Numerical Computation of Integrals of Highly Oscillating Function by Block – Pulse and Chebyshev Wavelet Method

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Abstract—This paper presents, numerical integration of highly oscillating function of modified Filon type problem by wavelet based quadrature method like Chebyshev and Block – pulse method, development of the numerical integration wavelet methods are illustrate with examples

Keywords—Numerical Integration, Chebyshev wavelet, Block – pulse functions.

I. INTRODUCTION

The problem is considered in this paper is the numerical integration of highly oscillating integral or modified Filon type problem

\[ I = \int e^{i(x/\sigma)} f(x) \, dx \]  

where \( f(x) \) is non oscillatory smooth function and \( w \) and \( \sigma \) are frequency parameter, eq,(1) is also rewritten as

\[ Re\, I = \frac{b}{a} \left( \int f(x) \cos \left( \frac{wx}{\sigma} \right) \, dx \right) \]

\[ Im\, I = \frac{b}{a} \left( \int f(x) \sin \left( \frac{wx}{\sigma} \right) \, dx \right) \]

Analytical/numerical evaluation of these integral are difficult when frequency parameter \( w \) and \( \sigma \) is large, and its fails to evaluate for lower order quadrature methods like Simpson’s, Trapezoidal, Gaussian quadrature etc., oscillating integrals are widely occurs in past Fourier transform, image analysis, quantum chemistry, electrodynamics, and many applications related to science and technology. From the literature of review we may realized that many authors have been worked in numerical integration of oscillating function, Filon [1,2], clenshaw – curis [3], asymptotic [4], Complex integration method [5], generalized Gaussian quadrature rule [6,7], Haar wavelet and hybrid function method[9]. In this paper, the wavelet approach based on Chebyshev and Block – pulse method are used to evaluate the modified Filon type problem, the necessary computer program has been developed in symbolic mathematics capabilities of MAPLE this paper is demonstrate as follows. In Section 2. Mathematical preliminaries required for understanding the derivation. In Section 3. Selecting the highly oscillating integrals of the modified Filon type problem, In Section 4. We also compare numerical results to verify the exact value.

II. MATHEMATICAL PRELIMINARIES

A. Block - pulse functions

The Block pulse function is defined in the interval \([0, T)\) as follows

\[ \phi_i(t) = \frac{1}{h} \quad \text{if} \quad i h \leq t < (i + 1) h \]

\[ \phi_i(t) = 0 \quad \text{otherwise} \]

Where \( h = \frac{T}{m} \), \( t \in [0, T) \) and \( i = 0,1,2,3,\ldots,(m - 1) \)

The arbitrary function \( f(t) \) is integrable in the interval[0,T] can be approximately represented by finite series form as

\[ f(t) = f_0 \phi_0(t) + f_1 \phi_1(t) + f_2 \phi_2(t) + \ldots + f_{m-1} \phi_{m-1}(t) \]
Where \( f \) is the \( m \) - segment piecewise constant approximation of \( f \) over the interval \([0, T)\) to calculate the coefficients \( f_i \), to consider the nodal points \( \tau_k \) as

\[
t_k = \frac{2k-1}{2m}, \quad k = 1, 2, 3, \ldots, m
\]

eqn. (3) rewritten as \( f \) \( \tau_k = \frac{1}{m} \sum_{i=1}^{m} f_i \delta_{\tau_i} \tau_k = f_k \) , \( k = 1, 2, 3, \ldots, m \)

Numerical integral based Block-pulse function is

\[
\int_0^T f(t)\, dt = \frac{1}{m} \sum_{i=1}^{m} f_i \left( \frac{2i-1}{2m} \right)^2
\]

In generally

\[
\int_a^b f(x)\, dx = \frac{b-a}{m} \sum_{i=1}^{m} f(a + b - a \frac{(2i-1)}{2m})
\]

\( B. \) Chebyshev wavelets method

The chebyshev wavelet \( \theta_{k,m} \) is defined in the interval \([0,1]\) as follows

\[
\theta_{k,m} = \frac{\sin((k+1/2)m \pi t)}{\sin(m \pi/2)} \quad \text{for} \quad 0 \leq t < \frac{1}{m}, \quad \text{otherwise}
\]

Where \( T_m(t) \) is the chebyshev polynomial of degree \( m \) into the weight function \( 1 - t^2 \) on \([-1, 1]\) and given by

\[
T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t)
\]

Where \( m = 1, 2, 3, \ldots \)

The arbitrary function \( f(t) \) is integrable in the interval \( a \leq t \leq b \) can be expressed as

\[
f(t) = \sum_{n=0}^{\infty} c_n \theta_{n}(t)
\]

The truncated series in eqn.(6) can be expressed as

\[
f(t) = \sum_{n=0}^{m} c_n \theta_{n}(t)
\]

Since

\[
\int_0^T T_m(t)\, dt = \begin{cases} \frac{2}{m} & \text{if} \quad m \text{ is odd} \\ \frac{1}{m^2} & \text{if} \quad m \text{ is even} \end{cases}
\]

We have

\[
\int_0^T \theta_{n,m}\, dt = \begin{cases} \frac{1}{m} & \text{if} \quad m \text{ is odd} \\ \frac{1}{m^2} & \text{if} \quad m \text{ is even} \end{cases}
\]

We consider the nodal points \( x_P \) as

\[
x_P = \frac{x_m - x_{m-1}}{x_m - x_{m-1}} \quad \text{for} \quad i = 1, 2, 3, \ldots, M
\]

eqn.(7) reduced to

\[
f \left( \frac{x_{m-1} + x_{m+1}}{2} \right) = \frac{2}{2m} \sum_{i=1}^{m} c_n \theta_{n}(t)
\]

\( \text{III. HIGHLY OSCILLATING FUNCTIONS} \)

The following highly oscillating functions over the region \( 0 \leq x \leq 1 \) are tested in figure.

\( I_1 = \sin(x) \), \( I_2 = \cos(x) \), \( I_3 = \sin(3\pi x) \), \( I_4 = \cos(3\pi x) \), \( I_5 = \sin(5\pi x) \), \( I_6 = \cos(5\pi x) \), \( I_7 = \sin(7\pi x) \), \( I_8 = \cos(7\pi x) \)
IV. NUMERICAL EXAMPLES

We have computed the numerical results obtained using Block – pulse function and chebyshev wavelet method of various order $n$ and $m$ are shown in the table.

Table 1

<table>
<thead>
<tr>
<th>Integral with exact value</th>
<th>order</th>
<th>Block pulse function</th>
<th>Chebyshev wavelet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int_{0}^{\infty} \frac{x^3}{\sin x} \cos x , dx$</td>
<td>n=600</td>
<td>0.0001018449260</td>
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<tr>
<td>$\int_{0}^{\infty} \frac{x^3}{\sin x} \cos x , dx$</td>
<td>n=400</td>
<td>0.0000452753296170</td>
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<tr>
<td>$\int_{0}^{\infty} \frac{x^3}{\sin x} \cos x , dx$</td>
<td>n=400</td>
<td>0.00067714143129</td>
<td></td>
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<td>n=300</td>
<td>0.034983780775</td>
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</tr>
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</table>

Table 2

<table>
<thead>
<tr>
<th>Integral with exact value</th>
<th>order</th>
<th>Chebyshev wavelet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int_{0}^{\infty} \frac{x^3}{\sin x} \cos x , dx$</td>
<td>m=3, n=15</td>
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<tr>
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<tr>
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<tr>
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<tr>
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<td>m=4, n=15</td>
<td>0.22.004949534930</td>
</tr>
</tbody>
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V. CONCLUSIONS

This paper presents, numerical integration methods are applied to evaluate the highly oscillating function by block-pulse functions and chebyshev wavelet method, the numerical examples are provided to show the effectiveness of the wavelet methods.

References
