International Journal of Pure and Applied Mathematics
Volume 120 No. 5 2018, 1273-1294
ISSN: 1314-3395 (on-line version)
url: http://www.acadpubl.eu/hub/
Special Issue
http://www.acadpubl.eu/hub/

STRONG DOMINATION IN PSEUDO REGULAR AND COMPLETE FUZZY GRAPHS

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Abstract

In this article, the perception of strong domination constant number is introduced by using membership values of strong arcs in fuzzy graphs. The strong domination constant number γ_{sc} of pseudo regular fuzzy graph and complete fuzzy graph is determined. Further the relationship between the strong domination constant number of a pseudo regular fuzzy graph and complete fuzzy graph are discussed and theorems related to these concepts are stated and proved.

Keywords:

Pseudo regular fuzzy graph, Totally pseudo regular fuzzy graph, Complete fuzzy graph, Strong arcs, Weight of arcs, Strong domination constant number.

AMS Subject Classification: *05C12, 03E72,05C72*

1.Introduction

The primitive idea of domination appear in the game of chess where the problem was to place minimum number of chess pieces so as to dominate all the squares of the chess board. Mathematical research on the theory of domination for crisp graphs was initiated by Ore [18]. Cockayne and Hedetnieme [4] further developed the concept. Fuzzy graphs were introduced by Rosenfeld [20], who has described the fuzzy analogue of different graph theoretic concepts like paths, cycles, trees and connectedness and established some of their properties [20]. Some important works in fuzzy graph theory can be found in [1,2,3,14,15,16] Bhutani and Rosenfeld have introduced the concept of strong arcs [3]. Domination in fuzzy graphs using effective edges was introduced by Somasundaram and Somasundaram [22]. Pathinathan and Jesintha Rosline [19] defined relationship between various types of edges in both regular and totally regular fuzzy graph. Santhi Maheswari and Sekar [21] introduced on pseudo regular fuzzy graphs. Kalaiarasi [8] defined Optimization of fuzzy integrated vendor-buyer inventory models.

In this article, our aim is to introduce and study the theory of domination in the setting of pseudo regular and complete fuzzy graphs. Also, a comparison study is made between pseudo regular and totally pseudo regular fuzzy graphs with reference to strong domination constant number.

2.Preliminaries

Definition 2.1

A fuzzy graph G is a pair of function $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(V,E)$ where $E\subseteq V\times V$.

Definition 2.2

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The 2-degree of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$. That is , $t_G(v)=\sum d_G(u)$, where $d_G(u)$ is the degree of the vertex u which is adjacent with the vertex v.

Definition 2.3

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. A pseudo (average) degree of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v)=\frac{t_G(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v.

Definition 2.4

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. If $d_a(v)=k$, for all v in V then G is called k-pseudo regular fuzzy graph.

Definition 2.5

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The total pseudo degree of a vertex v is G is denoted by $td_a(v)$ and is defined as $td_a(v)=d_a(v)+\sigma(v)$ for all $v\in V$.

Definition 2.6

Let G be a fuzzy graph on $G^*:(V,E)$. If all the vertices of G have the same total pseudo degree k, then G is said to be a totally k-pseudo regular fuzzy graph.

Definition 2.7

A fuzzy graph G is said to be complete if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all $u,v\in\sigma^*$.

Definition 2.8

An arc (u,v) of a fuzzy graph is called an effective arc (M-strong arc) if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$.

Definition 2.9

The order p and size q of a fuzzy graph $G:(\sigma,\mu)$ are defined to be $p=\sum_{x\in V}\sigma(x)$ and $q=\sum_{(x,y)\in V\times V}\mu(x,y)$.

Definition 2.10

An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted.

Definition 2.11

An arc (x, y) in G is α-strong if $\mu(x, y) > CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is β-strong if $\mu(x, y) = CONN_{G-(x,y)}(x, y)$. An arc (x, y) in G is δ-arc if $\mu(x, y) < CONN_{G-(x,y)}(x, y)$.

Definition 2.12

A path P is called strong path if P contains only strong arcs. If $\mu(u,v) > 0$, then u and v are called neighbors. The set of all neighbors of u is denoted by $N(u)$. Also v is called strong neighbors of u if arc (u,v) is strong.

Definition 2.13

A node u is said to be isolated if $\mu(u,v) = 0$ for all $v \neq u$.

Definition 2.14

A set S of vertices of G is a dominating set of G if every vertex of $V(G) - S$ is adjacent to some vertex in S.

Definition 2.15

A minimum dominating set in a graph G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the domination number of G and is denoted by $\gamma(G)$.

Definition 2.16

A set D of nodes of G is a strong dominating set of G if every node of $V(G) - D$ is a strong neighbor of some node in D.

Definition 2.17

A minimum strong dominating set as a strong dominating set of minimum scalar cardinality. The scalar cardinality of a minimum strong dominating set is called the strong domination number of G.

Definition 2.18

The weight of a strong dominating set D is defined as $W(D)=\sum_{u\in D}\mu(u,v)$, where $\mu(u,v)$ is the minimum of the membership values (weight) of the strong arcs incident on u. The strong domination number of a fuzzy graph G is defined as the minimum weight of strong dominating sets of G and it is denoted by $\gamma_s(G)$ or simply γ_s. A minimum strong dominating set in a fuzzy graph G is a strong dominating set of minimum weight.

Definition 2.19

Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph on $G^*:(V,E)$. The strong domination constant number of an G is defined as the weight of all strong dominating sets of G are same and it is denoted by $\gamma_{sc}(G)$ or simply γ_{sc}.

Example 2.1

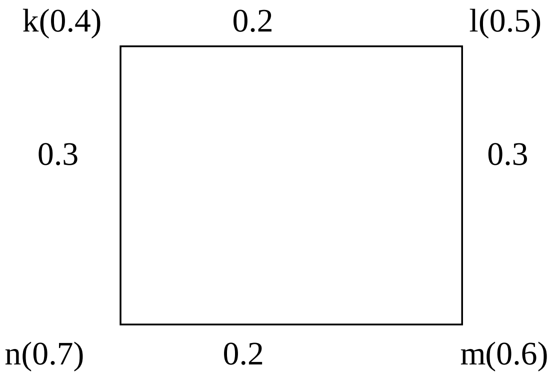

Fig.1. Strong domination constant number in Pseudo regular fuzzy graph

In this pseudo regular fuzzy graph, strong arcs are (k,l), (k,n), (n,m) and (m,l). The strong dominating sets are $D_1=(k,l)$, $D_2=(k,m)$, $D_3=(k,n)$, $D_4=(n,m)$, $D_5=(n,l)$, $D_6=(m,l)$

Where $W(D_1)=0.2+0.2=0.4$, $W(D_2)=0.2+0.2=0.4$, $W(D_3)=0.2+0.2=0.4$,

$W(D_4)=0.2+0.2=0.4$, $W(D_5)=0.2+0.2=0.4$, $W(D_6)=0.2+0.2=0.4$

Hence $\gamma_{sc}(G)=0.4$

Remark 2.1

A pseudo regular fuzzy graph need not be a strong domination constant number of fuzzy graph.

Example 2.2

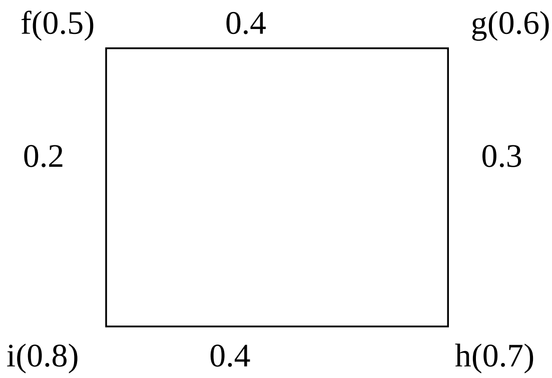

Fig.2. Pseudo regular fuzzy graph with out strong domination constant number

The graph G is a pseudo regular fuzzy graph.

In this pseudo regular fuzzy graph, strong arcs are (f,g), (g,h), and (h,i). The strong dominating sets are $D_1 = (f,g)$, $D_2 = (f,h)$, $D_3 = (g,h)$, $D_4 = (g,i)$, $D_5 = (h,i)$

Where $W(D_1) = 0.4 + 0.3 = 0.7$, $W(D_2) = 0.4 + 0.3 = 0.7$, $W(D_3) = 0.3 + 0.3 = 0.6$,

$W(D_4) = 0.3 + 0.4 = 0.7$, $W(D_5) = 0.3 + 0.4 = 0.7$

But $W(D_1) \neq W(D_3)$

Hence G is not a strong domination constant number of fuzzy graph .

Remark 2.2

If all the nodes are isolated, then V is the only strong dominating set of G of order p and $\gamma_{sc}(G) = 0$.

Remark 2.3

A strong domination constant number of fuzzy graph need not be a pseudo regular fuzzy graph.

Example 2.3

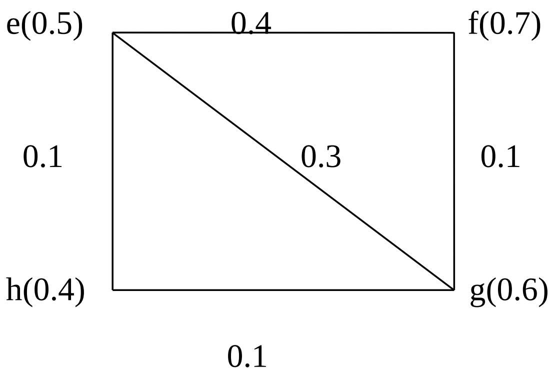

Fig. 3. Strong domination constant number of fuzzy graph

In this fuzzy graph, strong arcs are (e,f), (e,g), (e,h), (f,g) and (g,h). The strong dominating sets are $D_1=(e,f)$, $D_2=(e,g)$, $D_3=(e,h)$, $D_4=(f,g)$, $D_5=(f,h)$ and $D_6=(g,h)$

Where $W(D_1)=0.1+0.1=0.2$, $W(D_2)=0.1+0.1=0.2$, $W(D_3)=0.1+0.1=0.2$,

$W(D_4)=0.1+0.1=0.2$, $W(D_5)=0.1+0.1=0.2$, $W(D_6)=0.1+0.1=0.2$

Hence $\gamma_{sc}(G)=0.2$

The fuzzy graph G is strong domination constant number of fuzzy graph. But $d_a(e)\neq d_a(f)$. Hence G is not a pseudo regular fuzzy graph.

Definition 2.20

Let $G:(\sigma,\mu)$ be a complete fuzzy graph on $G^*:(V,E)$. The strong domination constant number of an G is defined as the weight of all strong dominating sets of G are same and it is denoted by $\gamma_{sc}(G)$

Example 2.4

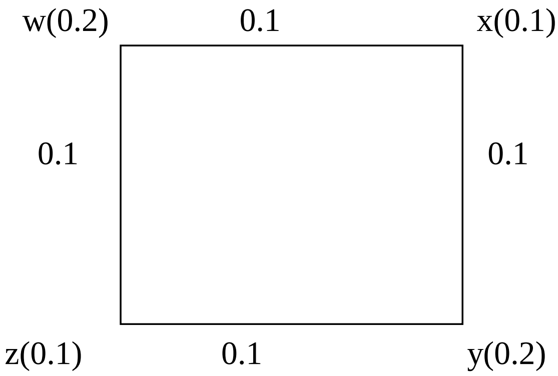

Fig.4. Strong domination constant number in complete fuzzy graph

In this complete fuzzy graph, strong arcs are (w,x), (w,z), (z,y) and (y,x). The strong dominating sets are $D_1=(w,x)$, $D_2=(w,y)$, $D_3=(w,z)$, $D_4=(z,y)$, $D_5=(z,x)$, $D_6=(y,x)$

Where $W(D_1)=0.1+0.1=0.2$, $W(D_2)=0.1+0.1=0.2$, $W(D_3)=0.1+0.1=0.2$,

$W(D_4)=0.1+0.1=0.2$, $W(D_5)=0.1+0.1=0.2$, $W(D_6)=0.1+0.1=0.2$

Hence $\gamma_{sc}(G)=0.2$

Remark 2.4

A complete fuzzy graph need not be a strong domination constant number of fuzzy graph.

Example 2.5

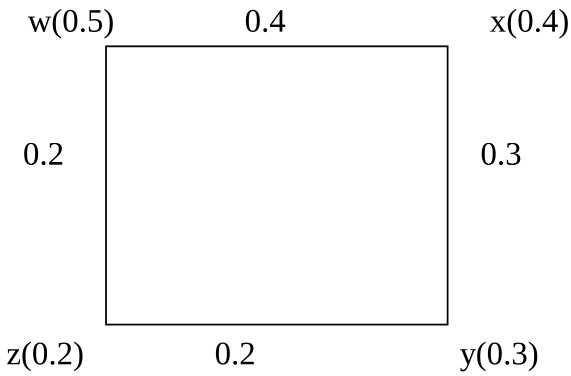

Fig.5. Complete fuzzy graph with out strong domination constant number

The graph G is a complete regular fuzzy graph.

In this complete fuzzy graph, strong arcs are (w,x), (w,z), (z,y) and (y,x). The strong dominating sets are $D_1=(w,x)$, $D_2=(w,y)$, $D_3=(w,z)$, $D_4=(z,y)$, $D_5=(z,x)$, $D_6=(y,x)$

Where $W(D_1)=0.2+0.3=0.5$, $W(D_2)=0.2+0.2=0.4$, $W(D_3)=0.2+0.2=0.4$,

$W(D_4)=0.2+0.2=0.4$, $W(D_5)=0.2+0.3=0.5$, $W(D_6)=0.2+0.3=0.5$

Here the weight of all strong dominating sets of G are not same.

Hence G is not a strong domination constant number of fuzzy graph .

Remark 2.5

A strong domination constant number of fuzzy graph need not be a complete fuzzy graph.

Example 2.6

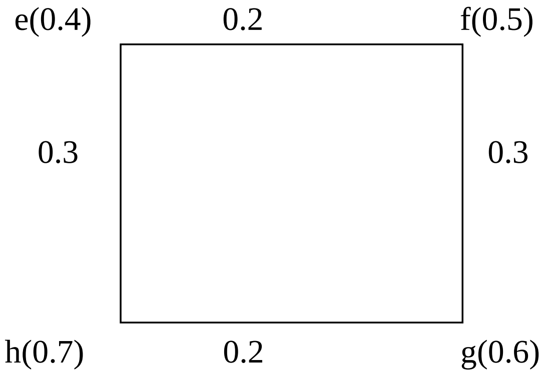

Fig.6. Strong domination constant number of fuzzy graph

In this fuzzy graph, strong arcs are (e, f), (e,h), (h, g) and (g, f). The strong dominating sets are $D_1 = (e, f)$, $D_2 = (e, g)$, $D_3 = (e,h)$, $D_4 = (h, g)$, $D_5 = (h, f)$, $D_6 = (g, f)$

Where $W(D_1) = 0.2 + 0.2 = 0.4$, $W(D_2) = 0.2 + 0.2 = 0.4$, $W(D_3) = 0.2 + 0.2 = 0.4$,

$W(D_4) = 0.2 + 0.2 = 0.4$, $W(D_5) = 0.2 + 0.2 = 0.4$, $W(D_6) = 0.2 + 0.2 = 0.4$

Hence $\gamma_{sc}(G) = 0.4$

The fuzzy graph G is strong domination constant number of fuzzy graph. But G is not a complete fuzzy graph.

3. Pseudo regular fuzzy graph with strong domination constant number

Theorem 3.1

If $G : (\sigma, \mu)$ be a pseudo regular fuzzy graph, then $\gamma_{sc}(G) = 2\{\wedge(\mu(u,v) / u, v \in \sigma^*)\}$

Proof:

Since G is a pseudo regular fuzzy graph, all arcs are strong and each node is adjacent to all other nodes. Hence , $D = \{u, v\}$ is a strong dominating sets for each $u, v \in \sigma^*$.

Hence $\gamma_{sc}(G) = 2\{\wedge(\mu(u,v) / u, v \in \sigma^*)\}$

Example 3.1

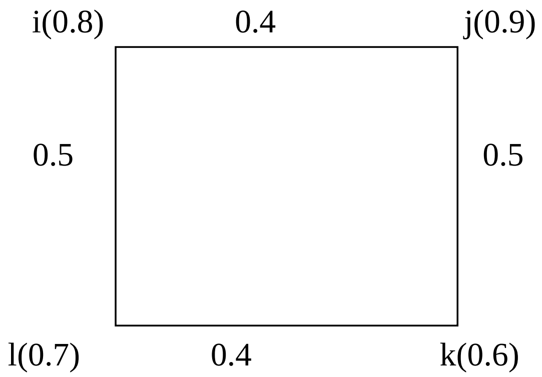

Fig.7. Pseudo regular fuzzy graph with strong domination constant number

$$\gamma_{sc}(G) = 0.8$$

Remark 3.1

The above condition is also true for totally pseudo regular fuzzy graph.

Example 3.2

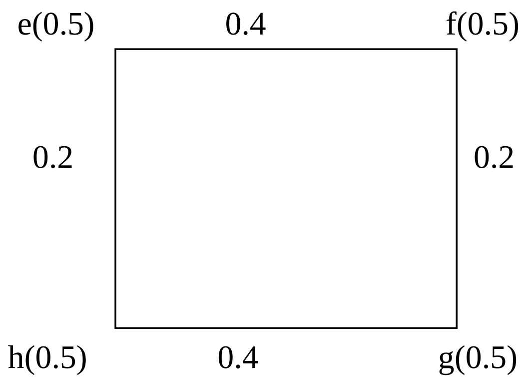

Fig.8. Totally Pseudo regular fuzzy graph with Strong Domination constant number

$$\gamma_{sc}(G) = 0.4$$

Theorem 3.2

Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph of size q. Then $\gamma_{sc}(G)=\frac{q}{2}$ if and only if all edges have same membership value.

Proof:

If all edges have same membership value, then the strong dominating set of G is a set D containing nodes. Hence, strong domination constant number is exactly $\gamma_{sc}(G)=\sum_{u\in D}\mu(u,v)=\frac{q}{2}$.

Conversely, suppose that $\gamma_{sc}(G)=\frac{q}{2}$. To prove that all edges have same membership value. If the alternative edges have same membership value then $\gamma_{sc}(G)\neq\frac{q}{2}$, which is a contradiction. Hence, all edges have same membership value.

Remark 3.2

The above condition is also true for totally pseudo regular fuzzy graph.

Example 3.3

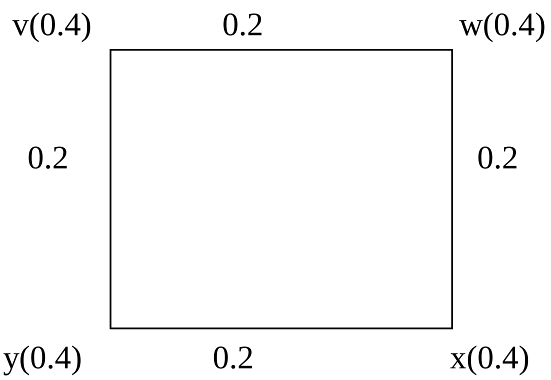

Fig.9. Totally Pseudo regular fuzzy graph with Strong Domination constant number

$\gamma_{sc}(G)=0.4$

Theorem 3.3

A pseudo regular fuzzy graph $G:(\sigma,\mu)$ with its crisp graph $G^*:(V,E)$ as even cycle is both pseudo regular and totally pseudo regular then G contains strong domination constant number.

Proof:

Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph. Then its crisp graph $G^*:(V,E)$ as even cycle and G be both pseudo regular and totally pseudo regular fuzzy graph. There are two cases arise.

Case (i)

Let G be both pseudo regular and totally pseudo regular fuzzy graph with constant values in σ and μ. In G all arcs are strong and each strong dominating sets of G having same weight. Then by the definition [2.19] G contains strong domination constant number.

Example 3.4

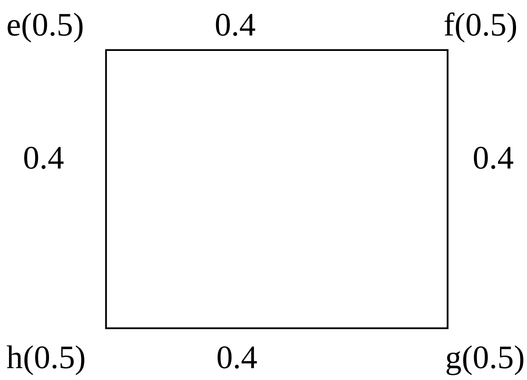

Fig.10 . Pseudo regular and totally pseudo regular fuzzy graph with strong domination constant number

$$\gamma_{sc}(G)=0.8$$

The graph G is pseudo regular and totally pseudo regular fuzzy graph with strong domination constant number.

Case (ii)

Let G be both pseudo regular and totally pseudo regular fuzzy graph with constant values in σ and with same alternative values in μ. In G all arcs are strong and each strong dominating sets of G having same weight. Then by the definitions [2.19] G contains strong domination constant number.

Example 3.5

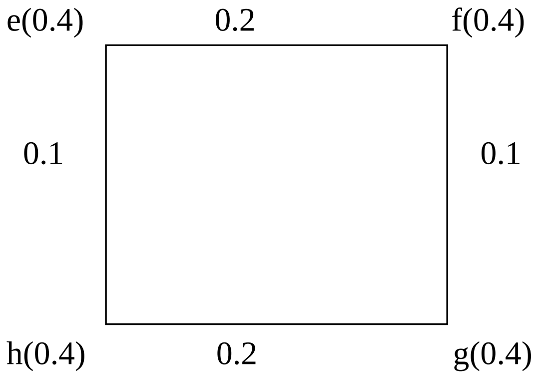

Fig.11 . Pseudo regular and totally pseudo regular fuzzy graph with strong domination constant number

$$\gamma_{sc}(G) = 0.2$$

The graph G is pseudo regular and totally pseudo regular fuzzy graph with strong domination constant number.

Theorem 3.4

Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph , $\gamma_{sc}(G) = \frac{p}{2}$ if and only if the following conditions hold

1) G is a totally pseudo regular fuzzy graph
2) All arcs are M-strong arcs.

Proof: If G is a totally pseudo regular fuzzy graph and all arcs are M-strong arcs, then the strong dominating set of G is a set D containing nodes. Hence , strong domination constant number is exactly $\gamma_{sc}(G) = \frac{p}{2}$.

Conversely, suppose that $\gamma_{sc}(G) = \frac{p}{2}$. To prove that G is a totally pseudo regular fuzzy graph and all arcs are M-strong arcs. If possible and some nodes say u and v have different weights, then the arc weight corresponding to these nodes is $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

If $\mu(u,v) < \sigma(u) \wedge \sigma(v)$, then obviously $\gamma_{sc}(G) < \frac{p}{2}$, a contradiction and if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$, then clearly $\gamma_{sc}(G) < \frac{p}{2}$, a contradiction. Hence, all the conditions are sufficient.

Example 3.6

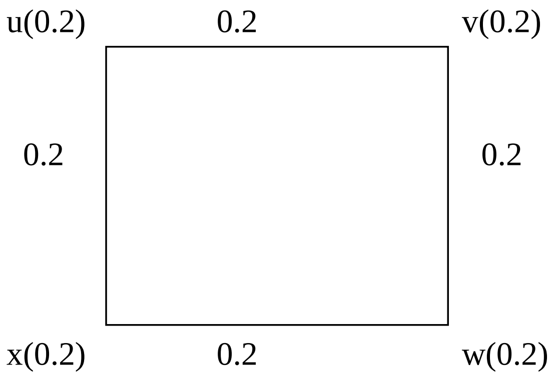

Fig.12. Pseudo regular and totally pseudo regular fuzzy graph with strong domination constant number $\gamma_{sc}(G) = 0.4$

The graph G is pseudo regular and totally pseudo regular fuzzy graph with strong domination constant number.

4. Complete fuzzy graph with strong domination constant number

Theorem 4.1

Let $G:(\sigma,\mu)$ be a complete fuzzy graph of size q. Then $\gamma_{sc}(G)=\frac{q}{2}$ if and only if alternative nodes have same weight.

Proof: If alternative nodes have same weight, then all the edges have same membership value and all arcs are strong, then the strong dominating set of G is a set D containing nodes. Hence, strong domination constant number is exactly $\gamma_{sc}(G)=\sum_{x\in D}\mu(x,y)=\frac{q}{2}$.

Conversely, suppose that $\gamma_{sc}(G)=\frac{q}{2}$. To prove that alternative nodes have same weight. If possible all nodes have different weight and all arcs are strong , then the arcs weight corresponding to nodes is $\mu(x,y)\le\sigma(x)\wedge\sigma(y)$.

If $\mu(x,y)<\sigma(x)\wedge\sigma(y)$, then obviously $\gamma_{sc}(G)=0$ or $\gamma_{sc}(G)\le\frac{q}{2}$, but G is not a complete fuzzy graph which is a contradiction and if $\mu(x,y)=\sigma(x)\wedge\sigma(y)$, then clearly $\gamma_{sc}(G)=0$, which is a contradiction. Hence, alternative nodes have same weight.

Example 4.1

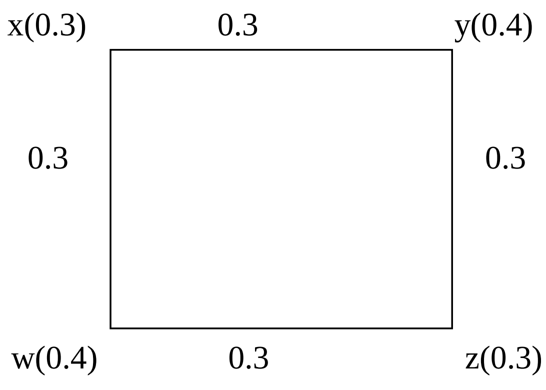

Fig.13 . Complete fuzzy graph with strong domination constant number

$\gamma_{sc}(G)=0.6$

Theorem 4.2 Let $G:(\sigma,\mu)$ be a complete fuzzy graph , $\gamma_{sc}(G) \geq \frac{p}{3}$ if and only if the following conditions hold

1) Alternative nodes have same weight.
2) All edges have same membership value.

Proof:

If alternative nodes have same weight and all edges have same membership value, then the strong dominating set of G is a set D containing nodes. Hence, strong domination constant number $\gamma_{sc}(G) \geq \frac{p}{3}$.

Conversely, suppose that $\gamma_{sc}(G) \geq \frac{p}{3}$. To prove that alternative nodes have same weight and all edges have same membership value. If all nodes have same weight and alternative edges have same membership value then $\gamma_{sc}(G) \geq \frac{p}{3}$ or $\gamma_{sc}(G) < \frac{p}{3}$, but G is not a complete fuzzy graph, which is a contradiction. Hence, all the conditions are sufficient.

Example 4.2

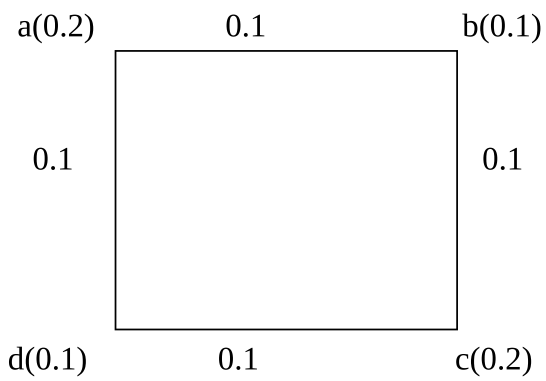

Fig.14. Complete fuzzy graph with Strong Domination constant number

$$\gamma_{sc}(G) = \frac{p}{3} = 0.2$$

Example 4.3

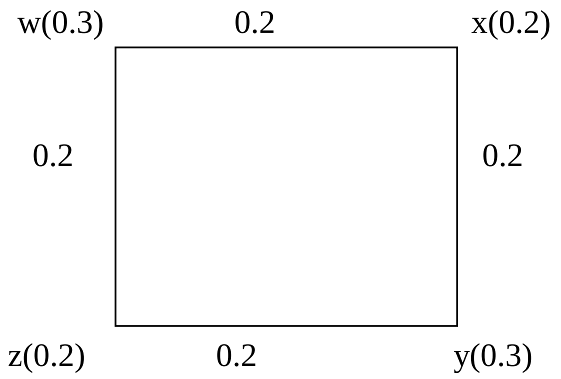

Fig.15. Complete fuzzy graph with Strong Domination constant number

$$\gamma_{sc}(G) > \frac{p}{3}$$

0.4 >0.33

Conclusion:

Research in the area of domination theory is interesting due to the diversity of applications and vast variety of domination parameters that can be defined. In this article, the perception of strong domination constant number have been introduced for pseudo regular fuzzy graph and complete fuzzy graph and some interesting results have been proved. Other domination parameters can be defined and investigated in the similar setting as future work.

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