



Ranking of Trapezoidal Intuitionistic Fuzzy Sets based on Improved Accuracy Function and solving Fuzzy MCDM Problems Using TOPSIS method

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Abstract This research paper offers an improved accuracy function for the adequate ranking order of trapezoidal intuitionistic fuzzy sets and by using trapezoidal intuitionistic fuzzy TOPSIS method to solve the multicriteria decision-making problems. The alternatives can be ranked on the basis of the values of the closeness coefficients and the most enviable one(s) can be chosen in the decision-making process. In the end, some illustrative examples are given to show the potency of the proposed method.

Keywords Interval valued intuitionistic fuzzy sets, Trapezoidal Intuitionistic Fuzzy Number, Accuracy Function, TOPSIS Method, Fuzzy Multi Criteria Decision Making.

1. Introduction

In order to find the most suitable alternative among a set of executable alternatives, decision-making is the procedure for it. Hwang and Yoon [1], developed an eminent method for multicriteria decision-making. On the basis of fuzzy arithmetic operations, Triantaphyllou and Lin [2], developed the fuzzy edition of the TOPSIS access by leading to a fuzzy relative closeness for every alternatives. Between any two fuzzy numbers Chen [3] defined a crisp Euclidean distance in order to stretch out the TOPSIS approach to fuzzy group decision-making circumstances. Fuzzy multicriteria decision-making problem converted into a crisp one by Tsaur et al. [4]. Nowadays, numerous researchers have paid nice care to interval-valued fuzzy sets (IVFSs) [5, 6], intuitionistic fuzzy sets (IFSs)/vague sets (VSs) [7,8] and interval-valued intuitionistic fuzzy sets (IVIFSs) [9], which are all the generalization of the fuzzy set proposed by Zadeh [10], and applied them in decision-making (DM) problems. For our purpose, we extended these IVIFN into TrIFN. Ashtiani et al. [11] has given an interval-valued fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) for solving multi-criteria decision-making problems. Li [12] has given multi-attribute decision-making models and methods using IFSs, and then Li [13] extended the generalized-ordered, weighted, averaging operators to look into multi attribute decision-making problems using the accuracy function and the score function to rank the IFSs. Then, Ye [15] conferred a multicriteria decision-making technique using an improved accuracy function of VSs. Chen [16] constituted versatile algorithms with SAW and TOPSIS strategies by taking into account of each subjective and objective data to work out best multicriteria choices. Rudvan Sahin [17] developed an improved accuracy function for interval-valued intuitionistic fuzzy [18] sets to solve fuzzy MCDM problems. The basic characteristic of the IVIFS is that the values of its membership function and nonmembership function are intervals instead of precise numbers. So as to create correlations between two IVIFSs, some metric strategies were proposed by accuracy functions and score functions [15, 19,20] and were applied to multicriteria decision-making (MCDM) problems.

However, our studies proclaimed that these functions make some important defect in some places. Therefore, during this paper; we have a tendency to propose an improved accuracy function and developed trapezoidal intuitionistic fuzzy TOPSIS methodology supported the improved accuracy function to resolve multicriteria decision-making problems within which the performance rating values are expressed by TrIFSs.

The rest of this paper is unionized as follows. In Section 2, we succinctly introduce TrIFSs and its accuracy functions and score functions. Section 3 proposes an improved accuracy function of TrIFSs and makes comparisons with existing functions. Section 4 develops a TOPSIS method based on the improved accuracy function to solve trapezoidal intuitionistic fuzzy MCDM problems. Section 5 investigates two illustrative examples to show the potency of the proposed method. In Section 6, the paper is consummated.

2. Preliminaries

Definition 2.1 [9] Let the set of all closed subintervals of the interval [0, 1] be expression of an IVIFS in X is given by $\{(\underline{a}, \underline{c}), (\overline{a}, \overline{c})\} \in X$, where with the condition $0 < \underline{a} < \underline{c} < \overline{a} < \overline{c} \leq 1$. Let $(\neq \emptyset)$ be a given set $A: \rightarrow [0, 1], \cdot \rightarrow [0, 1]$ set A are denoted by (\cdot) and (\cdot)

The degree of belongingness and non-belongingness of the element x to the respectively.

Thus we denote A by $A = \{ (x, y) \mid (x, y) \in J \text{ where } 0 < (x, y) \leq 1, (x, y) \geq 0, (x, y) \geq 0 \}$.

The unknown degree (hesitancy degree) for each $x \in A$ defined as follows $(x) = 1 - (\mu(x) - \nu(x)) = 1 - (\mu(x) - \nu(x))$.

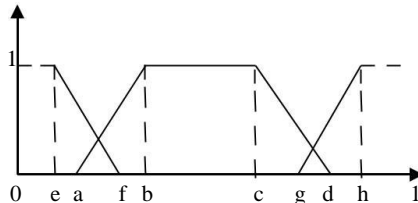
Let $IVIFS(X)$ denotes the set of all IVIFSs in X . An IVIFS value is denoted by (μ, ν, η) and the degree of indeterminacy by $(\eta) = [1 - \mu - \nu]$ for convenience. And we will denote Trapezoidal Intuitionistic Fuzzy Number by $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ where (μ, ν, η) denotes the degree of membership and (ν, μ, η) denotes degree of nonmembership.

Definition 2.2 [9] Let $P, Q \in IVIFS(X)$. A subset relation is defined by $P \subseteq Q$ if and only if $(\mu_P(x) \leq \mu_Q(x), \nu_P(x) \geq \nu_Q(x) \text{ and } (\eta_P(x) \geq \eta_Q(x))$, for all $x \in X$.

Definition 2.3 [9] The complement of A is defined by $A^c = \{ (x, y) \mid (x, y) \in X \}$.

Definition 2.4 [20] Let $A = ([1, 1, 1, 1], [1, 1, 1, 1])$ and $B = ([2, 2, 2, 2], [2, 2, 2, 2])$ are two TrIFN. A subset relation is defined by $A \subseteq B$ if and only if $1 \leq 2, 1 \leq 2, 1 \leq 2, 1 \leq 2$ and $1 \geq 2, 1 \geq 2, 1 \geq 2, 1 \geq 2$ respectively.

Definition 2.5 Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be a trapezoidal intuitionistic fuzzy number. Then the and cuts of the membership and non membership functions are defined by $(\mu_{\alpha}, (\nu_{\alpha})$ and $(\mu_{\beta}), (\nu_{\beta})$.



Definition 2.6 [19] Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be an IVIFN. The accuracy function H is defined by $H(x) = \mu(x) + \nu(x)$.

We can extend this accuracy function for TrIFN by the following proposition.

Proposition 2.6.1 Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be a TrIFN. The accuracy function H_T is defined from $H(A)$ as follows

$$H_T(A) = \frac{1}{2} [(b-a) + a(c-d) + d(e-f) + f(h-g) + g] d$$

Hence, $(\mu) = \frac{-(1-\mu) + -(1-\nu)}{2} = \mu + \nu + \dots$

Definition 2.7 [15] Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be an IVIFN. The novel accuracy function M is defined by

$$(M) = \frac{-(1-\mu) + -(1-\nu)}{2} = \mu + \nu + \dots$$

We can extend this novel accuracy function for TrIFN by the following proposition.

Proposition 2.7.1 Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be a TrIFN. The novel accuracy function M_T is defined from $M(A)$ as follows

$$M_T(A) = \frac{1}{2} [(b-a) + a(c-d) + d] d + \frac{1}{2} [(e-f) + f(h-g) + g] d$$

Hence, $(\mu) = \frac{-(1-\mu) + -(1-\nu)}{2} = \mu + \nu + \dots$

Definition 2.8 [18] Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be an IVIFN. The new novel accuracy function L is defined by

We can extend this new novel accuracy function for TrIFN by the following proposition.

Proposition 2.8.1 Let $([a, b, c, d], [\alpha, \beta, \gamma, \delta])$ be a TrIFN. The new novel accuracy function L_T is defined from $L(A)$ as follows

$$L_T(A) = \frac{1}{2} [(b-a) + a(c-d) + d] d + \frac{1}{2} [(e-f) + f(h-g) + g] d + ((b-a) + a)(e-f) + f(c-d) + d(h-g) + g] d$$

Hence, $(\mu) = \frac{-(1-\mu) + -(1-\nu)}{2} = \mu + \nu + \dots$

3. Ranking by Improved Accuracy function for TrIFN

Consider that the socioeconomic environment becomes more complex, the membership and nonmembership values gathered from the information may be intervals rather than exact numbers. Also that interval may be of the trapezoidal intuitionistic fuzzy number form. So in order to solve decision problem when the given information is of TrIFNs, we introduce the improved accuracy function to rank the alternatives.

Example 3.1 Let $A_1 = ([0.1, 0.2, 0.2, 0.4], [0.2, 0.2, 0.3, 0.3])$ and $A_2 = ([0.1, 0.1, 0.3, 0.4], [0.1, 0.2, 0.3, 0.4])$ be two trapezoidal intuitionistic fuzzy values for two alternatives.

By applying Proposition 2.6.1 we obtain $Hr(A_1) = 0.475$ and $Hr(A_2) = 0.475$.

By applying Proposition 2.7.1 we obtain $M_T(A_1) = -0.3$ and $M_T(A_2) = -0.3$.

In both the above two Proposition 2.6.1 and Proposition 2.7.1, we have noticed that the novel accuracy function and the accuracy function are inversely proportional to the hesitation. Since we came to know that both are directly proportional to membership and nonmembership values, respectively. In order to overcome this illogical concept the new improved accuracy function is defined as follows

Definition 3.1.1 Let $([a, b, c, d], [e, f, g, h])$ be a TrIFN, the improved accuracy function K_T of a TrIF value, based on the hesitancy degree is defined from $K(A)$ [17] as follows

Since

$$\frac{1}{2} \left(\frac{a+b}{c+d} + \frac{a+b}{d+c} + \frac{1}{2} \left(\frac{e+f}{g+h} + \frac{e+f}{h+g} \right) \right) d$$

Hence, $K_T(A) = \dots$

Note:

- If we put $b = a, c = d$ and $f = e, g = h$, we can define the improved accuracy function for an IVIFN. [17]

By applying Definition 3.1.1 to Example 3.1, we obtain $K_T(A_1) = 0.3542$ and $K_T(A_2) = 0.3708$

Example 3.2 Let $A_1 = ([0.2, 0.3, 0.4, 0.4], [0.1, 0.2, 0.3, 0.5])$ and $A_2 = ([0.1, 0.3, 0.4, 0.5], [0.2, 0.2, 0.3, 0.4])$ be two trapezoidal intuitionistic fuzzy values for two alternatives. If our intention is to choose the best alternative, according to the accuracy function Hr , we got $Hr(A_1) = 0.6$ and $Hr(A_2) = 0.6$. And according to the novel accuracy function M_T , we got $M_T(A_1) = -0.075$ and $M_T(A_2) = -0.075$. And according to the new novel accuracy function L_T , $L_T(A_1) = 0.1733$ and $L_T(A_2) = 0.1733$.

So we cannot say which alternative is better for these cases. But when we apply Definition 3.1.1 we can obtain $K_T(A_1) = 0.4692$ and $K_T(A_2) = 0.4808$. Thus the alternative A_2 is more suitable than the alternative A_1 . But other functions which we derived from the existing functions cannot rank properly and so, we tend to do not recognize that which alternative is more good. Therefore, we will say that our proposed function for TrIFN is more practical and more sensible than the others.

Proposition 3.1.1 For any trapezoidal intuitionistic fuzzy subset $A = ([a, b, c, d], [e, f, g, h])$, the improved accuracy function $K_T(A) \in [0, 1]$.

Proposition 3.1.2 For a fuzzy subset $A = a = ([a, a, a, a], [1 - a, 1 - a, 1 - a, 1 - a])$, the improved accuracy function is $K_T(A) = a$.

In particular

If $A = ([1, 1, 1, 1], [0, 0, 0, 0])$, $K_T(A) = 1$

If $A = ([0, 0, 0, 0], [1, 1, 1, 1])$, $K_T(A) = 0$

Theorem 3.1.1 For any two TrIFSs $A \leq B$ then $K_T(A) \leq K_T(B)$

Proof $K_T(A) - K_T(B) =$

$$\frac{a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2}{2} - \frac{a_1 d_2 + a_2 d_1 + b_1 c_2 + b_2 c_1}{3} - \frac{a_2 c_2 + a_1 c_1 + b_2 d_2 + b_1 d_1 + c_2 e_2 + c_1 e_1 + d_2 f_2 + d_1 f_1 + a_2 g_2 + a_1 g_1 + b_2 h_2 + b_1 h_1}{6} - \frac{a_2 h_2 + a_1 h_1 + b_2 g_2 + b_1 g_1 + c_2 f_2 + c_1 f_1}{2} - \frac{d}{e_1 + e_2}$$

Hence,

Example 5.1 A leading company has to select one among four candidates for their company for the HR post, according to the following three criteria: (1) C_1 is the Academic Records, (2) C_2 is his/her IQ, (3) C_3 is the Fluency in English. The criterion weight is given by $\mathbf{W} = (0.35, 0.25, 0.40)$. The alternatives A_i ($i = 1, 2, 3, 4$) is to be assessed using the TrIFN by the decision maker under the above three criteria as listed in the following DM, $D_{4 \times 3} (X_{ij})$

$$D_{4 \times 3} (X_{ij})$$

Then we use the proposed approach to select the most enviable one(s). Then the accuracy matrix $R_{4 \times 3} ()$ is obtained by using (1).

$$R_{4 \times 3} K_{T_j} (x_{ij}) = \begin{matrix} & & 0.4367 & 0.4983 & 0.5167 \\ & & 0.6075 & 0.5600 & 0.4233 \\ & & 0.5650 & 0.3200 & 0.5225 \\ & & 0.5608 & 0.3450 & 0.5300 \end{matrix}$$

We can determine the values of positive and negative ideal solutions by using (2) as follows

$$d_1(A_1, A_1) = 0.3033, \quad d_2(A_1, A_2) = 0.2902, \quad d_3(A_1, A_3) = 0.2977, \quad d(A_1, A_4) = 0.2928$$

$$d_1(A_2, A_1) = 0.2857, \quad d_2(A_2, A_2) = 0.3057, \quad d_3(A_2, A_3) = 0.2986, \quad d(A_2, A_4) = 0.3014$$

We can get the closeness coefficient $C_i(A_i)$ ($i = 1, 2, 3, 4$) by using (3): $C_1(A_1) = 0.4851$, $C_2(A_2) = 0.5130$, $C_3(A_3) = 0.5008$, $C_4(A_4) = 0.5072$

Thus the four alternatives are ranked as follows A_2, A_4, A_3 and A_1 . So clearly A_2 is preferable amongst them.

Example 5.2 let us take the same problem as in [18] and make another example. The alternative A_i ($i = 1, 2, \dots, 5$) the appropriate criterion C_j ($j = 1, 2, \dots, 6$) and the criterion weight is given by $\mathbf{W} = (0.2, 0.1, 0.25, 0.1, 0.15, 0.2)$. The alternatives A_i ($i = 1, 2, 3, 4, 5$) is to be assessed using the TrIFN by the decision maker under the above six criteria as listed in the following DM, $D_{5 \times 6} (X_{ij})$

$$D_{5 \times 6} (X_{ij}) = \begin{matrix} & & 2,2,3,3 & 4,4,5,5 & 6,6,7,7 & 2,2,3,3 & 4,4,5,5 & 2,2,4,4 & 7,7,8,8 & 1,1,2,2 & 1,1,3,3 & 5,5,6,6 & 5,5,7,7 & 2,2,3,3 \\ & & 6,6,7,7 & 2,2,3,3 & 5,5,6,6 & 1,1,3,3 & 6,6,7,7 & 2,2,3,3 & 6,6,7,7 & 1,1,2,2 & 3,3,4,4 & 5,5,6,6 & 4,4,7,7 & 1,1,2,2 \\ & & 4,4,5,5 & 3,3,4,4 & 7,7,8,8 & 1,1,2,2 & 5,5,6,6 & 3,3,4,4 & 6,6,7,7 & 1,1,3,3 & 4,4,5,5 & 3,3,4,4 & 3,3,5,5 & 1,1,3,3 \\ & & 6,6,7,7 & 2,2,3,3 & 5,5,7,7 & 1,1,3,3 & 7,7,8,8 & 1,2,2,2 & 3,3,4,4 & 1,1,2,2 & 5,5,6,6 & 1,1,3,3 & 7,7,8,8 & 1,1,2,2 \\ & & 5,5,6,6 & 3,3,4,4 & 3,3,4,4 & 3,3,5,5 & 6,6,7,7 & 1,1,3,3 & 6,6,8,8 & 1,1,2,2 & 6,6,7,7 & 2,2,3,3 & 5,5,6,6 & 2,2,4,4 \end{matrix}$$

Then we use the proposed approach to get the most desirable alternative(s). Then the accuracy matrix $R_{5 \times 6} ()$ is

obtained by using (1).

$$R_{5 \times 6} K_{T_j} (x_{ij}) = \begin{matrix} & & 0.3300 & 0.7200 & 0.5700 & 0.8300 & 0.2650 & 0.7050 \\ & & 0.7200 & 0.6950 & 0.7200 & 0.7850 & 0.3900 & 0.7450 \\ & & 0.5450 & 0.8300 & 0.6100 & 0.7550 & 0.5450 & 0.5800 \\ & & 0.7200 & 0.7400 & 0.8100 & 0.5300 & 0.6950 & 0.8475 \\ & & 0.6100 & 0.4450 & 0.7550 & 0.8200 & 0.7200 & 0.6400 \end{matrix}$$

We can determine the values of positive and negative ideal solutions by using (2) as follows

$$d_1(A_1, A_1) = 0.2154, \quad d_2(A_1, A_2) = 0.1428, \quad d_3(A_1, A_3) = 0.1745, \quad d_4(A_1, A_4) = 0.1064, \quad d_5(A_1, A_5) = 0.1424$$

$$d_1(A_2, A_1) = 0.2414, \quad d_2(A_2, A_2) = 0.2995, \quad d_3(A_2, A_3) = 0.2607, \quad d_4(A_2, A_4) = 0.3311, \quad d_5(A_2, A_5) = 0.2954$$

We can get the closeness coefficient $C_i(A_i)$ ($i = 1, 2, 3, 4$) by using (3): $C_1(A_1) = 0.5285$, $C_2(A_2) = 0.6771$, $C_3(A_3) = 0.5990$, $C_4(A_4) = 0.7568$, $C_5(A_5) = 0.6747$

Thus the five alternatives are ranked as A_4, A_2, A_5, A_3 and A_1 . So clearly A_4 is preferable amongst them.

6. Conclusion

In this paper we have extended the improved accuracy function for IVIFSs into improved accuracy function for TrIFSs. And a Trapezoidal intuitionistic fuzzy TOPSIS method based on the proposed accuracy function is applied to rank the alternatives and the most suitable one(s) can be chosen in the DM process. In the end of this paper for giving the applications and the potency of the developed approach, two illustrative examples are illustrated. In the future we will continue functioning on the application of our proposed technique to different domains.

References

- [1] C. L. Hwang and K. Yoon, *Multiple Attribute Decision Making: Methods and Applications*, Springer, Berlin, Germany, 1981.
- [2] E. Triantaphyllou and C. T. Lin, "Development and evaluation of five fuzzy multiattribute decision-making methods," *International Journal of Approximate Reasoning*, vol. 14, no. 4, pp. 281–310, 1996.
- [3] C. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, vol. 114, no. 1, pp. 1–9, 2000.
- [4] S.H. Tsaur, T. Y. Chang, and C.H. Yen, "The evaluation of airline service quality by fuzzy MCDM," *Tourism Management*, vol. 23, pp. 107–115, 2002.
- [5] M. B. Gorzalczyński, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," *Fuzzy Sets and Systems*, vol. 21, no. 1, pp. 1–17, 1987.
- [6] I. B. Turksen, "Interval valued fuzzy sets based on normal forms," *Fuzzy Sets and Systems*, vol. 20, no. 2, pp. 191–210, 1986.
- [7] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
- [8] H. Bustince and P. Burillo, "Vague sets are intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 79, no. 3, pp. 403–405, 1996.
- [9] K. Atanassov and G. Gargov, "Interval-valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, pp. 1–17, 1987.
- [10] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
- [11] B. Ashtiani, F. Haghighirad, A. Makui, and G. A. Montazer, "Extension of fuzzy TOPSIS method based on interval valued fuzzy sets," *Applied Soft Computing Journal*, vol. 9, no. 2, pp. 457–461, 2009.
- [12] D. Li, "Multiattribute decision making models and methods using intuitionistic fuzzy sets," *Journal of Computer and System Sciences*, vol. 70, no. 1, pp. 73–85, 2005.
- [13] D. Li, "Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets," *Expert Systems with Applications*, vol. 37, no. 12, pp. 8673–8678, 2012.
- [14] J. Ye, "Improved method of multicriteria fuzzy decision-making based on vague sets," *Computer-Aided Design*, vol. 39, no. 2, pp. 164–169, 2007.
- [15] J. Ye, "Multicriteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment," *Expert Systems with Applications*, vol. 36, no. 3, pp. 6899–6902, 2009.
- [16] T. Chen, "Multi-criteria decision-making method with leniency reduction based on interval-valued fuzzy sets," *Journal of the Chinese Institute of Industrial Engineers*, vol. 28, no. 1, pp. 1–19, 2011.
- [17] R. Sahin, "Fuzzy multicriteria decision making method based on the improved accuracy function for interval-valued intuitionistic fuzzy sets," *Soft Comput.* 1–7 (2015).
- [18] S. Priyadarshini, A. Kousalya, "WEB SERVICE A SURVEY ON QOS BASED SELECTION USING RANKING METHODOLOGIES "International Journal of Innovations in Scientific and Engineering Research (IJISER)" ,vol4 ,no1,pp1-3,2017.
- [19] V. L. G. Nayagam, S. Muralikrish, and G. Sivaraman, "Multicriteria decision-making method based on interval-valued intuitionistic fuzzy sets," *Expert Systems with Applications*, vol. 38, no. 3, pp. 1464–1467, 2011.
- [20] Z. Xu, "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making," *Control and Decision*, vol. 22, no. 2, pp. 1179–1187, 2007.
- [21] V.L.G. Nayagam, S. Jeevaraj, Geetha Sivaraman, Ranking of incomplete trapezoidal information, *Soft comput.* (2016), DOI 10.1007/s00500-016-2256-1.

