

ON THE COMPLEMENT CONNECTED GEODETIC NUMBER OF A GRAPH

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Abstract

For a connected graph $G = (V, E)$ of order $p \geq 2$, a set $S \subseteq V$ is called a geodetic set of G if every vertex of G is contained in a geodesic joining some pair of vertices in S . The geodetic number $g(G)$ of G is the minimum cardinality of its geodetic sets. A geodetic set S of a connected graph G is said to be a *complement connected geodetic set* if $S = V$ or the subgraph $\langle V - S \rangle$ is connected. The minimum cardinality of a complement connected geodetic set of G is the *complement connected geodetic number* of G and is denoted by $g_{cc}(G)$. The complement connected geodetic number of certain classes of graphs is determined. Necessary conditions for the complement connected geodetic number to be 2 are given. It is shown that for a connected graph of order p and diameter d , $g_{cc}(G) \leq p - d + 1$. Also, it is proved that for a tree T , $g_{cc}(G) = p - d + 1$ if and only if T is a caterpillar. It is also proved that for every pair of integers a, b with $2 \leq a \leq b$, there exists a connected graph G with $g(G) = a$ and $g_{cc}(G) = b$.

Keywords : geodetic number, complement connected geodetic set, complement connected geodetic number.

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1 Introduction

We consider finite graphs with neither loops nor multiple edges. For any graph G , we denote the vertex set by $V(G)$ and the edge set by $E(G)$. We denote the order of G by $p = |V(G)|$ and the size by $q = |E(G)|$. For every vertex $v \in V$, the *open neighborhood* $N(v)$ is the set $\{u \in G/uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N[v] = N(v) \cup v$. The degree of a vertex $v \in V$ is $\deg_G(v) = \deg(v) = |N(v)|$. The *minimum* and *maximum degrees* of a graph G are denoted by $\delta = \delta(G)$ and $\Delta = \Delta(G)$, respectively. The subgraph induced by a set S of vertices of a graph G is denoted by $\langle S \rangle$ with $V(\langle S \rangle) = S$ and $E(\langle S \rangle) = \{uv \in E(G): u, v \in S\}$. For basic graph theoretic terminology we refer to [5]. For a vertex v of G , the *eccentricity* $e(v)$ is the distance between v and a vertex farthest from v . The minimum eccentricity among the vertices of G is the radius, $radG$ and the maximum eccentricity is its diameter, $diamG$ of G . Two vertices x and y are *antipodal* if $d(x, y) = diamG$. A vertex v is an *extreme vertex* of a graph G if the subgraph induced by its neighbors is complete. If $e = \{u, v\}$ is an edge of a graph G with $d(u) = 1$ and $d(v) > 1$, then we call e a pendant edge, u a leaf or end vertex and v a support.

The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest uv path in G . An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. It is known that the distance is a metric on the vertex set $V(G)$. The interval $I[u, v]$ consists of all vertices lying on some $u - v$ geodesic of G . For $S \subseteq V$, $I[S] = \bigcup_{u, v \in S} I[u, v]$. A set S of vertices is a geodetic set if $I[S] = V$, and the minimum cardinality of a geodetic set is the geodetic number $g(G)$. A geodetic set of cardinality $g(G)$ is called a g -set. The geodetic number of a graph was introduced in [2, 6] and further studied by several authors. In this paper we study the complement connected geodetic number of a connected graph.

The following theorems are used in the sequel.

Theorem 1.1. ([3]) *Every geodetic set of a graph contains its extreme vertices.*

Theorem 1.2. ([3]) *The geodetic number of a tree T is the number of end-vertices in T . In fact, the set of all end-vertices of T is the unique minimum geodetic set of T .*

Throughout the following G denotes a connected graph with at least two vertices.

2 The Complement Connected Geodetic Number of a Graph

Definition 2.1. A geodetic set S of a connected graph G is said to be a complement connected geodetic set if $S = V$ or the subgraph $\langle V - S \rangle$ is connected. The minimum cardinality of a complement connected geodetic set of G is the complement connected geodetic number of G and is denoted by $g_{cc}(G)$. A complement connected geodetic set of minimum cardinality is called the $g_{cc}(G)$ -set of G .

Example 2.2. For the graph G given in Figure 2.1, the set $S = \{v_1, v_3\}$ is a g -set of G and so $g(G) = 2$. Since the subgraph $G[V - S]$ induced by $V - S$ is disconnected, S is not a complement connected geodetic set of G . Let $S_1 = \{v_1, v_2, v_3\}$. Then S_1 is a complement connected geodetic set of G and so $g_{cc}(G) = 3$. Thus the geodetic number and the complement connected geodetic number of a graph are different.

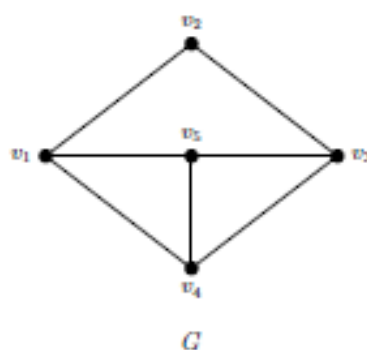


Figure 2.1

Theorem 2.3. *Each extreme vertex of a graph G belongs to every complement connected geodetic set of G .*

Proof. Since every complement connected geodetic set is a geodetic set, the result follows from Theorem 1.1.

Corollary 2.4. For the complete graph K_p ($p \geq 2$), $g_{cc}(K_p) = p$.

Proof. Since every vertex of the complete graph K_p ($p \geq 2$) is an extreme vertex, the vertex set of K_p is the unique complement connected geodetic set of K_p . Thus $g_{cc}(K_p) = p$.

Theorem 2.5. For any tree T with k end vertices, $g_{cc}(T) = k$.

Proof. Let S be the set of all end vertices of T . By Theorem 1.2, $g(T) = |S|$. Since the subgraph $\langle V - S \rangle$ is connected, S is the unique minimum complement connected geodetic set of T . Therefore $g_{cc}(T) = |S| = k$.

Corollary 2.6. For any non-trivial path P_p ($p \geq 3$), $g_{cc}(P_p) = 2$.

Proof. This follows from Theorem 2.5.

Theorem 2.7. For a connected graph G of order p , $2 \leq g(G) \leq g_{cc}(G) \leq p$.

Proof. Any geodetic set needs at least two vertices and so $g(G) \geq 2$. Since every complement connected geodetic set is also a geodetic set, $g(G) \leq g_{cc}(G)$. Also, since $V(G)$ is a complement connected geodetic set, it is clear that $g_{cc}(G) \leq p$. Thus $2 \leq g(G) \leq g_{cc}(G) \leq p$.

Remark 2.8. The bounds in Theorem 2.7 are sharp. For any non-trivial path P_p , the set of two end vertices is the unique geodetic set so that $g(P_p) = 2$. For any non-trivial tree T , the set of all end vertices is the unique geodetic set as well as unique complement connected geodetic set and so $g(T) = g_{cc}(T)$. For the complete graph K_p , $g_{cc}(K_p) = p$. Also all the inequalities in the Theorem are strict. For the graph G given in Figure 2.2, $S_1 = \{v_1, v_3, v_6\}$ and $S_2 = \{v_2, v_4, v_6\}$ are the only two minimum geodetic sets of G so that $g(G) = 3$. Also $S' = \{v_1, v_2, v_3, v_6\}$ is a minimum complement connected geodetic set of G so that $g_{cc}(G) = 4$. Thus $2 < g(G) < g_{cc}(G) < p$.

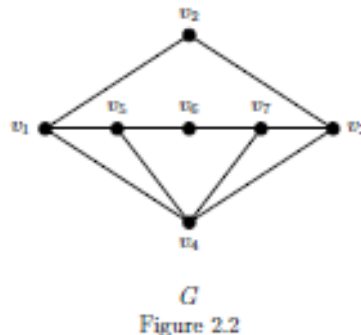


Figure 2.2

Theorem 2.9. For the cycle $G = C_p$ ($p \geq 3$), $g_{cc}(G) = \begin{cases} \frac{p}{2} + 1, & \text{if } p \text{ is even;} \\ \left\lceil \frac{p}{2} \right\rceil + 1, & \text{if } p \text{ is odd.} \end{cases}$

Proof. Case 1. Suppose that p is even. Let $p = 2n$. Let $C_{2n} : v_1, v_2, \dots, v_{2n}, v_1$ be the cycle of order $2n$. Let $S = \{u, v\}$ be a set of antipodal vertices of C_{2n} . Since every pair of antipodal vertices of C_{2n} is a geodetic set of G , S is a geodetic set of G . Since the subgraph $\langle V - S \rangle$ is disconnected, S is not a complement connected geodetic set of G . Let $S' = \{v_1, v_2, \dots, v_{n+1}\}$. Then $\langle V - S' \rangle$ is connected. It is evident that S' is a complement connected geodetic set of C_p and so $g_{cc}(G) = n + 1 = \frac{p}{2} + 1$.

Case 2. Suppose that p is odd. Let $p = 2n + 1$. Let $C_{2n+1} : v_1, v_2, \dots, v_{2n+1}, v_1$ be the cycle of order $2n + 1$. For any vertex u , let v and w be the antipodal vertices. Then $S = \{u, v, w\}$ is a geodetic set of C_{2n+1} . Since the subgraph $\langle V - S \rangle$ is disconnected, S is not a complement connected geodetic set of G . Let $S' = \{v_1, v_2, \dots, v_{n+1}, v_{n+2}\}$. It is obvious that S' is a complement connected geodetic set of G so that $g_{cc}(G) = \left\lceil \frac{p}{2} \right\rceil + 1$.

Theorem 2.10. For the complete bipartite graph $G = K_{m,n}$, $g_{cc}(G) = \begin{cases} n, & \text{if } m = 1, n \geq 2; \\ 3, & \text{if } m = n = 2; \\ 4, & \text{if } 2 < m \leq n. \end{cases}$

Proof. Let $G = K_{m,n}$. When $m = 1$ and $n \geq 2$, $G = K_{m,n}$ is a tree with n end vertices. Therefore by Theorem 2.5, $g_{cc}(K_{1,n}) = n$. When $m = n = 2$, $G = K_{2,2}$ is the cycle C_4 . Hence by Theorem 2.9, $g_{cc}(K_{2,2}) = 3$. Now, let $2 < m \leq n$. Let $X = \{x_1, x_2, \dots, x_m\}$, $Y = \{y_1, y_2, \dots, y_n\}$ be a bipartition of G . Then $S = \{x_i, x_j, y_r, y_s\}$, where $i \neq j, r \neq s$ is a minimum geodetic set of G . Since the subgraph $\langle V - S \rangle$ is connected, S is a minimum complement connected geodetic set of G and so $g_{cc}(G) = 4$.

Corollary 2.11. For the star $G = K_{1,p-1}$, $g_{cc}(G) = p - 1$.

Proof. This follows from Theorem 2.10.

Theorem 2.12. For the wheel $W_p = K_1 + C_{p-1}$, ($p \geq 5$), $g_{cc}(W_p) = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even;} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd.} \end{cases}$

Proof. Let $G = W_p$. Let v be the vertex of K_1 and let $v_1, v_2, \dots, v_{p-1}, v_1$ be the cycle C_{p-1} . It is clear that any geodetic set of W_p , ($p \geq 5$) is a subset of $V(C_{p-1})$.

Case 1. Let p be even. Let $S = v_1, v_3, \dots, v_{p-2}, v_{p-1}$ be a set of $\frac{p}{2}$ vertices of G . Then S is a geodetic set of G . Also, since the subgraph $\langle V - S \rangle$ is connected, S is a complement connected geodetic set of G . We prove that S is a minimum complement connected geodetic set of G . Suppose $g_{cc}(G) \geq \frac{p}{2}$. Then there exists a complement connected geodetic set S' such that $|S'| > \frac{p}{2}$. Since $S' \subseteq V(C_{p-1})$, $S'' = S' - \{x\}$ is a geodetic set of G for some $x \in S'$. Since $v \notin S''$, $\langle V - S'' \rangle$ is connected and so S'' is a complement connected geodetic set of G , which is a contradiction. Thus S is a minimum complement connected geodetic set of G so that $g_{cc}(G) = \frac{p}{2}$.

Case 2. Let p be odd. Let $S = v_1, v_3, \dots, v_{p-4}, v_{p-2}$ be a set of $\frac{p-1}{2}$ vertices of G . By the similar argument as in Case 1, we can prove that S is a complement connected geodetic set of G and so $g_{cc}(G) = \frac{p-1}{2}$.

Theorem 2.13. For a connected graph G , $g_{cc}(G) = 2$ if and only if there exists peripheral vertices u and v such that any one of the following holds:

- (i) every vertex of G is on a diametral path joining u and v .
- (ii) either u or v or both are extreme vertices.
- (iii) degree of at least one vertex in $\langle V - \{u, v\} \rangle$ is more than 3 in G .

Proof. Let u and v be peripheral vertices of G such that any one of (i), (ii) and (iii) holds. Since $g_{cc}(G) \geq 2$ and since P is a geodesic joining u and v such that each vertex of G is on a geodesic joining u and v , $S = \{u, v\}$ is a geodetic set of G . Also, since the subgraph $\langle V - S \rangle$ is connected, we have $g_{cc}(G) = 2$. Conversely, let $g_{cc}(G) = 2$ and let $S = \{u, v\}$ be a

geodetic basis for G . Then every vertex of G lies on a geodesic joining u and v . We claim that $d(u, v) = d(G)$. If $d(u, v) < d(G)$, then let x and y be two vertices of G such that $d(x, y) = d(G)$. Now, it follows that x and y lie on distinct geodesics joining u and v . Hence

$$d(u, v) = d(u, x) + d(x, v) \quad (1)$$

and

$$d(u, v) = d(u, y) + d(y, v) \quad (2)$$

By the triangle inequality,

$$d(x, y) \leq d(x, u) + d(u, y) \quad (3)$$

Since $d(u, v) < d(x, y)$, (3) becomes

$$d(u, v) < d(x, u) + d(u, y) \quad (4)$$

Using (4) in (1), we get $d(x, v) < d(x, u) + d(u, y) - d(u, x) = d(u, y)$. Thus,

$$d(x, v) < d(u, y) \quad (5)$$

Also, by triangle inequality, we have

$$d(x, y) \leq d(x, v) + d(v, y) \quad (6)$$

Now, using (5) and (2), (6) becomes $d(x, y) < d(u, y) + d(v, y) = d(u, v)$. Thus, $d(G) < d(u, v)$ which is a contradiction. Hence $d(u, v) = d(G)$ and since $S = \{u, v\}$ is a geodetic basis for G , it follows that each vertex of G is on a diametral path joining u and v satisfying (i). Next we show that $\langle V - S \rangle$ is connected. If G is a path, then we are done. If G is not a path and u and v are not extreme vertices and degree of all vertices in $\langle V - S \rangle$ is less than or equal to 2, then it follows that G is an even cycle which implies $\langle V - S \rangle$ is disconnected, which is a contradiction. Therefore either u or v or both should be extreme vertices in G thus satisfying (ii) or degree of at least one vertex in $\langle V - \{u, v\} \rangle$ is more than 3 in G , satisfying (iii).

Corollary 2.14. For the hypercube Q_n ($n \geq 3$), $g_{cc}(Q_n) = 2$.

Proof. Q_n has 2^n vertices, which may be labeled $a_1 a_2 a_3 \dots a_n$, where each a_i ($1 \leq i \leq n$) is either 0 or 1. It is easily seen that $S = \{(0, 0, 0, \dots, 0), (1, 1, 1, \dots, 1)\}$ is a $g(Q_n)$ set for $n \geq 3$. Since the subgraph $\langle V - S \rangle$ is connected, S is a complement connected geodetic set of Q_n . Hence $g_{cc}(Q_n) = 2$.

Theorem 2.15. Let G be a connected graph with cut-vertices and S be a complement connected geodetic set of G . If v is a cut-vertex of G , then every component of $G - v$ contains an element of S .

Proof. Suppose that there is a component G_1 of $G - v$ such that G_1 contains no vertex of S . By Theorem 2.3, G_1 does not contain any end vertex of G . Let $u \in V(G_1)$. Since S is a complement connected geodetic set, there exists vertices $x, y \in S$ such that u lies on the $x - y$ geodesic $P: x = u_0, u_1, u_2, \dots, u, \dots, u_t = y$ in G . Let P_1 be a $x - u$ subpath of P and P_2 be a $u - y$ subpath of P . Since v is a cut vertex of G , both P_1 and P_2 contain v so that P is not a path, which is a contradiction. Thus every component of $G - v$ contains an element of S .

Theorem 2.16. No cut - vertex of a connected graph G belongs to any minimum complement connected geodetic set of G .

Proof. Let S be a minimum complement connected geodetic set of G , and $v \in S$ be any vertex of G . We claim that v is not a cut vertex of G . Suppose that v is a cut vertex of G . Let $G_1, G_2, \dots, G_r$ ($r \geq 2$) be the components of $G - v$. By Theorem 2.15, each component

$G_i (1 \leq i \leq r)$ contains an element of S . We claim that $S_1 = S - \{v\}$ is also a complement connected geodesic set of G . Let x be a vertex of G . Since S is a complement connected geodesic set, x lies on a geodesic joining a pair of vertices u and v of S . Without loss of generality, we assume that $u \in G_1$. Since v is adjacent to at least one vertex of each $G_i (1 \leq i \leq r)$, assume that v is adjacent to z in $G_k, k \neq 1$. Since S is a complement connected geodesic set, z lies on a geodesic Q joining v and a vertex w of S such that w must necessarily belong to G_k . Thus $w \neq v$. Now, since v is a cut vertex of G , $P \cup Q$ is a path joining u and w in S and thus the vertex x lies on this geodesic joining two vertices u and w of S_1 . Thus we have proved that every vertex that lies on a geodesic joining a pair of vertices u and v of S also lies on a geodesic joining two vertices of S_1 . Hence it follows that every vertex of G lies on a geodesic joining two vertices of S_1 , and since the subgraph $\langle V - S_1 \rangle$ is connected, S_1 is a complement connected geodesic set of G . Since $|S_1| = |S| - 1$, this contradicts the fact that S is a minimum complement connected geodesic set of G . Hence $v \notin S$ so that no cut vertex of G belongs to any minimum complement connected geodesic set of G .

Theorem 2.17. *If G is a non complete connected graph such that it has a minimum cutset U and $\langle U \rangle$ is connected, then $g_{cc}(G) \leq p - \kappa(G)$, where $\kappa(G)$ is the vertex connectivity of G .*

Proof. Since G is non complete, it is clear that $1 \leq \kappa(G) \leq p - 2$. Let $U = \{u_1, u_2, \dots, u_\kappa\}$ be a minimum cutset of vertices of G . Let $G_1, G_2, \dots, G_r (r \geq 2)$ be the components of $G - U$ and let $S = V(G) - U$. Then every vertex $u_i (1 \leq i \leq \kappa)$ is adjacent to at least one vertex of $G_j (1 \leq j \leq r)$. It is clear that S is a geodesic set of G . Since $\langle U \rangle$ is connected, the subgraph $\langle V - U \rangle$ is connected. Therefore S is a complement connected geodesic set of G so that $g_{cc}(G) \leq p - \kappa(G)$.

Theorem 2.18. *Let G be a connected graph with diameter d and order p . Then $g_{cc}(G) \leq p - d + 1$.*

Proof. If $G = K_p$, then $g_{cc}(G) = p$ so that the result is trivial. We assume that G is non complete. Let u and v be two vertices of G such that $d(u, v) = d \geq 2$. Let $P : u = v_0, v_1, v_2, \dots, v_{d-1}, v_d = v$ be a diametral path in G . Let $S = V - \{v_1, v_2, \dots, v_{d-1}\}$. Then every vertex $v_i (1 \leq i \leq d - 1)$ lies on the $u - v$ geodesic so that S is a geodesic set of G . Since the subgraph $\langle V - S \rangle$ is connected, S is a complement connected geodesic set of G so that $g_{cc}(G) \leq p - d + 1$.

Remark 2.19. The bound in Theorem 2.18 is sharp. For the cycle $G = C_4, g_{cc}(G) = 3$. Also $d = 2, p = 4$ and $p - d + 1 = 3$. Thus $g_{cc}(G) = p - d + 1$.

Theorem 2.20. *For every non-trivial tree $T, g_{cc}(T) = p - d + 1$ if and only if T is a caterpillar.*

Proof. Let T be a non-trivial tree. Let $d = d(u, v)$ and let $P : u = v_0, v_1, \dots, v_{d-1}, v_d = v$ be a diametral path in T . Let k be the number of end vertices in T . Let l denote the number of internal vertices in T other than $v_1, v_2, \dots, v_{d-1}$. Then $d - 1 + l + k = p$. By Theorem 2.5, $g_{cc}(T) = k$ and so $g_{cc}(T) = p - d + 1 - l$. Hence $g_{cc}(T) = p - d + 1$ if and only if $l = 0$, if and only if all the internal vertices of T lie on the diametral path P , (i.e.) if and only if T is a caterpillar.

Theorem 2.21. *Let G be a connected graph. Then $g_{cc}(G) = p$ if and only if $G = K_p$.*

Proof. Let $G = K_p$. Then by Corollary 2.4, $g_{cc}(G) = p$. Conversely, let $g_{cc}(G) = p$. If $G \neq K_p$, then $d \geq 2$ and by Theorem 2.18, $g_{cc}(G) \leq p - 1$, which is a contradiction. Therefore $G = K_p$.

Theorem 2.22. Let G be a connected graph with diameter d . If $g_{cc}(G) = p - 1$, then $d = 2$.

Proof. Let $g_{cc}(G) = p - 1$. Since G is a connected graph with diameter d , by Theorem 2.18, we have $g_{cc}(G) \leq p - d + 1$. This implies that $d \leq 2$. If $d = 1$, then $G = K_p$. Therefore by Theorem 2.21, $g_{cc}(G) = p$, which is a contradiction to our assumption. Thus $d = 2$.

Theorem 2.23. Let G be a connected graph of order p with exactly one vertex of degree $p - 1$ which is not a cut vertex. Then $g_{cc}(G) \leq p - 2$.

Proof. Let x be the vertex of degree $p - 1$ which is not a cut vertex of G . Since $\langle N(x) \rangle$ is non complete, there exist at least two non adjacent vertices say y and z such that y and z belong to $\langle N(x) \rangle$. Let x_1 be a vertex in $y - z$ geodesic. Then $S = V - \{x, x_1\}$ is a geodetic set of G . Since the subgraph $\langle V - S \rangle$ is connected, S is a complement connected geodetic set of G so that $g_{cc}(G) \leq p - 2$.



Figure 2.3

Remark 2.24. The bound in Theorem 2.23 is sharp. For the graph G given in Figure 2.3, $S = \{v_1, v_3, v_4\}$ is a minimum complement connected geodetic set of G so that $g_{cc}(G) = 3 = p - 2$.

Theorem 2.25. Let G be a graph of order $p \geq 3$. If G contains a cut vertex of degree $p - 1$, then $g_{cc}(G) = p - 1$.

Proof. Let v be the cut vertex of degree $p - 1$ in G and S be a minimum complement connected geodetic set of G . By Theorem 2.16, $v \notin S$. Since v is a full degree cut vertex, it is clear that $g_{cc}(G) = p - 1$.

In view of Theorem 2.7, we give the following realization result for the geodetic number and the complement connected geodetic number of a graph.

Theorem 2.26. If a and b are positive integers such that $2 \leq a \leq b$, then there exists a connected graph G with $g(G) = a$ and $g_{cc}(G) = b$.

Proof. **Case 1.** $a = b$. For $a = 2$, let $G = P_p$ be a non-trivial path. By Theorem 1.5, $g(G) = 2$ and by Corollary 2.6, $g_{cc}(G) = 2$. Now let $3 \leq a = b$. Let G be a tree with a end vertices. By Theorems 1.2 and 2.5, we have $g(G) = a$ and $g_{cc}(G) = a$.

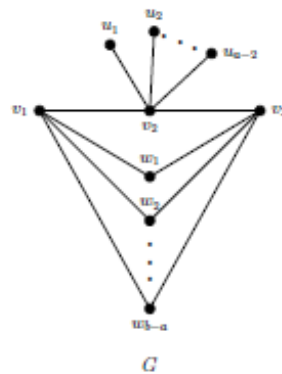


Figure 2.4

Case 2.2 $a < b$. Let G be the graph in Figure 2.4 obtained from the path on three vertices $P : v_1, v_2, v_3$ by adding $b - 2$ new vertices $w_1, w_2, \dots, w_{b-a}, u_1, u_2, \dots, u_{a-2}$ and joining each $u_i (1 \leq i \leq b - 2)$ with v_2 and joining each $w_i (1 \leq i \leq b - a)$ with v_1 and v_3 . Let $U = \{u_1, u_2, \dots, u_{a-2}\}$ be the set of all end vertices of G . By Theorem 1.1, U is a subset of every geodetic set of G . Since $I[U] \neq V(G)$, U is not a geodetic set of G . It is clear that $S = U \cup \{v_1, v_3\}$ is a geodetic set of G so that $g(G) = a - 2 + 2 = a$. Since the subgraph $(V - S)$ is disconnected, S is not a complement connected geodetic set of G . Let $S' = S \cup \{w_1, w_2, \dots, w_{b-a}\}$. Then S' is a complement connected geodetic set of G so that $g_{cc}(G) = a + b - a = b$.

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