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A NEW ALGORITHM TO SOLVE MULTI-OBJECTIVE ASSIGNMENT PROBLEM

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Abstract: A new method namely optimal flowing method has been proposed to solve multi-objective assignment problem (MOAP). In this method, using the optimal solution of the 'k' single objective assignment problem (AP), the proposed method provides ideal and set of all efficient solutions. Using numerical examples the above method is described.

Key Words: Assignment problem, multi-objective decision making, optimal assignment, optimal flowing method.

1. Introduction

The assignment problem is to find an optimal [6] assignment in which 'n' jobs are allocated to 'n' workers and each worker accepts exactly just one job so that the total assignment cost must be minimum. Hungarian method is the most standard to solve AP presented by Kuhn [4]. But in the real life problem most of the AP where we have to optimize more than one objective function at the same time. In literature survey, most of the AP models are discussed with the single objective, there are only few research paper available in MOAP in my information sight. Geetha and Nair [3] delivered a solution for an AP that minimizes both time and cost. Tsai et al. [8] attempt to solve a MOAP in decision making problem associated with cost, time and quality by fuzzy concept. Marler [7] narrated a report on multi-objective optimization methods for engineering field. Bao et al. [ ] developed 0-1 programming method to convert MOAP into single objective AP. Kayvan Salehi [4] presented MOAP with interval parameter by converting MOAP into crisp AP by applying weighted min-max method. Ventepala Yadaiah and Haragopal [9] gave a working procedure to solve MOAP in which the objective is to minimize the total cost, time and quality. Medvedeva and Medvedev [5] used the properties of the primal and dual problems in MOAP.

2. Formulation of Multi-objective Assignment Problem

Assume there are n works to be completed and n persons are available for doing the works. Assume that each person can do each work at a time, though with unreliable grade of efficiency. Let d_{ij}^{k} be the 'kth' cost if the i[th] person is assigned the j[th] work, the problem is to find an ideal solution and set of all efficient solution to the given MOAP.

Mathematically, the multi-objective assignment problem can be stated as:

(P_t) $$\text{Minimize } Z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^{1}\, y_{ij};$$

$$\text{Minimize } Z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^{2}\, y_{ij};$$

...

$$\text{Minimize } Z_k = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^{k}\, y_{ij}$$

subject to

$$\sum_{i=1}^{n} y_{ij} = 1,\ j = 1,2,...,\mathrm{n}\ ;\ \sum_{j=1}^{n} y_{ij} = 1,\ i = 1,2,...,\mathrm{n} \text{ and} \tag{1}$$

$$y_{ij} = \begin{cases} 1, & \text{if i}^{\text{th}} \text{ person is assigned j}^{\text{th}} \text{ work} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Where all objective function coefficients d_{ij}^{k} are non-negative integers and $y = (y_{11},..., y_{nn})$ is the matrix of decision variables.

Definition 2.1 A solution $y^{*} \in \Re^{n}$ is said to be a Pareto optimal solutions of the problem MOAP if there exist no other $y \in \Re^{n}$ satisfying $Z_i(y) \le Z_i(y^{*})$, $(i = 1,2,..., k)$.

3. Optimal Flowing Method

In this section we proposed a new method namely optimal flowing method to find an efficient solution and ideal solution for MOAP.

Algorithm:

Step 1: First test whether the given FIAP is a balanced one or not. If it is balanced one (i.e, number of persons identical to the number of works) then go to step 3. If it is an unbalanced one (i.e, number of persons are not equal to the number of works) then go to step 2.

Step 2: Introduce dummy row/column with zero cost in the given MOAP.

Step 3: Construct 'k' single objective AP from the given MOAP as:

$$(P_k) \quad \text{Minimize } \mathrm{Z}_k = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}^{k}\ y_{ij},$$

subject to

$$\sum_{i=1}^{n} y_{ij} = 1\ ,\ j = 1,2,...,\mathrm{n}\ ;\ \sum_{j=1}^{n} y_{ij} = 1,\ i = 1,2,...,\mathrm{n}\ ;$$

$$y_{ij} = \begin{cases} 1, & \text{if i}^{\text{th}} \text{ person is assigned j}^{\text{th}} \text{ work} \\ 0, & \text{otherwise} \end{cases} .$$

Step 4: Obtain the optimal assignment for the problem $(P_t) = P_1, P_2,.., P_s,..., P_k$ for $t = 1,2,...,\mathrm{k}$ using existing method. Let its optimal assignment solution be Y_t°, for $t = 1,2,...,\mathrm{k}$ with minimum cost Z_t°, which is an Ideal solution to the given MOAP.

Step 5: Use the optimal solution of the problem (P_s) obtained in step 4 in the problem $P_1, P_2,.., P_{s-1}, P_{s+1}..., P_k$.

Step 6: Repeat the step 5 for all the problem in (P_t) which provides all its efficient solution for the given MOAP.

Now, the proposed method optimal flowing method is described with the numerical examples.

Example 3.1

Let us consider an example that

Min $Z_1 = 13y_{11} + 8y_{12} + 16y_{13} + 18y_{14} + 19y_{15} + 9y_{21} + 15y_{22} + 24y_{23} + 9y_{33}$

Min $Z_2 = 13y_{11} + 15y_{12} + 8y_{13} + 10y_{21} + 20y_{22} + 12y_{23} + 15y_{31} + 10y_{32} + 12y_{33}$

Subject to

$$\sum_{i=1}^{3} y_{ij} = 1 \ , j = 1,2,3 ; \ \sum_{j=1}^{3} y_{ij} = 1 , \ i = 1,2,3 .$$

$$y_{ij} = \begin{cases} 1 , \ \text{if i}^{\text{th}} \text{ person is assigned j}^{\text{th}} \text{ work} \\ 0 , \ \text{otherwise} \end{cases}$$

Solution: The example 3.1 is a balanced AP.

Using the step 3, the optimal assignment for the single objective AP is:

(P_1): $Y_1^{\circ} = (0, 1, 0, 0, 0, 1, 1, 0, 0)$ with Min $Z_1^{\circ} = 29$ and

(P_2): $Y_2^{\circ} = (0, 0, 1, 1, 0, 0, 0, 1, 0)$ with Min $Z_2^{\circ} = 28$.

Now, by step 4, using the optimal assignment of the problem (P_1) in the problem (P_2), its efficien solution is $Z_1, Z_2 = (29, 38)$ and

Now, by step 5, using the optimal assignment of the problem (P_2) in the (P_1), its efficient solution is $Z_1, Z_2 = (28, 42)$.

Therefore, for the example 3.1, the Ideal solution is min $Z_1^{\circ}, Z_2^{\circ} = (29, 28)$ and the set of all efficient solutions for Z_1, Z_2 are $(29, 38)$ and $(28, 42)$.

Example 3.2

Min $Z_1 = 7y_{11} + 7y_{12} + 12y_{13} + 8y_{21} + 4y_{22} + 6y_{23} + 5y_{31} + 10y_{32} + 4y_{33}$

Min $Z_2 = 21y_{11} + 20y_{12} + 25y_{13} + 9y_{21} + 12y_{22} + 14y_{23} + 9y_{31} + 15y_{32} + 16y_{33}$

Min $Z_3 = 29y_{11} + 57y_{12} + 56y_{13} + 16y_{21} + 35y_{22} + 28y_{23} + 22y_{31} + 20y_{32} + 19y_{33}$

Subject to

$$\sum_{i=1}^{3} y_{ij} = 1 \ , \ j = 1,2,3 \ ; \ \sum_{j=1}^{3} y_{ij} = 1 \ , \ i = 1,2,3 \ .$$

$$y_{ij} = \begin{cases} 1 \ , \ \text{if i}^{\text{th}} \text{ person is assigned j}^{\text{th}} \text{ work} \\ 0 \ , \ \text{otherwise} \end{cases}$$

Solution: The example 3.2 is a balanced AP.

Using the step 3, the optimal assignment for the single objective AP is:

(P_1): $Y_1^{\circ} = (1, 0, 0, 0, 1, 0, 0, 0, 1)$ with Min $Z_1^{\circ} = 15$,

(P_2): $Y_2^{\circ} = (0, 1, 0, 0, 0, 1, 1, 0, 0)$ with Min $Z_2^{\circ} = 43$ and

(P_3): $Y_3^{\circ} = (1, 0, 0, 0, 0, 1, 0, 1, 0)$ with Min $Z_3^{\circ} = 77$.

Now, by step 4 and step 5, using the optimal assignment of the problem (P_1) in the problem (P_2) and (P_3), its efficient solution is $Z_1, Z_2, Z_3 = (15, 49, 83)$

Simillarly, using the optimal assignment of the problem (P_2) in the problem (P_1) and (P_3), its efficien solution is $Z_1, Z_2, Z_3 = (18, 43, 107)$ and by the optimal assisgnment of the problem (P_3) in the problem (P_1) and (P_2), its efficient solution is $Z_1, Z_2, Z_3 = (23, 50, 77)$.

Therefore, for the example 3.2, the Ideal solution is min $Z_1^{\circ}, Z_2^{\circ}, Z_3^{\circ} = (15, 43, 77)$ and the set of all efficient solutions for Z_1, Z_2, Z_3 are (15, 49, 83), (18, 43, 107) and (23, 50, 77).

4. Conclusion

In this paper we proposed a new algorithm to solve MOAP using the optimal solution of single AP problem. Since the proposed method is purely based on Hungarian method, it is very easy to apply and to solve. The ideal solution and the set of all Pareto optimal solution (or) efficient solution were obtained in the proposed method will help the decision maker who were handling the MOAP in the real life situation.

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