = SS(SE(F,A) \cup SE(G,A))$.

(b) elementary intersection of (F,A) and (G,A) is denoted by $(F,A)\Cap(G,A)$ and is defined by $(F,A)\Cap(G,A) = SS(\mathscr{B})$, where, $\mathscr{B} = \{\tilde{x}\tilde{\in}(\tilde{X},A) : \tilde{x}\tilde{\in}(F,A)$ and $\tilde{x}\tilde{\in}(G,A)\}$ i.e. $(F,A)\Cap(G,A) = SS(SE(F,A) \cap SE(G,A))$.

Definition 2.7. *[5] For any soft set$(F,A) \in S(\tilde{X})$, the elementary complement of (F,A) is denoted by $(F,A)^{\mathbb{C}}$ and is defined by $(F,A)^{\mathbb{C}} = SS(\mathscr{B})$, where, $\mathscr{B} = \{\tilde{x}\tilde{\in}(\tilde{X},A) : \tilde{x}\tilde{\in}(F,A)^C\}$ and $(F,A)^C$ is the complement of (F,A).*

Remark 2.8. *[5] It can be easily verified that if* $(F,A),(G,A) \in S(\tilde{X})$*, then* $(F,A) \Cup (G,A)$ *and* $(F,A) \Cap (G,A)$ *and* $(F,A)^{\mathbb{C}}$ *are members of* $S(\tilde{X})$.

Proposition 2.9. *[5] For any two soft sets* $(F,A),(G,A) \in S(\tilde{X})$.

(a) $(F,A) \Cup (G,A) = (F,A)\tilde{\cup}(G,A)$

(b) $(F,A) \Cap (G,A) = (F,A)\tilde{\cap}(G,A)$*, if* $(F,A) \Cap (G,A) \neq (\tilde{\Phi},A)$.

Remark 2.10. *[2] The above result can be extended easily to arbitrary union and arbitrary intersection.*

Proposition 2.11. *[5] For any soft set* $(F,A)\in S(\tilde{X})$*, in general* $(F,A)^{\mathbb{C}}\tilde{\subseteq}(F,A)^C$ *and* $(F,A)^{\mathbb{C}} = (F,A)^C$*, if* $(F,A)^{\mathbb{C}} \neq (\tilde{\Phi},A)$ *i.e. if* $(F,A)^C \in S(\tilde{X})$.

Proposition 2.12. *[5] For any soft set* $(F,A)\in S(\tilde{X})$.

(a) $(F,A) \Cap (F,A)^{\mathbb{C}}=(\tilde{\Phi},A)$.

(b) *In general* $(F,A) \Cup (F,A)^{\mathbb{C}}\tilde{\subseteq}(\tilde{X},A)$ *but if* $(F,A)^{\mathbb{C}} \neq (\tilde{\Phi},A)$*, then* $(F,A) \Cup (F,A)^{\mathbb{C}} = (\tilde{X},A)$.

(c) *In general* $((F,A)^{\mathbb{C}})^{\mathbb{C}} \neq (F,A)$. *If* $(F,A)^{\mathbb{C}} \neq (\tilde{\Phi},A)$*, then* $((F,A)^{\mathbb{C}})^{\mathbb{C}} = (F,A)$.

Proposition 2.13. *[2] Let* $\{(F_i,A) : i \in \Delta\}$ *be any collection of soft sets in* $S(\tilde{X})$*, then,*

(a) $[\Cup\{(F_i,A) : i \in \Delta\}^{\mathbb{C}} = \Cap\{(F_i,A)^{\mathbb{C}} : i \in \Delta\}$.

(b) $[\Cap\{(F_i,A) : i \in \Delta\}]^{\mathbb{C}} = \Cup\{(F_i,A)^{\mathbb{C}} : i \in \Delta\}$.

Definition 2.14. *[2] Let* τ *be a collection of soft sets of* $S(\tilde{X})$*. Then* τ *is said to be a soft topology on* $(\tilde{X},A)$ *if*

(i) $(\tilde{\Phi},A)$ *and* $(\tilde{X},A)$ *belong to* τ.

(ii) *the elementary intersection of any two soft sets of* τ *belong to* τ.

(iii) *the elementary union of any number of soft sets of* τ *belong to* τ .

The triplet $(\tilde{X},\tau,A)$ *is called a soft topological space.*

Definition 2.15. *[2] In a soft topological space* $(\tilde{X},\tau,A)$*, the members of* τ *are called soft open sets in* $(\tilde{X},\tau,A)$.

Definition 2.16. *[2] Let $(\tilde{X},\tau,A)$ be a soft topological space. A soft set $(F,A)\in S(\tilde{X})$ is called a soft closed set in $(\tilde{X},\tau,A)$ if its relative complement $(F,A)^C \in S(\tilde{X})$ and $(F,A)^{\mathbb{C}} \in \tau$.*

Definition 2.17. *[2] Let $(\tilde{X},\tau,A)$ be a soft topological space. Then a subcollection $\mathscr{B}$ of τ, consisting $(\tilde{\Phi},A)$, is said to be a soft base for τ if $\forall \tilde{x}\tilde{\in}(\tilde{X},A)$ and for any soft open set (F,A) consisting the soft element $\tilde{x}$, there exists $(G,A)\in\mathscr{B}$ such that $\tilde{x}\tilde{\in}(G,A)\tilde{\subseteq}(F,A)$.*

Definition 2.18. *[2] Let $(\tilde{X},\tau,A)$ be a soft topological space. Then $(F,A)[\neq(\tilde{\Phi},A)]\in S(\tilde{X})$ is said to be a soft neighbourhood (soft nbd) of a soft element $\tilde{x}$ if there exists a soft open set (G,A) such that $\tilde{x}\tilde{\in}(G,A)\tilde{\subseteq}(F,A)$. The soft nbd system at a soft element $\tilde{x}$, denoted by $\aleph_\tau(\tilde{x})$, is the family of all its soft nbds.*

Definition 2.19. *[2]A mapping $\nu : SE(\tilde{X})\to P(S(\tilde{X}))$ is said to be a soft nbd operator on $SE(\tilde{X})$ if the following conditions hold:*
$(N1)$ $\nu(\tilde{x})\neq\phi, \forall\tilde{x}\in SE(\tilde{X})$
$(N2)$ $\tilde{x}\tilde{\in}(F,A), \forall(F,A)\in\nu(\tilde{x})$
$(N3)$ $(F,A)\in\nu(\tilde{x}),(F,A)\tilde{\subseteq}(G,A)\Rightarrow(G,A)\in\nu(\tilde{x})$
$(N4)$ $(F,A),(G,A)\in\nu(\tilde{x})\Rightarrow(F,A)\Cap(G,A)\in\nu(\tilde{x})$
$(N5)$ $(F,A)\in\nu(\tilde{x})\Rightarrow\exists(G,A)\in\nu(\tilde{x})$ *such that* $(G,A)\tilde{\subseteq}(F,A)$ *and* $(G,A)\in\nu(\tilde{y}),\forall\tilde{y}\tilde{\in}(G,A)$.

Definition 2.20. *[2] Let X and Y be two non-empty set and $\{f_\lambda : X\to Y,\lambda\in A\}$ be a collection of functions. Then a function $f : SE(\tilde{X})\to SE(\tilde{Y})$ defined by $[f(\tilde{x})](\lambda)=f_\lambda(\tilde{x}(\lambda)),\forall\lambda\in A$ is called a soft function.*

Definition 2.21. *[2] Let $f : SE(\tilde{X})\to SE(\tilde{Y})$ be a soft function. Then*

(i) image of a soft set (F,A) over X under the soft function f, is denoted by $f[(F,A)]$, is defined by $f[(F,A)]=SS\{f(SE(F,A))\}$ i.e. $f[(F,A)](\lambda)=f_\lambda((F(\lambda)),\forall\lambda\in A$.

(ii) inverse image of a soft set (G,A) over Y under the soft function f, denoted by $f^{-1}[(G,A)]$, is defined by $f^{-1}[(G,A)]=SS\{f^{-1}(SE(G,A))\}$ i.e. $f^{-1}[(G,A)](\lambda)=f_\lambda^{-1}(G(\lambda)),\forall\lambda\in A$.

6

Definition 2.22. *[2] Let* $(\tilde{X},\tau,A)$ *and* $(\tilde{Y},\nu,A)$ *be two soft topological spaces and* $f : SE(\tilde{X}) \to SE(\tilde{Y})$ *be a soft function associated with the family of functions* $\{f_\lambda : X \to Y, \lambda \in A\}$*. Then we denote this soft function as* $f : (\tilde{X},\tau,A) \to (\tilde{Y},\nu,A)$*.*
Now $f : (\tilde{X},\tau,A) \to (\tilde{Y},\nu,A)$ *is said to be soft continuous at* $\tilde{x_\circ}\tilde{\in}(\tilde{X},A)$*, if for every* $(V,A) \in \nu$ *such that* $f(\tilde{x_\circ})\tilde{\in}(V,A)$*, there exists* $(U,A) \in \tau$ *such that* $\tilde{x_\circ}\tilde{\in}(U,A)$ *and* $f[(U,A)]\tilde{\subseteq}(V,A)$*.*
f *is said to be soft continuous on* $(\tilde{X},\tau,A)$ *if it is soft continuous at each soft element* $\tilde{x_\circ}\tilde{\in}(\tilde{X},A)$*.*

Definition 2.23. *[2] Let* $f : SE(\tilde{X}) \to SE(\tilde{Y})$ *be a soft function associated with the family of functions* $\{f_\lambda : X \to Y, \lambda \in A\}$*. Then* f *is said to be*

(a) injective if $\tilde{x} \neq \tilde{y}$ *implies* $f(\tilde{x}) \neq f(\tilde{y})$*.*

(b) surjective if $f(\tilde{X},A) = (\tilde{Y},A)$*.*

(c) bijective if both injective and surjective.

Proposition 2.24. *[2] Let* X *and* Y *be two non-empty sets and* A *be the set of parameters. Also let* $f : SE(\tilde{X}) \to SE(\tilde{Y})$ *be a soft function associated with the family of functions* $\{f_\lambda : X \to Y, \lambda \in A\}$*. If* $(F,A) \in S(\tilde{Y})$*, then*

(i) $ff^{-1}(F,A)\tilde{\subseteq}(F,A)$*.*

(ii) $(F,A)\tilde{\subseteq}f^{-1}f(F,A)$*.*

Proposition 2.25. *[2] Let* X *and* Y *be two non-empty sets and* A *be the set of parameters. Also let* $f : SE(\tilde{X}) \to SE(\tilde{Y})$ *be a soft function associated with the family of functions* $\{f_\lambda : X \to Y, \lambda \in A\}$*. If* $(F_1,A),(F_2,A) \in S(\tilde{X})$ *then*

(i) $(F_1,A)\tilde{\subseteq}(F_2,A) \Rightarrow f[(F_1,A)]\tilde{\subseteq}f[(F_2,A)]$*.*

(ii) $f[(F_1,A) \Cup (F_2,A)] = f[(F_1,A)] \Cup f[(F_2,A)]$*.*

(iii) $f[(F_1,A) \Cap (F_2,A)]\tilde{\subseteq}f[(F_1,A)] \Cap f[(F_2,A)]$*.*

(iv) $f[(F_1,A) \Cap (F_2,A)] = f[(F_1,A)] \Cap f[(F_2,A)]$*, if* f *is one-one.*

Proposition 2.26. *[2] Let X and Y be two non-empty sets and A be the set of parameters. Also let $f : SE(\tilde{X}) \to SE(\tilde{Y})$ be a soft function associated with the family of functions $\{f_\lambda : X \to Y, \lambda \in A\}$. If $(F_1,A),(F_2,A) \in S(\tilde{Y})$ then*

(i) $(F_1,A)\tilde{\subseteq}(F_2,A) \Rightarrow f^{-1}[(F_1,A)]\tilde{\subseteq}f^{-1}[(F_2,A)]$.

(ii) $f^{-1}[(F_1,A) \Cup (F_2,A)] = f^{-1}[(F_1,A)] \Cup f^{-1}[(F_2,A)]$.

(iii) $f^{-1}[(F_1,A) \Cap (F_2,A)] = f^{-1}[(F_1,A)] \Cap f^{-1}[(F_2,A)]$.

Definition 2.27. *[2] A soft function $f : (\tilde{X},\tau,A) \to (\tilde{Y},\nu,A)$ is said to be*

(a) soft open if f maps soft open sets of τ to soft open sets of ν.

(b) soft closed if f maps soft closed sets of τ to soft closed sets of ν.

Definition 2.28. *[2] Let $(\tilde{X},\tau,A)$ be a soft topological space. If for $\tilde{x},\tilde{y} \in SE(\tilde{X})$ with $\tilde{x}(\tilde{\lambda}) \neq \tilde{y}(\lambda), \forall \lambda \in A$,*

(a) there exist $(F,A) \in \tau$ such that $[\tilde{x}(\lambda) \in (F,A)(\lambda)$ and $\tilde{y}(\lambda) \notin (F,A)(\lambda)]$ or $[\tilde{y}(\lambda) \in (F,A)(\lambda)$ and $\tilde{x}(\lambda) \notin (F,A)(\lambda)], \forall \lambda \in A$, then $(\tilde{X},\tau,A)$ is called a soft T_0 space.

(b) there exist $(F,A),(G,A) \in \tau$ such that $[\tilde{x}(\lambda) \in (F,A)(\lambda),\ \tilde{y}(\lambda) \notin (F,A)(\lambda)]$ and $[\tilde{y}(\lambda) \in (G,A)(\lambda),\ \tilde{x}(\lambda) \notin (G,A)(\lambda)], \forall \lambda \in A$, then $(\tilde{X},\tau,A)$ is called a soft T_1 space.

(c) there exist $(F,A),(G,A) \in \tau$ such that $\tilde{x}\tilde{\in}(F,A)$ and $\tilde{y}\tilde{\in}(G,A)$ and $(F,A)\tilde{\cap}(G,A) = (\tilde{\Phi},A)$, then $(\tilde{X},\tau,A)$ is called a soft T_2 space.

3 Strong soft neighbourhood operator

Proposition 3.1. *Let $(\tilde{X},\tau,A)$ be a soft topological space, $\tilde{x}_i \in S(\tilde{X})$, $(F_i,A) \in \aleph(\tilde{x}_i)\ \forall i \in \Delta$ and $(G,A) = SS\{\tilde{x}_i : i \in \Delta\}$, $(F,A) = \underset{i\in\Delta}{\Cup}(F_i,A)$ then $(F,A) \in \aleph(\tilde{y})\ \forall\ \tilde{y}\tilde{\in}(G,A)$.*

Proof. Given, $(F_i,A) \in \aleph(\tilde{x})\ \forall\ i \in \Delta$.
Then $\exists(G_i,A) \in \tau$ such that, $\tilde{x}_i\tilde{\in}(G_i,A)\tilde{\subset}(F_i,A)\ \forall\ i \in \Delta$.
Let $(G_0,A) = \underset{i\in\Delta}{\Cup}(G_i,A)$.

Then $(G,A)\tilde{\subset}(G_0,A) \in \tau$ and $(G_0,A)\tilde{\subset}(F,A)$.
Now for any $\tilde{y}\tilde{\in}(G,A)$, $\tilde{y}\tilde{\in}(G_0,A)\tilde{\subset}(F,A)$ and $(G_0,A) \in \tau$.
Hence $(F,A) \in \aleph(\tilde{y})$. □

Definition 3.2. *A mapping $\nu : SE(\tilde{X}) \to P(S(\tilde{X}))$ is said to be a strong soft neighbourhood operator if the following conditions hold:*

(N1) $\nu(\tilde{x}) \neq \phi,\ \forall \tilde{x} \in SE(\tilde{X})$

(N2) $\tilde{x}\tilde{\in}(F,A),\ \forall (F,A) \in \nu(\tilde{x})$

(N3) $(F,A) \in \nu(\tilde{x}), (F,A)\tilde{\subset}(G,A) \Rightarrow (G,A) \in \nu(\tilde{x})$

(N4) $(F,A),(G,A) \in \nu(\tilde{x}) \Rightarrow (F,A) \Cap (G,A) \in \nu(\tilde{x})$

(N5) $(F,A) \in \nu(\tilde{x}) \Rightarrow \exists (G,A) \in \nu(\tilde{x})$ *such that* $(G,A)\tilde{\subset}(F,A)$ *and* $(G,A) \in \nu(\tilde{y}),\ \forall \tilde{y}\tilde{\in}(G,A)$

(N6) $(F_i,A) \in \nu(\tilde{x}_i),\ \forall i \in \Delta \Rightarrow \Cup_{i\in\Delta}(F_i,A) \in \nu(\tilde{y}),\ \forall\ \tilde{y}\tilde{\in}SS\{\tilde{x}_i : i \in \Delta\}$

Remark 3.3. *A strong soft neighbourhood operator is also a soft neighbourhood operator since, it satisfies all the properties of soft neighbourhood operator.*

Example 3.4. *If $(\tilde{X},\tau,A)$ is a soft topological space, then the mapping $\nu : SE(\tilde{X}) \to P(S(\tilde{X}))$ defined by $\nu(\tilde{x}) = \aleph_\tau(\tilde{x})$, where $\aleph_\tau(\tilde{x})$ is the soft nbd system at the soft element $\tilde{x}$, is a strong soft nbd operator on $SE(\tilde{X})$.*

Proposition 3.5. *Let, X be a universal set and A be a set of parameters, $\nu : SE(\tilde{X}) \to P(S(\tilde{X}))$ be a strong nbd operator on $SE(\tilde{X})$ then there exists a soft topological space $(\tilde{X},\tau,A)$ such that $\nu(\tilde{x}) = \aleph_\tau(\tilde{x}),\ \forall \tilde{x}\tilde{\in}(\tilde{X},A)$.*

Proof. Let $\tau = \{(G,A) : (G,A) \in \nu(\tilde{x}),\ \forall \tilde{x}\tilde{\in}(G,A)\}$. Then,

(i) $(\tilde{\Phi},A),(\tilde{X},A) \in \tau$

(ii) Let $(F,A),(G,A) \in \tau$ be such that $(F,A) \Cap (G,A) \neq (\tilde{\Phi},A)$.
Let $\tilde{x}\tilde{\in}(F,A) \Cap (G,A)$
$\Rightarrow \tilde{x}\tilde{\in}(F,A)\ \&\ \tilde{x}\tilde{\in}(G,A)$
$\Rightarrow (F,A) \in \nu(\tilde{x})\ \&\ (G,A) \in \nu(\tilde{x})$

$\Rightarrow (F,A) \Cap (G,A) \in \nu(\tilde{x})$.
Now since it is true for any $\tilde{x}\tilde{\in}(F,A) \Cap (G,A)$, $(F,A) \Cap (G,A) \in \tau$.

(iii) Let $(F_i,A)(\neq (\tilde{\Phi},A) \in \tau$, $\forall i \in \Delta$.
Let, $\tilde{y}\tilde{\in} \underset{i\in\Delta}{\Cup} (F_i,A)$, Then there exists $\tilde{x}_i\tilde{\in}(F_i,A)$ such that $\tilde{y}\tilde{\in}SS\{\tilde{x}_i : i \in \Delta\}$ but
$(F_i,A) \in \nu(\tilde{x}_i)$, $\forall i \in \Delta$. Hence $\underset{i\in\Delta}{\Cup}(F_i,A) \in \nu(\tilde{y})$, $\forall \tilde{y}\tilde{\in} \underset{i\in\Delta}{\Cup}(F_i,A) \Rightarrow$
$\underset{i\in\Delta}{\Cup}(F_i,A) \in \tau$

This proves that $(\tilde{X},\tau,A)$ is a soft topological space.
Now we will show that for any $\tilde{x}\tilde{\in}SE(\tilde{X})$, $\nu(\tilde{x})$ is the soft nbd system at $\tilde{x}$.
Let $(F,A) \in \nu(\tilde{x})$, then by $(N5)$ $\exists(G,A) \in \nu(\tilde{x})$ such that $(G,A)\tilde{\subset}(F,A)$ and $(G,A) \in \nu(\tilde{y})$, $\forall \tilde{y}\tilde{\in}(G,A)$
$\Rightarrow \tilde{x}\tilde{\in}(G,A)\tilde{\subset}(F,A)$ & $(G,A) \in \tau$
$\Rightarrow (F,A) \in \aleph(\tilde{x})$
Thus $\nu(\tilde{x}) \subset \aleph(\tilde{x})$.
Conversely suppose $(F,A) \in \aleph(\tilde{x})$ then $\exists(G,A) \in \tau$ such that $\tilde{x}\tilde{\in}(G,A)\tilde{\subset}(F,A)$.
Since $(G,A) \in \tau$ & $\tilde{x}\tilde{\in}(G,A)$, $(G,A) \in \nu(\tilde{x})$.
Thus $(F,A)(\tilde{\supset}(G,A)) \in \nu(\tilde{x})$ by $(N3)$. So $\aleph(\tilde{x}) \subset \nu(\tilde{x})$.
Hence $\aleph(\tilde{x}) = \nu(\tilde{x})$. This completes the proof. □

4 Weak soft base of a soft topology

Definition 4.1. *Let $(\tilde{X},\tau,A)$ be a soft topological space. Then $\mathscr{B}$, a sub collection of members of τ containing $(\tilde{\Phi},A)$ is said to be an weak open base of τ iff $\forall \tilde{x} \in SE(\tilde{X})$ and for any soft open set (F,A) containing $\tilde{x}$ and for each $\alpha \in A$, $\exists(G_\alpha,A) \in \mathscr{B}$ such that $\tilde{x}(\alpha) \in G_\alpha(\alpha)$ & $(G_\alpha,A)\tilde{\subset}(F,A)$.*

Remark 4.2. *Every soft open base is a weak soft open base.*

Proposition 4.3. *For any soft topological space $(\tilde{X},\tau,A)$, a sub collection $\mathscr{B}$ of members of τ is a weak open base for τ iff every member of τ can be expressed as elementary union of some members of $\mathscr{B}$.*

Proof. $(\tilde{X},\tau,A)$ is a soft topological space. Let $\mathscr{B}$ be a weak open base for τ, and $(F,A)(\neq (\tilde{\Phi},A)) \in \tau$. Now for any $\tilde{x}\tilde{\in}(F,A)$ and for each $\alpha \in A$, $\exists(G_\alpha,A) \in \mathscr{B}$ such that $\tilde{x}(\alpha) \in G_\alpha(\alpha)$ and $(G_\alpha,A)\tilde{\subset}(F,A)$.

Therefore $\tilde{x}\tilde{\in}(G_{\tilde{x}},A) = \Cup_{\alpha\in A}(G_\alpha,A)\tilde{\subset}(F,A)$ and $(F,A) = \Cup_{\tilde{x}\tilde{\in}(F,A)}(G_{\tilde{x}},A)$.
Conversely, Let $\mathscr{B}$ be a sub collection of members of τ such that every member of τ can be expressed as elementary union of some members of $\mathscr{B}$.
Let $(F,A) \in \tau$, then $\exists(G_i,A) \in \mathscr{B},\ i \in \Delta$ such that $(F,A) = \Cup_{i\in\Delta}(G_i,A)$.
Let $\tilde{x}\tilde{\in}(F,A)$ then $\tilde{x}\tilde{\in} \Cup_{i\in\Delta}(G_i,A)$. Then for each $\alpha \in A$, $\exists i_\alpha \in \Delta$ such that $\tilde{x}(\alpha) \in G_{i_\alpha}(\alpha)$.
Therefore $\tilde{x}(\alpha) \in G_{i_\alpha}(\alpha)$ and $(G_{i_\alpha},A)\tilde{\subset}(F,A) \in \tau$. Hence $\mathscr{B}$ is an weak open base for τ. □

Remark 4.4. *Converse of Remark 4.2 is not true, bellow is an example.*

Example 4.5. *Let* $X = \{x,y,z,t\}$, $A = \{\alpha,\beta\}$ *and*
$\tau = \{(\tilde{\Phi},A),(\tilde{X},A),(F_1,A),(F_2,A),(F_3,A),(F_4,A),(F_5,A),(F_6,A),(F_7,A)\}$
where
$F_1(\alpha) = \{x\}$, $F_1(\beta) = \{t\}$; $F_2(\alpha) = \{y\}$, $F_2(\beta) = \{z\}$;
$F_3(\alpha) = \{t\}$, $F_3(\beta) = \{x\}$; $F_4(\alpha) = \{x,y\}$, $F_4(\beta) = \{z,t\}$;
$F_5(\alpha) = \{y,t\}$, $F_5(\beta) = \{z,x\}$; $F_6(\alpha) = \{x,t\}$, $F_6(\beta) = \{x,t\}$;
$F_7(\alpha) = \{x,y,t\}$, $F_7(\beta) = \{x,z,t\}$.
Then τ *is a soft topology on* $(\tilde{X},A)$.
Let $\mathscr{B} = \{(\tilde{\Phi},A),(\tilde{X},A),(F_1,A),(F_2,A),(F_3,A)\}$.
Then $\mathscr{B}$ *forms an weak soft open base for* τ*, since every member of* τ *can be expressed as elementary union of members of* $\mathscr{B}$*. But* $\mathscr{B}$ *fails to be a soft open base.*

Proposition 4.6. *Let* $(\tilde{X},\tau,A)$ *be a soft topological space and* $\mathscr{B}$ *is a weak soft open base for* τ*, then* $\mathscr{B}$ *has the following properties:*

(i) $\mathscr{B}$ *covers* $(\tilde{X},A)$.

(ii) If $(F_1,A),(F_2,A) \in \mathscr{B}$ *and* $\tilde{x}\tilde{\in}(F_1,A) \Cap (F_2,A)$ *then for each* $\alpha \in A$, $\exists(F_\alpha,A) \in \mathscr{B}$ *such that* $\tilde{x}(\alpha) \in F_\alpha(\alpha)$ *and* $(F_\alpha,A)\tilde{\subset}(F_1,A) \Cap (F_2,A)$.

Proof. (i) Let $\tilde{x}\tilde{\in}(\tilde{X},A)$ then for each $\alpha \in A$, $\exists(G_\alpha,A) \in \mathscr{B}$ such that $\tilde{x}(\alpha) \in G_\alpha(\alpha)$ and $(G_\alpha,A)\tilde{\subset}(\tilde{X},A)$.
Then $\tilde{x}\tilde{\in}(G_{\tilde{x}},A) = \Cup_{\alpha\in A}(G_\alpha,A)\tilde{\subset}(\tilde{X},A)$.

Therefore $\Cup_{\tilde{x}\in(\tilde{X},A)}(G_{\tilde{x}},A) = (\tilde{X},A)$.
Thus $\mathscr{B}$ covers $(\tilde{X},A)$.

(ii) Let $(F_1,A),(F_2,A) \in \mathscr{B}$ and $\tilde{x}\tilde{\in}(F_1,A)\Cap(F_2,A) = (F,A)$.
Clearly $(F,A) \in \tau$ then for any $\tilde{x}\tilde{\in}(F,A)$ and for each $\alpha \in A$, $\exists(F_\alpha,A) \in \mathscr{B}$ such that $\tilde{x}(\alpha) \in F_\alpha(\alpha)$ and $(F_\alpha,A)\tilde{\subset}(F,A)$.

□

5 Soft Net

Definition 5.1. *Let D be a directed set with order relation $\geq$ and $(\tilde{X},\tau,A)$ be a soft topological space. A function $\tilde{S}: D \to SE(\tilde{X})$ is said to be a soft net.*

Example 5.2. *Given a soft element $\tilde{x}$ in a soft topological space $(\tilde{X},\tau,A)$, let $\aleph_{\tilde{x}}$ denote the set of all neighbourhoods containing $\tilde{x}$. Then $\aleph_{\tilde{x}}$ is a directed set, where the direction is given by reverse inclusion, so that $(G,A) \geq (F,A)$ if and only if $(G,A)\tilde{\subset}(F,A)$. Consider a function $\tilde{S}: \aleph_{\tilde{x}} \to SE(\tilde{X})$, so that for (G,A) in $\aleph_{\tilde{x}}$, $\tilde{S}((G,A))$ is a soft element in (G,A). Then $\tilde{S}$ is a soft net.*

Definition 5.3. *Let D be a directed set with order relation $\geq$ and $(\tilde{X},\tau,A)$ be a soft topological space and $(G,A)\tilde{\subset}(\tilde{X},A)$. $\tilde{S}: D \to SE(\tilde{X})$, a soft net is said to be eventually in (G,A) if $\exists\, n \in D$ so that for all $m \in D$ with $m \geq n$, the soft element $\tilde{S}(m)\tilde{\in}(G,A)$.*

Definition 5.4. *If $\tilde{S}$ is a soft net in a soft topological space $(\tilde{X},\tau,A)$ and $\tilde{x}$ is a soft element of $(\tilde{X},\tau,A)$, we say that the soft net $\tilde{S}$ converges towards $\tilde{x}$ or has limit $\tilde{x}$ and will write $\tilde{S} \to \tilde{x}$ if and only if for every neighbourhood (G,A) of $\tilde{x}$, $\tilde{S}$ is eventually in (G,A).*

Example 5.5. *The example soft net given above on the neighbourhood system of a soft element $\tilde{x}$ converge towards $\tilde{x}$ according to this definition.*

Proposition 5.6. *Let $(\tilde{X},\tau_1,A)$ and $(\tilde{Y},\tau_2,A)$ be two soft topological spaces, $\tilde{x}$ be a soft element in $(\tilde{X},A)$, $f: (\tilde{X},\tau_1,A) \to (\tilde{Y},\tau_2,A)$ be a soft function. Then for any soft net $\tilde{S}: (D,\geq) \to SE(\tilde{X})$, $\tilde{S} \to \tilde{x} \Rightarrow f\circ\tilde{S} \to f(\tilde{x})$ iff the soft function $f: (\tilde{X},\tau_1,A) \to (\tilde{Y},\tau_2,A)$ is soft continues at $\tilde{x}$.*

Proof. $(\tilde{X},\tau_1,A)$ and $(\tilde{Y},\tau_2,A)$ are two soft topological spaces. Let for two soft elements $\tilde{x}\tilde{\in}(\tilde{X},A)$ and $\tilde{y}\tilde{\in}(\tilde{Y},A)$, $\aleph_{\tau_1}(\tilde{x})$ and $\aleph_{\tau_2}(\tilde{y})$ denote the soft neighbourhood system of $\tilde{x}$ and $\tilde{y}$ with respect to τ_1 and τ_2 respectively.

Assume that $f:(\tilde{X},\tau_1,A)\to(\tilde{Y},\tau_2,A)$ is soft continues at $\tilde{x}$ and $\tilde{S}:(D,\geq)\to SE(\tilde{X})$ be a soft net so that $\tilde{S}\to\tilde{x}$. Then, for any $(F,A)\in\aleph_{\tau_2}(f(\tilde{x}))$, $\exists(G,A)\in\aleph_{\tau_1}(\tilde{x})$ such that $f(G,A)\tilde{\subset}(F,A)$.
Also since $(G,A)\in\aleph_{\tau_1}(\tilde{x})$ and $\tilde{S}\to\tilde{x}$,
$\tilde{S}$ is eventually in (G,A)
$\Rightarrow f\circ\tilde{S}$ is eventually in $f(G,A)$
$\Rightarrow f\circ\tilde{S}$ is eventually in (F,A), since $f(G,A)\tilde{\subset}(F,A)$.
Now (F,A) being an arbitrary soft nbd of soft element $f(\tilde{x})$, $f\circ\tilde{S}\to f(\tilde{x})$.

Conversely assume that $f:(\tilde{X},\tau_1,A)\to(\tilde{Y},\tau_2,A)$ is a soft function such that for any soft net $\tilde{S}:(D,\geq)\to SE(\tilde{X})$, $\tilde{S}\to\tilde{x}\Rightarrow f\circ\tilde{S}\to f(\tilde{x})$.
If possible let $f:(\tilde{X},\tau_1,A)\to(\tilde{Y},\tau_2,A)$ is not soft continuous at $\tilde{x}$, then there exists $(F,A)\in\aleph_{\tau_2}(f(\tilde{x}))$ such that $f^{-1}(F,A)\notin\aleph_{\tau_1}(\tilde{x})$.
We take $(\aleph_{\tau_1}(\tilde{x}),\geq)$ as our directed set, where the direction is given by reverse inclusion, so that for (G,A), $(H,A)\in\aleph_{\tau_1}(\tilde{x})$, $(G,A)\geq(H,A)$ if and only if $(G,A)\tilde{\subset}(H,A)$.
Define a soft net $\tilde{S}:(\aleph_{\tau_1}(\tilde{x}),\geq)\to SE(\tilde{X})$ by,
$\tilde{S}((G,A))=\tilde{x_G}$ for each $(G,A)\in\aleph_{\tau_1}(\tilde{x})$ where $\tilde{x_G}\tilde{\in}SE((G,A))\setminus SE(f^{-1}(F,A))$.
Then for any $(G_1,A)\in\aleph_{\tau_1}(\tilde{x})$, $(G_2,A)\geq(G_1,A)$ will imply $(G_2,A)\tilde{\subset}(G_1,A)$ and this will imply $\tilde{S}((G_2,A))\tilde{\in}(G_1,A)$. Hence $\tilde{S}\to\tilde{x}$ and by our assumption this implies $f\circ\tilde{S}\to f(\tilde{x})$.
Now (F,A) is a soft nbd of $f(\tilde{x})$ but by definition of the net $\tilde{S}$, $f\circ\tilde{S}$ is not eventually in (F,A), which leads to a contradiction. Hence f must be soft continuous at $\tilde{x}$.
This completes the proof. □

Definition 5.7. *Let $(\tilde{X},\tau,A)$ be a soft topological space. If for $\tilde{x},\tilde{y}\in SE(\tilde{X})$ with $\tilde{x}\neq\tilde{y}$, there exist $(F,A),(G,A)\in\tau$ such that $\tilde{x}\tilde{\in}(F,A)$, $\tilde{y}\tilde{\in}(G,A)$ and $(F,A)\Cap(G,A)=(\tilde{\Phi},A)$, then $(\tilde{X},\tau,A)$ is called a soft $p-T_2$ space.*

Proposition 5.8. *Let $(\tilde{X},\tau,A)$ be a soft topological space. Then $(\tilde{X},\tau,A)$ is a soft $p-T_2$ space iff the limit of any convergent soft net $\tilde{S}:(D,\geq)\to SE(\tilde{X})$ is unique.*

Proof. $(\tilde{X},\tau,A)$ is a soft topological space.
First we assume that $(\tilde{X},\tau,A)$ is a soft $p-T_2$ space. Let $\tilde{S}:(D,\geq) \to SE(\tilde{X})$ be a convergent soft net. If possible let $\tilde{S}$ has two limits $\tilde{x}$ and $\tilde{y}$ such that $\tilde{x} \neq \tilde{y}$. Since $\tilde{x} \neq \tilde{y}$ there exists $(F,A) \in \aleph_\tau(\tilde{x})$ and $(G,A) \in \aleph_\tau(\tilde{y})$ such that $(F,A) \Cap (G,A) = (\tilde{\Phi},A)$. But since $\tilde{S} \to \tilde{x}$ and $\tilde{S} \to \tilde{y}$, $\tilde{S}$ is eventually in (F,A) and (G,A) which implies $\tilde{S}$ is eventually in $(F,A) \Cap (G,A)$, but $(F,A) \Cap (G,A) = (\tilde{\Phi},A)$, which leads us to a contradiction. Thus $\tilde{S}$ can not have two distinct limits i.e, limit of $\tilde{S}$ is unique.
Conversely assume that any convergent soft net $\tilde{S}:(D,\geq) \to SE(\tilde{X})$ has unique limit.
If possible let $(\tilde{X},\tau,A)$ is not soft $p-T_2$ and $\tilde{x}$, $\tilde{y} \tilde{\in} (\tilde{X},A)$ be two distinct soft elements. Then for every $((F,A),(G,A)) \in \aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y})$, $(F,A) \Cap (G,A) \neq (\tilde{\Phi},A)$. Now
$(\aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y}),\geq)$ is a directed set where for
$((F_1,A),(G_1,A))$, $((F_2,A),(G_2,A)) \in \aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y})$,
$((F_1,A),(G_1,A)) \geq ((F_2,A),(G_2,A))$ iff $(F_1,A) \tilde{\subset} (F_2,A)$ and $(G_1,A) \tilde{\subset} (G_2,A)$.
We define a soft net $\tilde{S}:(\aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y}),\geq) \to SE(\tilde{X})$ by
$\tilde{S}(((F_i,A),(G_i,A))) = \tilde{S}_i$, where $\tilde{S}_i \tilde{\in} (F_i,A) \Cap (G_i,A)$ for all $((F_i,A),(G_i,A)) \in \aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y})$.
Then for any two soft neighbourhoods $(F_n,A) \in \aleph_\tau(\tilde{x})$ and $(G_n,A) \in \aleph_\tau(\tilde{y})$, $((F_n,A),(F_n,A)) \in \aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y})$ and for any $((F_m,A),(G_m,A)) \in \aleph_\tau(\tilde{x}) \times \aleph_\tau(\tilde{y})$,
$((F_m,A),(G_m,A)) \geq ((F_n,A),(G_n,A)) \Rightarrow \tilde{S_m} \tilde{\in} (F_n,A) \Cap (G_n,A)$
i.e, $\tilde{S}$ is eventually in (F_n,A) & (G_n,A) which implies $\tilde{S} \to \tilde{x}$ and $\tilde{S} \to \tilde{y}$, which contradicts our assumption that $\tilde{S}$ has unique limit.
Hence, $(\tilde{X},\tau,A)$ must be a soft $p-T_2$ space. This completes the proof. □

6 Conclusion

In this paper, we have studied some topological properties of neighbourhoods, bases and nets on redefined soft topological spaces. Still there are many scopes to study some important properties like product space, uniformity, Urysohn's lemma, Tychonoff space, metrizability in this context.

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