



Fractal analysis as a financial market evaluation method

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Abstract

The article deals with the application of fractal analysis as a method to evaluate financial markets under intense market fluctuations. The algorithm of monofractal analysis is described to determine the Hurst exponent in order to make conclusions with respect to the riskiness of assets.

Key words: financial markets, fractal analysis, Hurst exponent.

1 Introduction

The increase in the number of financial crises and the growth of uncertainty, as well as the deterioration of confidence of financial market participants predetermine the interest of scientists regarding the research and development of a new system of indicators that would warn changes taking place in the world. Such systems of indicators are used on a real-time basis by both market economy entities and state institutions to develop and implement financial policy regulation methods [1, 2]

Therefore, there is a need to develop a system of financial indicators, which would be able to prevent the occurrence of negative phenomena and processes in the capital market, as well as would contribute to business solvency. Due to the fact that the financial and economic crisis of 2007-2008 has disproved the perfection of the existing system of financial indicators, which warned of the advent of the crisis, but limited the ability to determine the scale of the crisis and its spread rate, such indicators system should include not only quantitative but also qualitative indices.

The need for such a research is relevant with regard to the growth in number of participants of currency, exchange, and other financial organizations in Russia. Therefore, there was a need to develop and simulate new financial indicators.

Analysis, research of existing indices, and development and simulating the dynamics of new financial indicators will contribute to the improvement of the financial and monetary policy of the state, as well as trade in the financial market.

Modern processes in various spheres of human

activity associated with the development of science and technology can be reproduced by means of mathematical methods and models. However, the chosen model may not always be adequate. This is due to the fact that each phenomenon or process has its own specific course of development, as well as the tightness or depth of links, which are difficult to display mathematically. In addition, it is not always possible to obtain complete and reliable information that can be used to build the model [3].

Therefore, usually a certain process or phenomenon is described only partially for certain conditions, but this leads to the fact that the model is incomplete, and in cases, where the specified conditions are not met, the model becomes unusable.

Financial markets are usually analyzed employing linear models, but latter cannot describe the changes that may occur in the market. Methods of nonlinear dynamics can represent the market as a complex system that can be changed over time. [4]

The increase in the number of studies in the financial sphere carried out by foreign and domestic scientists is due to the increase in the number of financial shocks over the past 30 years.

Studies of portfolio risks are considered in scientific works of G. Markowitz [5]. The contribution of the scientist is that the level of risk can be reduced by combining risky assets in the investment portfolio. However, the scientist does not specify the existence of the relationship between the level of risk and expected income, whereas the Capital Asset Pricing Model (CAMP) developed by W. Sharp specifies this relationship [6].

Fractal analysis is one of the methods used to study the dynamics of financial time series. This tool was used by B. Mandelbrot in 1959 [7].

According to the fractal hypothesis of the market, the financial market functioning is based on fractal (self-similar) phenomena and processes. Such understanding of financial markets requires a new theoretical and methodological approach to their interpretation. In accordance with this key postulate, financial markets are understood as an aggregate of a significant number of diversified investment horizons

of irrational market participants (self-similar fractal market structure). The balance of these horizons is crucial for the market development and characterizes its state in strong form efficiency.

The purpose of the present article is to study the prospects of using R/S-analysis as a tool for the study of monofractal time series with respect to the financial markets' analysis, and the development of an algorithm for calculating the Hurst exponent.

2 Rationale for R/S analysis

There are different approaches to the financial market analysis, but two basic ones are used most often: fundamental and technical analysis. The first one is based on traditional theories that describe the market. However, financial markets are characterized by the emergence of situations, where the theory cannot foresee and describe the processes. Technical analysis is based on an empirical experience, though without a clear theoretical substantiation.

Due to the fact that the financial markets are currently explored as being both spatial and temporal fractals, this allowed expanding technical analysis tools. Market participants increasingly used methods and theories such as synergy, chaos, and fractal geometry.

Fractal analysis can be useful in cases, where market processes cannot be represented as classical models, because they can develop chaotically.

The analysis method examines the fractality of the time series. This technique was proposed by B. Mandelbrot and was based on the studies that were conducted by G. Hurst.

As of today, many calculating methods of the Hurst exponent are known. This exponent reflects the maximum range of currency quotes or stock prices for a certain period and is used as a measure of the long-term memory of time series, while its value ranges from 0.5 to 1.

$$(R/S)_n = c \cdot n^H \quad (1)$$

where R/S is the range of scatter of the variable x, which is a time series with the number of observations n, c is the constant.

The range of scatter is the interval (defined as maximum value minus minimum value) of the sums or partial sums of x (after subtracting the sample mean), which is divided by the standard deviation of the sample.

If x is a white noise (zero mean, or zero stability), then the range of perturbation sets that forms random walks is scaled to the standard deviation and grows as the square root of the series size, giving the expected value (0.5) of the Hurst exponent.

When the obtained value exceeds 0.5, then the conclusion is made about persistence, i.e. the series represents a generalized Brownian motion and is characterized by long-term memory, i.e. what is happening today affects the future.

If the Hurst exponent is within the range from 0 to 0.5, then a conclusion is made about the anti-persistence. This time series is much more variable than the random series, because it constantly fluctuates. While observing a decline in the previous

period, we should expect growth in the next period, and vice versa.

The value of $H=0.5$ characterizes the usual white Gaussian noise, random Brownian motion [8].

3 Hurst exponent calculating algorithm

The calculating algorithm for the Hurst exponent consists of the following stages.

1. We choose a time series consisting of M elements, and turn it into a series of length equal to $N = M-1$ by means of logarithmic correlations:

$$N_i = \log \left(\frac{M_{i+1}}{M_i} \right), i = 1, \dots, M-1 \quad (2)$$

2. The new time series N must be divided into I_a intervals ($a = 1, 2, \dots, A$), each of length n (subject to the condition $N = An$), and consisting of N_k elements ($k = 1, 2, \dots, n$), for $n \geq 10$. For all I_a we calculate the mean values e_a :

$$e_a = \frac{\sum_{k=1}^n N_{k,a}}{n} \quad (3)$$

3. For all I_a , we calculate the accumulated deviations from the mean $X_{k,a}$:

$$X_{k,a} = \sum_{i=1}^k (N_{i,a} - e_a), k = 1, 2, \dots, n \quad (4)$$

4. For all I_a we calculate the normalized range of scatter:

$$R_{l,a} = \max(X_{k,a}) - \min(X_{k,a}) \quad (5)$$

where $l \leq k \leq n$

and the sample standard deviation is

$$S_{l,a} = \sqrt{\left(\frac{l}{n} \right) \cdot \sum_{i=1}^k (N_{k,a} - e_a^2)} \quad (6)$$

5. We calculate the ratio R/S for all I_a intervals by the formula $R_{l,a}/S_{l,a}$, thus obtaining the average range of variation:

$$(R/S)_n = \frac{\sum_{a=1}^A (R_l / S_{l,a})}{A} \quad (6)$$

6. Next, we increase the length of the interval n to a value higher than the previous one and repeat the calculations 1-5 until n equals the value $(M-1)/2$.

The R/S parameter obtained from (1, 6) is called the standardized range because it has a zero mean value. The R/S changes the scale depending on the number of observations according to the power law, which is equal to H . The change of scale according to the power law is a sign of self-similarity, and consequently, of fractality of the time series.

We construct a linear regression equation:

$$\log(R/S_n) = \log(c) + H \log(n) \quad (7)$$

The segment, clipped off on the coordinate axis, represents the constant – estimation of $\log(c)$. The slope of the linear regression equation represents the estimate of H (Hurst exponent).

The value of H allows concluding about the randomness or persistence of the data considered.

The characteristic of the financial market at different

values of H is presented in Table 1 [2, 3].

H	Market characteristic
$H = 0$	"Dead" market, the lack of movements, or they are cyclic with high frequency oscillations.
$0 \leq H < 0.5$	A series is anti-persistent, unstable. The approach of H to zero characterizes more unstable price dynamics.
$H = 0.5$	A series is random, events are uncorrelated; they are already used and discounted by the market.
$0.5 < H \leq 1$	Persistent (trend-resistant) market, future values depend on the past ones. The closer the H to 1, the stronger the trend.

Table 1. Financial market characteristic

The H exponent also gives an opportunity to draw a conclusion about the degree of risk of investing in a certain asset [4, 8]:

- 1) if $H \rightarrow 1$, then the prices of the selected asset are more predictable, and therefore the risks associated with this asset, will be less;
- 2) if $H \rightarrow 0$, then the price behavior is more unpredictable and chaotic, and therefore the risks associated with the acquisition of the asset, will be higher;
- 3) if $H \rightarrow 0.5$, then the price dynamics are random.

4 Conclusion

In the context of deepening the effects of the crisis phenomena in the financial markets, the issues of the dynamics analysis and forecasting of these markets in general, as well as the threat of new crises become more urgent. In the context of the search for alternative concepts of the financial markets functioning as opposed to the efficient market hypothesis in the postcrisis period, the fractal market hypothesis becomes widely spread that makes it possible operating with new categories and using the so-called persistence property of their long-term memory to simulate the behavior of market indicators.

A key indicator for its evaluation is the Hurst exponent, while R/S-analysis is the most priority method for its calculation.

The R/S-analysis method is one of the most promising techniques to analyze financial market indicators in the period of intense fluctuations, and the Hurst exponent is an indicator, which is able to alert the emergence of negative trends in the market.

The results of the fractal analysis have

theoretical and applied value in the context of both timely identification and prevention of the crisis developments on the stock market revealed by its regulators, as well as from the standpoint of developing trading strategies by its participants, which take into account the influence of the uncertainty factor.

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