RELIABILITY OF A REPAIRABLE SYSTEM UNDER TWO MODES OF SERVICE

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SUMMARY
We study a repairable k-out-of- n reliability system in which two-repairman maintaining n identical components. We call the repairperson as server1 and server2. Server1 is available always and attends repair one at a time. Server2 will be brought to the system and activated on the accumulation of N failed components. It is assumed that the time between consecutive failures of components follow exponential distribution with parameter $\lambda$. Server1 repairs failed units with exponentially distributed service times at rate $\mu_1$ whereas server2 provides service, according to exponentially distributed service times at rate $\mu_2$ (again with single repair at a time). When all units are back to working state, server2 is switched off and again activated when the number of failed units accumulates to $N$. After repair, the components are considered as good as new. Probability distribution of the system size and server state in finite time and in the steady state is obtained by analyzing the underlying Markov process. In addition, we obtained several other characteristics of the model such as the average number of working components, the availability of system, average time durations server2 is continuously busy and idle, average down time of the system in a cycle etc. Optimal $N$ value is investigated under a suitably constructed profit function which is shown to be concave in $N$. Various numerical computations are also given.

Key words: Reliability, N-policy, two modes of service, Markov process, optimization

1. INTRODUCTION
We study a repairable k-out-of- n reliability system in which n identical components manned by two repairmen. Here the system is in a working mode only if at least k components of n must be working. In literature, these kinds of models have analyzed in the framework of calculating the system reliability, cost minimization problems, fixing repair facility for failed components etc. Such models provide a lot of application in industry as well as in real life situations. For details, see Akhtar (1994), Barlow, and Proschan (1965, 1975), Kapur and Lamberson (1997), Kullstam (1981), Nakagawa (1985) and Robinson and Neuts (1989).

In this model, at the time of starting of the system, we have assumed that all units are in working condition. As the system is in use, the components may deteriorate and fail one by one and when the number of failed units reaches a level $N - k + 1$, the system fails. If it is possible to allocate some service mechanism for the service of failed units, duration of functioning the system may extend to a much longer time by reducing the number of failed components and hence the system life cycle be enlarged. In addition, there are several occasions where more than one failed unit can be serviced simultaneously and it is possible only when there exist more than one service mechanism, which in turn give us a chance of extending the system operating time.

Here, we have considered the repairable system with two servers (or technicians) called Server1 and Server2 for fixing the failed components of the system. Failed components form a queue for repair. Server 1 is always available in the system and repair failed units one at a time whenever a failure occurs. Server2 will start for servicing of failed components only if the number of failed components of the queue reaches a level $N (1 < N \leq n - k)$ and starts repair of failed units one at a time. Server2 remain in the working mode until all the failed units become operational, at which epoch it is switched off to be hired again when the number of units in the waiting line accumulate to $N$. Serviced components assumed as good as new. Then a natural question is when should we activate server2? Hence our main goal is to find an optimal $N$ value to start the server2, which maximizes the net profit of the system, which in turn maximizes the system reliability.

Optimal number of repairs have analyzed by several researchers when analyzing systems subject to shocks and this can regard as optimal N-policy for replacement (see Lam Yeh (1991), Shen and Griffith (1996)). In addition, Stadje and Zuckerman studied the optimal strategies for replacement in (1990). The same model under N-policy for service
at a single service facility have studied by Krishnamoorthy et al. (2002). Also Ushakumari and Krishnamoorthy (1998) have generalized the model with arbitrary distribution for the service time. Krishnamoorthy and Ushakumari (2001) studied the model under D-policy for service. In the context of reliability analysis, the repair policies such as T-policy, (N,T) policy and max(N,T) policy have been investigated respectively by Rekha (2000), Chakravarthy et al. (2001) and Ushakumari and Krishnamoorthy (2004). In all those models, the T-policy refers to the idle time of server before starting repair, which is a random variable. In max(N,T) policy, the repairman starts repair when either failure of N components occur or the random variable T, which is the idle time of server realizes, whichever occurs first and in max(N,T) policy, the repairman starts repair at the epoch when the maximum of N or T happens and D-policy refers to the amount of workload (which is a random variable) accumulated to start service and has the maximum value D. In all the above frameworks, there was a single repair facility for repair of failed units and preventive maintenance policy adopted based on the functioning components of the system. Recently, Shekhar et al. (2017) studied an optimal (N,F) policy for machine redundant system.

This work is arranged as follows: Section 2 provides the model and its analysis and provides the joint distribution of the system size and server state in transient time and in the steady state. Section 3 is devoted to some measures of performances and in section 4, a cost analysis is carried out by a suitably defined net profit function and optimal N value is investigated. Some numerical investigations are carried out in Section 5.

2. Mathematical Model and Analysis

Consider a repairable reliability model in which two repairman maintaining n identical components. Here if at least k components of n must be working for the system to be operational. Life times of units (when the system is operating) have independent exponential distribution with parameter \( \lambda \), when t units are functioning. System is managed by two servers called server1 and server2. Server1 is always in the working mode and repairs failed units one at a time. Hire Server2 to the system for the service of units when the number of components accumulates to N for service, start service, and switched off when there is none in the queue of failed units. Again, server2 should brought back to the system when the number of units in the queue reaches N. This process continues. Repair times of server1 and server2 is according to two independent exponential distributions with rates \( \mu_1 \) and \( \mu_2 \) respectively. The serviced components are assumed as good as new and the repair and failure times of units are independent of each other.

Let \( N(t) \) and \( S(t) \) be the number of working components and the state of server2 at time \( t \). Then stochastic process \( Y(t) = \{(N(t),S(t)); t \geq 0\} \) forms a Markov process over a discrete state space \( D = \{(i,j); i \geq 0, j = 0, 1, \ldots, n\} \). All units are functional initially by assumption. Let \( P_i(j) \) be the system state probability.

\[
P_i(j) = \begin{cases} \frac{\mu}{\mu_2} & \text{if } j = k - 1 \\ \frac{\mu_2}{\mu} & \text{if } j = k \\ 0 & \text{otherwise} \end{cases}
\]

for \( i = k, k + 1, \ldots, n - 1 \).

\[
q_{i,j} = \begin{cases} \mu_i & \text{if } j = i - 1 \\ -\mu_i & \text{if } j = i + 1 \\ \mu_j & \text{if } j = i \\ 0 & \text{otherwise} \end{cases}
\]

for \( i, j = 0, 1, \ldots, n \).

\[
q_{i,j} = \begin{cases} \mu_{i,j} & \text{if } j = i - 1 \\ -\mu_{i,j} & \text{if } j = i + 1 \\ 0 & \text{otherwise} \end{cases}
\]

and for \( t = n - N + 1, n - N + 2, \ldots, n - 1 \).

\[
q_{i,j} = \begin{cases} \mu_i & \text{if } j = i - 1 \\ -\mu_i & \text{if } j = i + 1 \\ 0 & \text{otherwise} \end{cases}
\]
Then the system state distribution in transient time can be obtained as \( P(\mathbf{q}) = P(0)e^{\mathbf{q}\mathbf{\theta}} \), where \( P(0) = (0, 0, \ldots, 0, 1) \) is the initial probability vector, which by our assumption, 1 at the position corresponding to \( \mathbf{q}(n) \).

### 2.2 Steady State Probability Distribution

For the easiness of computation, let us henceforth that \( \lambda_i = \frac{1}{i} \) for \( i = k, k + 1, \ldots, n \). Also it is quite natural that the failure rate increases with decreasing number of functioning units. Here the Markov chain is finite and irreducible, the limit distribution exists. By setting \( \mathbb{P}_{i,j} = \lim_{t \to \infty} P_{ij}(t) = 0, \) the steady state probability vector \( \pi = (\pi_k, \pi_{k+1}, \ldots, \pi_n) \in \mathbb{D} \) can be obtained by solving the flow balance equations \( \pi = 0 \) together with the normalizing condition \( \sum \pi_i = 1 \). Then we have the steady state flow balance equations as:

\[
-\mu\pi_{k-1} + \lambda\pi_k = 0 
\]

(2.1)

\[
-\mu\pi_k + (\lambda + \mu)\pi_{k+1} + \lambda\pi_{k+1} = 0, \quad k \leq i \leq n - 1 
\]

with \( \pi = \pi_{n-N} + \lambda\pi_{n-N+1} + \lambda\pi_{n-N+2} = 0 \) (2.2)

\[
\mu\pi_{n-1} - (\lambda + \mu)\pi_{n-1} = 0 \quad (\lambda + \mu)\pi_n = \lambda\pi_{n-1} + \lambda\pi_{n-2} = 0 \quad (2.3)
\]

\[
\mu\pi_{n-2} - \lambda\pi_{n-2} - \mu\pi_{n-3} = 0, \quad n = N + 2 \leq i \leq n - 1 
\]

(2.4)

(2.5)

Solving the system of equations (2.1) to (2.6) recursively, we get

\[
\pi_{k+1} = \frac{\mu}{\lambda}^{k+1}, \quad k \leq i \leq n - N - 1 
\]

\[
\pi_{k+2} = \frac{1 - (\lambda/\mu)^{k+1}}{1 - (\lambda/\mu)^{n-N}} \cdot \pi_{k+1}, \quad n - N \leq i \leq n - 2
\]

\[
\pi_{n-k+1} = \frac{\mu}{\lambda}^{n-k+1}, \quad \pi_{n-k+2} = \theta_n \pi_{n-k+1}
\]

\[
\pi_{n-k+3} = \theta_n^{k+1} \pi_{n-k+2}
\]

(2.6)

where \( \theta_n = \frac{1}{\lambda} \left( \frac{\mu}{\lambda} \right)^{n-k+2} \).

Now, \( \pi_{k-1} \) can be computed using the fact that

\[
\sum_{k=1}^{n-1} \pi_{k+1} \pi_{k} + \sum_{k=1}^{n} \pi_{k+1} = 1, \quad \text{which gives us} \quad \pi_{k+1} = (\pi_k + \pi_{k+1})^{-1}, \quad \text{where} \quad \pi_1 = \frac{\mu}{\lambda}, \quad \pi_2 = \frac{\mu}{\lambda}^2
\]

(2.7)

(2.8)

(2.9)

(2.10)

Remark: In this system, when \( \mu_1 = 0 \) and \( \mu_2 = \mu \), the model reduces to Krishnamoorthy et al. (2002).

### 3. System Performance Measures

(i) Expected number of working components when server2 is idle is \( \mu_{\text{E}(W_1)} = \frac{1}{\mu(1-\rho)} \left[ \rho_{\text{E}(W_1)} N \rho_{\text{E}(W_1)} - N - 1 \right] \)

(3.1)

and average number of nonworking components when server2 is idle is \( \mu_{\text{E}(W_2)} = \frac{1}{\mu(1-\rho)} \rho_{\text{E}(W_2)} \rho_{\text{E}(W_2)} \)

(3.2)

(ii) Average number of functioning components when both servers are busy is \( \mu_{\text{E}(F_2)} = \mu_{\text{E}(F_2)} \left[ 1 - (\theta(1 + \theta) + 1) \delta^2 \right] + \left( \frac{\mu}{\lambda} - \delta \right) \mu_{\text{E}(F_2)} \left( 1 - (\theta(1 + \theta) + 1) \delta^2 \right) \}

(3.3)

(3.4)

(3.5)

where \( \theta = \frac{1}{\mu} \).

(iii) Fraction of time system is down in the steady

state is \( \mu_{\text{E}(W_1)} \) and the average time the system is down in the steady state is \( \mu_{\text{E}(W_1)} \).

(iv) System availability at any epoch in the steady state is \( 1 - \mu_{\text{E}(W_1)} \).
3.1 Distribution of Busy period of server1 and server2 simultaneously

To derive the distribution of time both servers are busy simultaneously, consider the set of states \( \{ (i, \emptyset) | 1 \leq i \leq n \} \cup \{ (n, D) \} \) of the Markov chain with \( (n, D) \) as an absorbing state. The transition rate matrix of this chain is given by \( \begin{pmatrix} \lambda & 0 \\ 0 & \mu_1 + \mu_2 \end{pmatrix} \). Here the matrix can be obtained by excluding the final row and final column of the Q-matrix corresponding to state \( (n, D) \). \( e_n \) is a column vector with last entry as 1 and \( e_0 \) is a vector of zeros. Now distribution of time until absorption is of phase type distribution \( F(x) = 1 - e^{-\lambda x} \frac{e_0^T e_n}{e_n^T e_n} \). where the row vector of initial probabilities is \( e_0 \) with 1 at \( n-k+1 \) position and zero at the remaining positions and \( e_n \) is vector of 1’s. For details see Neuts (1981).

3.2 Expected duration of time both servers are busy

The expected duration of time both servers are busy is the average time to reach the system with all units being functional once server2 started the repair and is obtained as \( E_2 = \frac{N}{\mu_1 + \mu_2} - \frac{1}{\mu_1} \). where \( \mu_1 = \mu_1 + \mu_2 \) see Krishnamoorthy et.al (2002) for reference.

3.3 Average to reach the failed state once server2 activated

Average to reach the system failed state once the server2 activated can be computed by taking the set of states \( \{ (i, D) | 0 \leq i \leq N-1 \} \cup \{ (n, D) \} \) of the Markov chain and it is the time to reach the failed state once server2 activated. This can be obtained as \( E_2 = \frac{N}{\mu_1 + \mu_2} - \frac{1}{\mu_1} \) as \( E_2 = \sum_{n=1}^{\infty} \frac{1}{\mu_1} \). Thus the expected time to reach \( \frac{n}{\mu_1} \) without visiting \( \frac{n}{\mu_1} \) is \( E_2 = \frac{N}{\mu_1 + \mu_2} - \frac{1}{\mu_1} \).

3.4 Expected idle period of server2

Expected idle period of server2 is the expected duration of time to start server2 from the starting of the system. This can be obtained by considering the set of states \( \{ (i, D) | 0 \leq i \leq N-1 \} \cup \{ (n, N, 1) \} \) with the state \( (n, N, 1) \) as absorbing. To compute this time, define \( T_{n-i} \) as the time to reach state \( n-i \) form \( n - i - 1 \). Then we get \( E(T_{n-i}) = \frac{1}{\mu_1 + \mu_2} \). The average time to reach server2 in a busy state is given by \( E(T_{n-i}) = \frac{1}{\mu_1 + \mu_2} E(T_{n-i+1}) \). Therefore the average time to reach server2 in a busy state is given by \( E(T_{n-i}) = \sum_{i=0}^{N} \frac{1}{\mu_1 + \mu_2} E(T_{n-i+1}) = \frac{E_2}{\mu_1 + \mu_2} \). Here \( E(T_{n-i}) \) is the expected busy period of server1 alone also.

3.5 Mean time to reach system failure (MTTF) state

MTTF is the time to reach the failed state once the system is started at \( (n, D) \). Therefore the expected time reach \( (k-1,1) \) from \( (n, D) \) can be obtained as the sum of (3.3.1) and (3.4.1) and is given by

\[
MTTF = \frac{N-k}{\mu_1 + \mu_2} \frac{1}{\mu_1} + \frac{1}{\mu_1} (1 - k) - \left( \frac{1}{\mu_1} \right)^{k+1} \left( \frac{N-k}{\mu_1 + \mu_2} \right)
\]

3.6 Expected length of a cycle

In this system, the length of a cycle is defined as the length of time to reach \( (n, D) \) starting from \( (n, D) \). Here it can happen in two ways: (i) without activating server2 or (ii) activating server2 in between. In case (i), the expected duration of such a cycle can be computed as follows. Define \( T_{n-i} \) as the time to visit states \( n-i \) from \( n-N-1 \) and \( T_{n-i} \) as the time to visit states \( n-i \) from state \( n-1 \). Then we can write \( E(T_{n-i}) = \frac{1}{\mu_1} + \frac{1}{\mu_1 + \mu_2} E(T_{n-i+1}) \) and \( E(T_{n-i}) = \frac{1}{\mu_1} + \frac{1}{\mu_1 + \mu_2} E(T_{n-i+1}) \) for i=0, 1...N-2. Thus the expected duration to reach \( (n, D) \) without visiting \( (n, N, 1) \) is \( E(C_i) = \sum_{i=0}^{N-1} [E(T_{n-i}) + E(T_{n-i})] \). Now we can compute recursively, \( E(T_{n-i}) = \frac{1}{\mu_1} \frac{\mu_1 + \mu_2}{\mu_1} \). and \( E(T_{n-i}) = \frac{1}{\mu_1} \frac{\mu_1 + \mu_2}{\mu_1} \). Therefore (3.6.1) gives \( E(C_i) = \frac{1}{\mu_1} \frac{\mu_1 + \mu_2}{\mu_1} \). In case (ii), a busy cycle is the average duration to reach the state where server2 is activated starting...
from a fully functional system together with the duration to return to \((R, U)\) from state \((R, N, 1)\). Therefore the expected duration of a cycle when server 2 is activated is given by 
\[
E(C_2) = E(R) + E(T_{R-N})
\]
which gives 
\[
E(C_2) = \frac{N}{\mu - \lambda} - \frac{1}{\mu - \lambda} + \frac{N - 1}{1 - \frac{\lambda}{\mu}}
\]

3.6.1 The average duration the system is idle in a cycle is given by 
\[
\frac{1}{\mu - \lambda}
\]
where \(\mu = \mu_1 + \mu_2\)

4. Cost Analysis
In this section, we try to find the optimal value of \(N\) by maximizing the net profit per cycle which simultaneously maximizing the system reliability. The following costs are considered. (a) \(K\)-the profit per unit time due to non-activation of server 2 and (b) \(C\)- the loss per unit time due to the system failure.

Therefore, the profit per cycle the server being not called \(= R^N\) Expected cycle length of server 2 not being called \(= K \cdot E(C_2)\). In addition, the average system down in a cycle in the steady state is 
\[
E(D) = \frac{1}{\mu - \lambda}
\]
Then we have the following theorem.

**Theorem 4.1:** The net profit per cycle is a function of \(N\) given by \(P(N) = K \cdot E(C_2) - C \cdot E(D)\) is concave in \(N\) when \(\lambda < \mu\) and the maximum value \(N^*\) of \(N\) can be computed and is obtained by solving the inequality \(P(N^*) \geq P(N^* + 1)\).

**Proof:** \(P(N)\) defined above is of the form
\[
P(N) = \frac{K}{\mu - \lambda} \sum_{n=0}^{\infty} \frac{\lambda^n \mu^{-n-1}}{n!} \cdot \left(1 - \frac{\lambda}{\mu}\right) \frac{N^n}{N^n - \lambda^{n+1}}
\]

Differentiating \(P(N)\) with respect to \(N\) twice, we can see that \(P'(N) < 0\) when \(\lambda < \mu\), which implies that \(P(N)\) is concave in \(N\). Then optimal \(N^*\) of \(N\) can be computed using (4.1).

5. Numerical Illustration
In this section we have computed the net profit of the system per cycle for the given input parameters \(\lambda = 2, \mu_1 = 3, \mu_2 = 4, n = 20, k = 8, K = 30\) and \(C = 1000\). The net profit per cycle different values of \(N\) is given in the following table 5.1.

<table>
<thead>
<tr>
<th>(N)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2N^*)</td>
<td>59</td>
<td>78.6</td>
<td>170</td>
<td>316.9</td>
<td>540.4</td>
<td>879.0</td>
<td>1385.1</td>
</tr>
<tr>
<td>(2N^*)</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P(N))</td>
<td>212</td>
<td>315</td>
<td>420</td>
<td>330</td>
<td>109</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

From the table, we can see that the net profit function increases first and reaches a maximum value and then decreases and the optimal \(N^*\) value is \(N^* = 11\) and the maximum profit is 4206.7244.

Also the mean time failure of the system is calculated for 
\(\lambda = 5, \mu_1 = 5.5, \mu_2 = 7.5, n = 80, k = 30\) and is given in table 5.2.

<table>
<thead>
<tr>
<th>(N)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTTF</td>
<td>17.76</td>
<td>16.36</td>
<td>15.36</td>
<td>13.95</td>
<td>12.38</td>
</tr>
<tr>
<td>(N)</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>MTTF</td>
<td>10.63</td>
<td>8.69</td>
<td>6.53</td>
<td>4.14</td>
<td>1.48</td>
</tr>
</tbody>
</table>
In this section, we have compared system reliability of the present model with the reliability of the single server model under N-policy studied by Krishnamoorthy et. al (2002). We can see that when $\mu_1 = 0$, the present model reduces to the model studied by Krishnamoorthy et. al (2002). We have calculated the system reliability for the present model for the parameter $\lambda = 10, \mu_1 = 3, \mu_2 = 12, n = 50, k = 30$ and various values of $N$ and also obtained the system reliability under single mode of service for the same parameters and when $\mu_1 = 0$ and is given in Table 5.4. We can see that the reliability of the present model is highly increased by the introduction of one more server to the system. Also we can see that the system reliability is seen to be decreasing as $N$ increases. The following table 5.4 gives reliability for various parameter values and different values of $N$.

### Table 5.3

<table>
<thead>
<tr>
<th>N</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>EB</td>
<td>1.93</td>
<td>2.90</td>
<td>3.87</td>
<td>4.82</td>
<td>5.77</td>
<td>6.70</td>
<td>7.62</td>
<td>8.53</td>
</tr>
<tr>
<td>EI</td>
<td>0.15</td>
<td>0.46</td>
<td>0.78</td>
<td>1.11</td>
<td>1.45</td>
<td>1.78</td>
<td>2.11</td>
<td>2.44</td>
</tr>
<tr>
<td>ED</td>
<td>0.00</td>
<td>0.08</td>
<td>0.01</td>
<td>0.016</td>
<td>0.021</td>
<td>0.028</td>
<td>0.033</td>
<td>0.034</td>
</tr>
</tbody>
</table>

### Table 5.4

<table>
<thead>
<tr>
<th>N</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>EB</td>
<td>9.42</td>
<td>10.2</td>
<td>11.1</td>
<td>11.93</td>
<td>12.70</td>
<td>13.42</td>
</tr>
<tr>
<td>EI</td>
<td>2.78</td>
<td>3.11</td>
<td>3.44</td>
<td>3.78</td>
<td>4.11</td>
<td>4.44</td>
</tr>
<tr>
<td>ED</td>
<td>0.56</td>
<td>0.99</td>
<td>0.30</td>
<td>0.105</td>
<td>0.128</td>
<td>0.156</td>
</tr>
</tbody>
</table>

### Conclusion:

In the present work, we have studied a repairable reliability system under two modes of service and two modes of service and presented a comparison of system reliability under single mode of service and two modes of service.
service under $N$-policy. There are two technicians, called repairman1 and repairman2. The repairman1 is always active and repairs units one at a time and repairman2 will activate only on the accumulation of $N$ components for service in the system and continue to work in the system until the service of all the failed units are completed. Server2 is again activated on the accumulation of $N$ failed components. Our main goal was to obtain the optimal value of $N$ which gives the maximum profit that in turn produce the maximum system availability. We have studied the model by analyzing a Markov process over the possible discrete set of states and obtained system state probability distributions in finite time and in the steady state. In addition, we have obtained various system characteristics like average number of failed components in the steady state, average system down time, average time required to bring the system to the position in which all components are working once the components started failure, average amount of time the server can keep idle, system reliability etc. Here we have obtained a net profit function per cycle, which showed to be a concave function in $N$ that gives the maximum net profit per cycle. In addition, we have compared the system reliability of the present model with the reliability of the system with single mode of service and found that for different values of $N$ and the given input parameters, the system reliability has been highly increased in the present model. Some numerical illustrations are also provided.

**Future work:** The model studied here can extent to various other service policies like T-policy, D-policy etc. studied under single mode of service and under general failure and service time distributions.

**References**


