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Heat and Mass transfer of a flow with variable Temperature and Mass Diffusion in the presence of transverse Magnetic field through Porous Mediam

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Abstract

This paper investigates heat and mass transfer effects of a flow in presence of magnetic field and mass diffusion through a porous space. The plate is exponentially accelerated with variable temperature and mass diffusion. Solutions of governing non-dimensional partial differential equations are obtained by using integral transform technique. Numerical results of governing equations are determined for fixed initial and boundary conditions. The velocity profiles are graphically discussed for different physical parameters and significant differences have been observed. Plots for the same have been produced in the manuscript.

Keywords *Variable Temperature, Porous Medium, Magneto Hydrodynamic Flow, Infinite, Vertical Plate.*

Introduction: Magneto-hydrodynamic has numerous practical uses in aerodynamics, metal industry, power generation and wave propagation etc. Heat and mass transfer plays important role in biological systems, biomedical industry, designing of space craft, supply of energy, information technology etc. Flow through porous media is a general phenomenon in nature and it has shown its importance in numerous engineering and industrial problems, such as, underground water resources, rain water harvesting, paper industry, membrane separation process, water purification processes, and flow of blood and in secondary and tertiary recovery of petroleum.

Soundalgekar [1] has reported an exact analysis to the Stokes and Reilaygh's problems related to flow past an impulsively started infinite isothermal vertical plate. It is observed that velocity decline on heating the plate and it rises on cooling the plate. Raptis et al. [2] studied the radiation effect on an electrically conducting fluid and to define the radiative heat flux. Rosseland approximation is used in the energy equation . Srinivas et al.[3] investigated the effect of electrically conducting fluid in presence of a uniform magnetic field. Bhatti et al. [11] analyzed the solution of two phase non-Newtonian Jeffrey fluid model with some application in bio-medical engineering.

Rafael [4] shows by numerical computation that the magnetic effect past a semi infinite impermeable stretched plate, the destructive chemical reaction vanishes the concentration boundary layer on electrically conducting second grade fluid in porous medium. Nayak et al. [10] obtained the closed form solution of second grade viscoelastic (Walter B) fluid for boundary layer equations. They obtained the concentration distribution and velocity enhanced by the interaction of the magnetic field. Jha [5] inspected free convection flow through porous space in presence of magnetic field force. the heat and mass transfer effect had been analyzedby Muthucumaraswamy et al. [6] when the plate was oscillating in its plane, the temperature varies with time. Saraswat et al. [9] when the platewas accelerated exponentially and the temperature varied with time. Rajesh [7] find the velocity field expression and skin fraction for electrically conducting fluid in presence of uniform magnetic field. Asogwa et al. [8] worked on a vertical plate which is exponentially accelerated with variable mass diffusion. they observed that as time, acceleration parameter and reaction parameter increases the temperature rises.

However, the study of heat transfer and temperature dependent mass diffusion for MHD flow has not been studied by any of the authors.

In the present study, an infinite vertical plate which is exponentially accelerated has been considered in presence of magnetic fields. Flow is taken along transverse direction. To begin with static state, the temperature $\underset{\sim}{T}$ of the fluid and temperature $\underset{\sim}{T}_w$ of the plate is invariable throughout the plate with concentration $\underset{\sim}{C}_\infty$. A crosswise transverse magnetic field $\underset{\sim}{B}$ is applied to the plate. At time $\tilde{\tau} > 0$, the plate accelerates exponentially in its own plane with a velocity $\underset{\sim}{u} = \underset{\sim}{u}_o e^{a\tilde{\tau}}$. The temperature at the plate rises exponentially with time while the concentration at the plate rises linearly. The magnetic lines of force are predetermined related to the plate. Then according to the Boussinesq's approximation, the equations governing the unsteady flow are given by

$$\frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{t}} = g\beta(\underset{\sim}{T} - \underset{\sim}{T}_\infty) + g\beta(\underset{\sim}{C} - \underset{\sim}{C}_\infty) + \nu \frac{\partial^2 \underset{\sim}{u}}{\partial \underset{\sim}{y}^2} - \nu\left(\frac{\underset{\sim}{u}}{\underset{\sim}{K}}\right) - \frac{\sigma B^2}{\rho}(\underset{\sim}{u} - \underset{\sim}{u}_0 e^{a\tilde{\tau}}) \quad (1)$$

$$\rho C_p \frac{\partial \underset{\sim}{T}}{\partial \underset{\sim}{t}} = \kappa \frac{\partial^2 \underset{\sim}{T}}{\partial \underset{\sim}{y}^2} - \frac{\partial q}{\partial \underset{\sim}{y}} \quad (2)$$

$$\frac{\partial \underset{\sim}{C}}{\partial \tilde{\tau}} = \underset{\sim}{D} \frac{\partial^2 \underset{\sim}{C}}{\partial \underset{\sim}{y}^2} - \underset{\sim}{Q}\underset{\sim}{C} \quad (3)$$

Conditions existing at the beginning and on the boundary are

$$\tilde{\tau} \le 0, \quad \underset{\sim}{u} = 0, \quad \underset{\sim}{T} = \underset{\sim}{T}_\infty \quad \underset{\sim}{C} = \underset{\sim}{C}_\infty \quad \text{for all } \underset{\sim}{y}$$

$$\tilde{\tau} > 0, \quad \underset{\sim}{u} = \underset{\sim}{u}_o e^{a\tilde{\tau}}, \quad \underset{\sim}{T} = \underset{\sim}{T}_\infty + (\underset{\sim}{T}_w - \underset{\sim}{T}_\infty)e^{a\tilde{\tau}}, \quad \underset{\sim}{C} = \underset{\sim}{C}_\infty + (\underset{\sim}{C}_w - \underset{\sim}{C}_\infty)\left(\frac{\underset{\sim}{u}_0^2}{\nu}\right)^{\frac{1}{3}} \tilde{\tau} \quad \text{at } \underset{\sim}{y} = 0$$

$$\underset{\sim}{u} = 0 \quad \underset{\sim}{T} \to \underset{\sim}{T}_\infty \quad \underset{\sim}{C} \to \underset{\sim}{C}_\infty \quad \text{as } \underset{\sim}{y} \to \infty \quad (4)$$

Using the dimensionless parameters,

$$\hat{u} = \left(\frac{\underset{\sim}{u}}{(\nu \underset{\sim}{u}_0)^{\frac{1}{3}}}\right), \quad \hat{\tau} = \tilde{\tau}\left(\frac{\underset{\sim}{u}_0^2}{\nu}\right)^{\frac{1}{3}}, \quad \hat{y} = \underset{\sim}{y}\left(\frac{\underset{\sim}{u}_0}{\nu}\right)^{\frac{1}{3}}, \quad \hat{\zeta} = \left(\frac{\underset{\sim}{T} - \underset{\sim}{T}_\infty}{\underset{\sim}{T}_w - \underset{\sim}{T}_\infty}\right), \quad \hat{C} = \left(\frac{\underset{\sim}{C} - \underset{\sim}{C}_\infty}{\underset{\sim}{C}_w - \underset{\sim}{C}_\infty}\right), \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B^2 \nu}{\rho \tilde{u}_0^2}, \quad Pr = \frac{\mu C_p}{\kappa},$$

$$Gr = \left(\frac{g\beta(\underset{\sim}{T}_w - \underset{\sim}{T}_\infty)}{\tilde{u}_0}\right), Gc = \left(\frac{g\beta(\underset{\sim}{C}_w - \underset{\sim}{C}_\infty)}{\tilde{u}_0}\right), \frac{1}{K} = \frac{\tilde{u}_0^2 \tilde{K}}{\nu^2}, R = \frac{4\sigma \underset{\sim}{T}_d^3}{\kappa a}. \quad (5)$$

equations (1-4) are transformed to

$$\frac{\partial \hat{u}}{\partial \tau} = Gr\hat{\zeta} + Gc\hat{C} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} - K\hat{u} - M(\hat{u} - e^{a\tau}) \quad (6)$$

$$\frac{\partial \hat{\zeta}}{\partial \tau} = \frac{1}{\alpha}\frac{\partial^2 \hat{\zeta}}{\partial \hat{y}^2} \quad (7)$$

$$\frac{\partial \hat{C}}{\partial \tau} = \frac{1}{Sc}\frac{\partial^2 \hat{C}}{\partial \hat{y}^2} - Q\hat{C} \quad (8)$$

$$\hat{u} = 0, \quad \hat{\zeta} = 0, \quad \hat{C} = 0, \quad \text{for all } \hat{y} \le 0, \quad \tau \le 0$$

$$\hat{u} = e^{a\tau}, \quad \hat{\zeta} = e^{a\tau}, \quad \hat{C} = \tau, \quad \text{at } \hat{y} = 0, \quad \tau > 0$$

$$\hat{u} = 0, \quad \hat{\zeta} \to 0, \quad \hat{C} \to 0 \quad \text{as } \hat{y} \to \infty \quad (9)$$

Solution:

Using integral transform techniques to equations (6-9), set of transformed equations are obtained as:

$$\frac{d^2\ddot{u}}{d\hat{y}^2} - (p + M^*)\ddot{u} = -Gr\ddot{\zeta} - Gc\ddot{C} - \frac{M}{p-a} \tag{10}$$

$$\frac{d^2\ddot{\zeta}}{d\hat{y}^2} - p\alpha\ddot{\zeta} = 0 \tag{11}$$

$$\frac{d^2\ddot{C}}{d\hat{y}^2} - (p + Q)Sc\ddot{C} = 0 \tag{12}$$

$$\ddot{u} = 0, \quad \ddot{\zeta} = 0, \quad \ddot{C} = 0 \qquad \text{for all } \hat{y}, \quad \tau \le 0$$

$$\ddot{u} = \frac{1}{p-a}, \quad \ddot{\zeta} = \frac{1}{p-a}, \quad \ddot{C} = \frac{1}{s^2} \qquad \text{at } \hat{y} = 0, \quad \tau > 0$$

$$\ddot{u} = 0, \quad \ddot{\zeta} \to 0, \quad \ddot{C} \to 0 \qquad \text{as } \hat{y} \to \infty, \quad \tau > 0 \tag{13}$$

which gives:

$$\ddot{u} = \left[\frac{1}{p-a} + \left\{\frac{Gr}{(p-a)\left\{p(\alpha-1)-M^*\right\}} + \frac{Gc}{p^2\left\{p(Sc-1)+(QSc-M^*)\right\}}\right\} - \frac{M}{(p-a)(p+M^*)}\right] e^{-y\sqrt{p+M^*}}$$
$$-\left\{\frac{Gre^{-y\sqrt{p\alpha}}}{(p-a)\left\{p(\alpha-1)-M^*\right\}} + \frac{Gce^{-y\sqrt{(p+Q)Sc}}}{p^2\left\{p(Sc-1)+(QSc-M^*)\right\}}\right\} + \frac{M}{(p-a)(p+M^*)} \tag{14}$$

$$\ddot{\zeta} = \frac{e^{-y\sqrt{p\alpha}}}{p-a} \tag{15}$$

$$\ddot{C} = \frac{e^{-y\sqrt{(Q+p)Sc}}}{p^2} \tag{16}$$

where, $\bar{u} = L[u(x,\tau),\tau]$, $\bar{c} = L[c(x,\tau),\tau]$, $\bar{\zeta} = L[\zeta(x,\tau),\tau]$ and s is the parameter of Laplace Transform.

Taking inverse Laplace transform of equations (14-16), expressions for velocity, temperature and concentration are obtained as:

$$\hat{u} = \left(1 - \frac{M}{a+M^*} + \frac{Gr}{(\alpha-1)(a-\varsigma)}\right)\frac{e^{a\tau}}{2}\left[e^{\hat{y}\sqrt{M^*+a}}\operatorname{erfc}\left\{\eta + \sqrt{(M^*+a)\tau}\right\} + e^{-\hat{y}\sqrt{M^*+a}}\operatorname{erfc}\left\{\eta - \sqrt{(M^*+a)\tau}\right\}\right]$$
$$+\left(\frac{Gre^{\varsigma\tau}}{2(\alpha-1)(a-\varsigma)}\right)\left[e^{\hat{y}\sqrt{\varsigma+M^*}}\operatorname{erfc}\left\{\eta + \sqrt{(\varsigma+M^*)\tau}\right\} + e^{-\hat{y}\sqrt{\varsigma+M^*}}\operatorname{erfc}\left\{\eta - \sqrt{(\varsigma+M^*)\tau}\right\}\right]$$
$$+\frac{Gc}{Sc-1}\left[\left(\frac{-1}{2q^2} - \frac{\tau}{2q} - \frac{\hat{y}}{4q\sqrt{M^*}}\right)e^{\hat{y}\sqrt{M^*}}\operatorname{erfc}(\eta + \sqrt{M^*\tau}) + \left(\frac{-1}{2q^2} - \frac{\tau}{2q} + \frac{y}{4q\sqrt{M^*}}\right)e^{-\hat{y}\sqrt{M^*}}\operatorname{erfc}(\eta - \sqrt{M^*\tau})\right]$$
$$+\frac{Gc}{Sc-1}\left(\frac{e^{q\tau}}{2q^2}\right)\left[e^{\hat{y}\sqrt{M^*+q}}\operatorname{erfc}(\eta + \sqrt{(M^*+q)\tau}) + e^{-\hat{y}\sqrt{M^*+q}}\operatorname{erfc}(\eta - \sqrt{(M^*+q)\tau})\right]$$
$$+\left(\frac{Gre^{\varsigma\tau}}{2(\alpha-1)(a-\varsigma)}\right)\left[e^{\hat{y}\sqrt{\alpha\varsigma}}\operatorname{erfc}\left\{\eta\sqrt{\alpha} + \sqrt{\varsigma\tau}\right\} + e^{-\hat{y}\sqrt{\alpha\varsigma}}\operatorname{erfc}\left\{\eta\sqrt{\alpha} - \sqrt{\varsigma\tau}\right\}\right]$$

$$-\left(\frac{Gr\,e^{a\tau}}{2(\alpha-1)(a-\varsigma)}\right)\left[e^{\hat{y}\sqrt{\alpha a}}\,\mathrm{erfc}\left\{\eta\sqrt{\alpha}+\sqrt{a\tau}\right\}+e^{-\hat{y}\sqrt{\alpha a}}\,\mathrm{erfc}\left\{\eta\sqrt{\alpha}-\sqrt{a\tau}\right\}\right]$$

$$+\frac{Gc}{Sc-1}\left[\left(\frac{1}{2q^2}+\frac{\tau}{2q}+\frac{\hat{y}\sqrt{Sc}}{4q\sqrt{Q}}\right)e^{y\sqrt{ScQ}}\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{Q\tau})+\left(\frac{1}{2q^2}+\frac{\tau}{2q}-\frac{\hat{y}\sqrt{Sc}}{4q\sqrt{Q}}\right)e^{-\hat{y}\sqrt{ScQ}}\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{Q\tau})\right]$$

$$-\frac{Gc}{Sc-1}\left(\frac{e^{q\tau}}{2q^2}\right)\left[e^{\hat{y}\sqrt{(Q+q)Sc}}\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{(Q+q)\tau})+e^{-\hat{y}\sqrt{(Q+q)Sc}}\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{(Q+d)\tau})\right]$$

$$+\frac{M}{a+M^*}\left[e^{at}+e^{-M^*\tau}\left\{\mathrm{erfc}(\eta)-1\right\}\right] \tag{17}$$

$$\hat{\zeta}=\frac{e^{a\tau}}{2}\left[e^{\hat{y}\sqrt{\alpha a}}\,\mathrm{erfc}\left\{\eta\sqrt{\alpha}+\sqrt{a\tau}\right\}+e^{-\hat{y}\sqrt{\alpha a}}\,\mathrm{erfc}\left\{\eta\sqrt{\alpha}-\sqrt{a\tau}\right\}\right] \tag{18}$$

$$\hat{C}=\left[\left(\frac{\tau}{2}+\frac{\hat{y}\sqrt{Sc}}{4\sqrt{Q}}\right)e^{\hat{y}\sqrt{ScQ}}\,\mathrm{erfc}(\eta\sqrt{Sc}+\sqrt{Q\tau})+\left(\frac{\tau}{2}-\frac{\hat{y}\sqrt{Sc}}{4\sqrt{Q}}\right)e^{-\hat{y}\sqrt{ScQ}}\,\mathrm{erfc}(\eta\sqrt{Sc}-\sqrt{Q\tau})\right] \tag{19}$$

Where $\varsigma=\frac{M^*}{\alpha-1}, q=\frac{M^*-Q\,Sc}{Sc-1}, \alpha=\frac{3Pr}{3+4R}, \eta=\frac{\hat{y}}{2\sqrt{\tau}}$

Results and Discussion

Velocity profile has been studied for different physical parameters, namely, acceleration parameter, parameter of radiation , parameter of chemical reaction , Grashof number , Prandtl number , Schmidt number , time , parameter of permeability and magnetic field parameter .

In figure 1,the variation of the dimensionless velocity respect to the distance of the plate for different values of acceleration parameter a(2,4,6,8) and Pr=0.71, Gr=-2, K=0.5, R=2, Gc=-5, τ =0.2, M=1, Q=2, Sc=0.2. It is observed that, with the increase of accelerating parameter a, the velocity raised. It decreases as the distance increases.

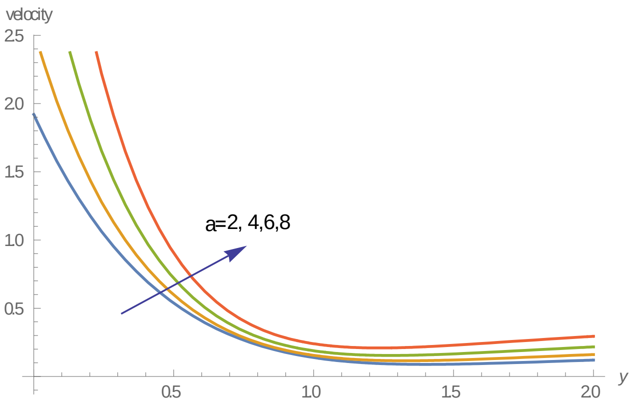

Figure - 1 : Dimensionless velocity for different values of acceleration 'a'.

Figure 2 displays the effect for different values of magnetic parameter M(=1,2,3,4) and K=0.5,Sc=0.2, a=2,Q=2, Gr=-2, R=2, Pr=0.71, Gc=-5, τ =0.2. Graph shows that the velocity is of the highest degree near the plate

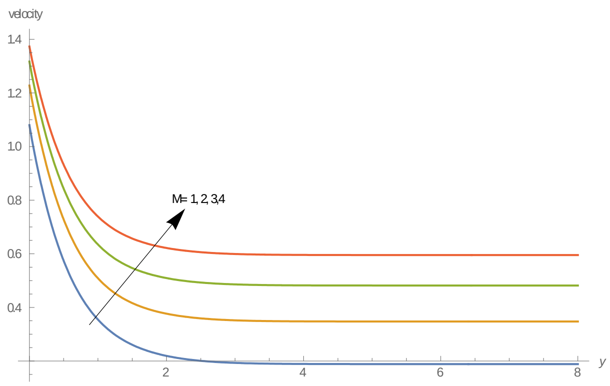

Figure - 2 : Dimensionless velocity for different values of magnetic parameter 'M'.

Figure -3 illustrates the variation in velocity with respect to time τ (= 0.2,0.4,0.6 ,0.8) and K=0.5, Sc=0.2,Q=2, Gr=-2, M=1, R=2, Pr=0.71, a=2, Gc=-5,. We observed that the variations in the velocity are not significant near the plate but as time increases velocity increases away from the plate.

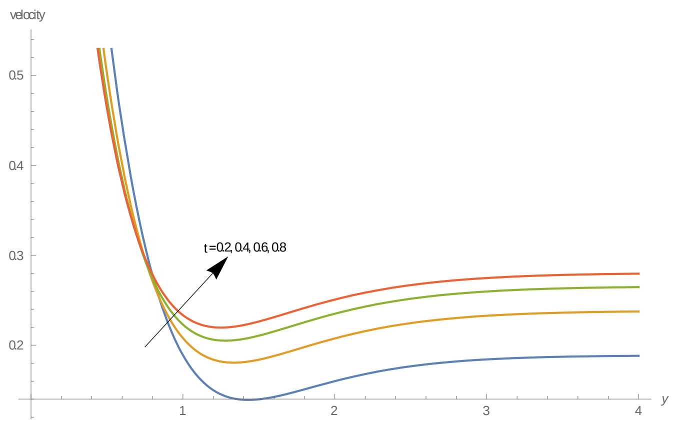

Figure -3 Dimensionless velocity at different time 'τ'

Figure 4 represents the change in velocity related to the permeability parameter K (= 0.25, 0.5, 0.75, 1) and M=1, τ =0.2, Sc=0.2, Gr=-2, R=2, a=2, Gc=-5, Q=2. It is noticed that velocity profile is higher near the plate as compared to the distance far away while permeability increases.

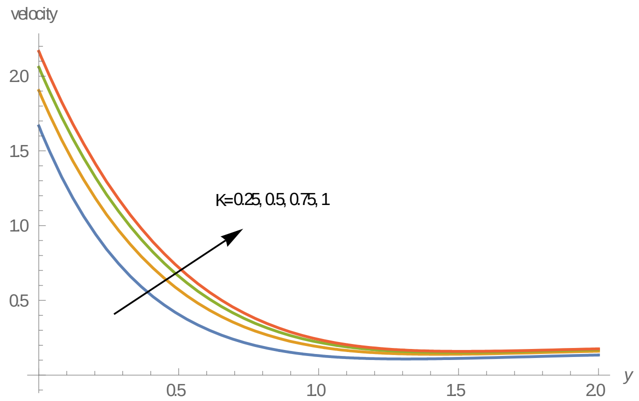

Figure-4 : Dimensionless velocity for different permeability parameter 'K'

Figure 5 displays the effect of radiation parameter R(= 2, 5, 10, 15) and M=1, K=0.5, τ =0.2, Gc=-5, Q=2, Gr=-2, a=2, Sc=0.2. It is noticed that the velocity profile shows a reverse effect with respect to radiation parameter as the velocity is maximum for small distances but it decreases as distance increases and radiation parameter increases.

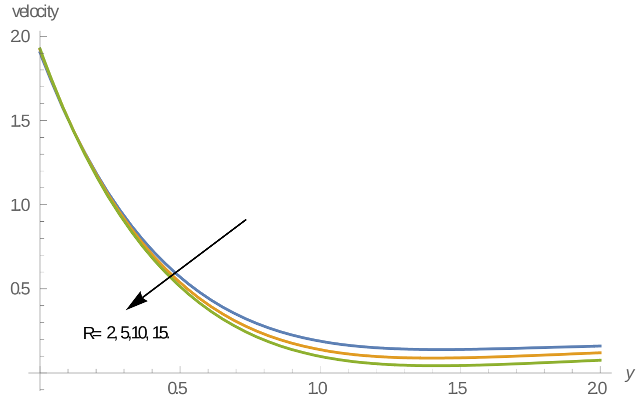

Figure -5: Dimensionless velocity with respect to y for different permeability parameter R

Figure 6 represents change in velocity due to mass Grashof number Gc(-20, -15, -10, -5) and a=2, Sc=0.2, Q=2, K=0.5, R=2, Gr=-2, τ =0.2, M=1, Pr=0.71. It is observed that as Grashof number (mass) increases while heating the plate the velocity increases. It is maximum at the plate and decreases as the distance increases.

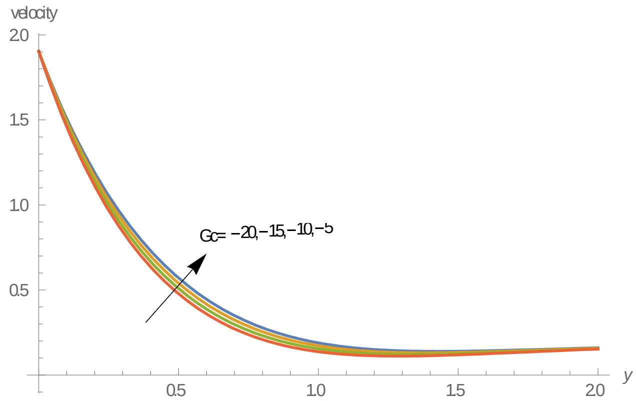

Figure-6 : Dimensionless velocity for different permeability parameter R

Conclusion:

A theoretical examination has been presented. The solutions for the present model has been obtained through integral transform

Some of the conclusion is given below:

1-The velocity increases as the acceleration parameter increases.

2. The velocity decreases as the radiation constant increases.

3. The velocity shows the reverse effect for smaller and larger values of y.

4. The velocity increases as the permeability and magnetic parameter increases.

The velocity decreases as radiation constant (R) increases. The effect of chemical reaction parameter (Q), Schmidt number (Sc) on velocity is negligible.

APPENDIX:

Nomenclature

Symbols

κ- Fluid Thermal conductivity, R- Parameter of Radiation, μ- Viscosity coefficient, $\tilde{C}$ - Fluid concentration, Gr -Grashof number(Thermal), ρ - Fluid density, Q-Parameter for chemical Reaction ,

M – Magnetic field parameter, ζ- non-dimensional temperature,Sc- Schmidt number, $\tilde{x}$, $\tilde{y}$ - Transverse and longitudinal coordinate, σ-Electric conductivity, τ- non – dimensional time, $\tilde{u}$ - Plate velocity, C- non-dimensional concentration, erf - Error function, y- non-dimensional perpendicular distance to the plate in horizontal direction, u- non-dimensional fluid velocity, Gm- Grashof number(Mass), Pr- Prandtl number, υ-Kinematic viscosity, Cp- Specific heat at constant pressure, $\tilde{T}$ - Fluid Temperature, α-Thermal diffusivity, g – Gravitational Acceleration, K- permeability parameter, t- non-dimensional time, q- Radiative heat flux, erfc- Complementary error function,. D-Chemical Molecular diffusivity.

υ-Kinematic viscosity.

Subscripts

w- wall, ∞- Free stream, 0- initial.

REFERENCES

[1] V.M. Soundalgekar., Free convection effects on the stokes problem for an infinite vertical plate, ASME Journal of Heat Transfer, vol.(99, pp. 499-501, 1977.

[2] A.A. Raptis, V.M. Soundalgekar , MHD flow past a steadily moving infinite vertical porous plate with constant heat flux, Nuclear Engineering and Design, vol.72, no.(3), pp. 373-379, 1982.

[3] S. Srinivas, M. Kothandapni, The influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls, Applied Mathematics and computation, vol. 213, no. 1, pp. 197-208, 2009.

[4] R. Cortell,MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species, Chemical Engineering Processing, vol.46,no. 8, pp.721-728, 2007.

[5] B.K. Jha, MHD free-convection and mass transform flow through a porous medium. Astrophysics and Space science. vol.175, pp. 283-289, 1991.

[6] R. Muthucumaraswamy,A. Vijaylakshmi, Effect of heat and mass transfer on flow past an oscillating vertical plate with variable temperature ,Int. J. of Appl. Math. and Mech. vol.1, pp.59-65, 2008.

[7] V. Rajesh, MHD effect of free convection and mass transform flow through a porous medium with variable temperature Int. J. of Appl. Math and Mech. vol.14, pp. 1-16, 2010.

[8] K.K Asogwa, et al., Flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion. International Journal of Computer Applications, vol. 2, Pp. 0975 – 0987, 2012

[9] A.Saraswat, R.K. Srivastava, heat and mass transfer past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion through a porous medium International Journal of Engineering, Business and Enterprise Applications, vol. 4,no.1, pp. 79-87, 2013.

[10]Nayak M.K, Dash G.C, Singh L.P, Heat and mass transfer effects on MHD viscoelastic fluid over a stretching sheet through porous medium in presence of chemical reaction, Propulsion and Power Research, vol. 5, no.1, pp.70-80,2016.

[11] M.M. Bhatti, Heat and mass transfer of two phase- flow with electric double layer effects induced due to peristaltic propulsion in the presence of transverse magnetic field, Journal of Molecular liquids. vol. 230, pp.237-24, 2017.