



EFFECTS OF STRATIFICATION IN RECTANGULAR CHANNEL PROBLEMS

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Abstract: In this paper we consider the flow of density stratified fluid flow through rectangular channels due to periodic suction and injection through the porous wall. The mathematical modelling of the problem is presented by considering the stratification in the unperturbed state as a linear function of y only. The various combinations of parameters like magnetohydrodynamics, narrow channel approximations and variable suction and injections with stratification are discussed in this paper. The effects of stratification on all these parameters are discussed through graphical representations of axial and transverse velocity profiles. The results of the discussion shows that stratification opposes the flow in vertical direction and reverses the flow at the center of the channel in order to realign the structure of the fluid.

1. INTRODUCTION

Motion of Fluid affected by the density and viscosity differences in the fluid is classified as stratified flow. The fluid in which the density ρ is a function of space (x , y & z) and (sometimes) time ' t ' is called as stratified fluid. Examples include stratification due to temperature differences of lakes and sea water, stratification due to salt contents in ocean river meeting places, groundwater reservoirs and sea, density variation mixtures in industrial, food and manufacturing processing, heterogeneity of the atmosphere etc. A report by Long et al [8], [9] contains many of the developments in this area of study.

Compared to non stratified flow, little research has been done on viscous, continuously non homogeneous fluid flow. In order to explain such phenomena as 'upstream-wakes' and 'blocking', experimental and theoretical work has been carried out by Long and others. Exact solution for the commencement of Hagen – Poiseuille flow of a homogeneous viscous fluid in a circular cylinder was obtained by Szymanski [16]. The solution shows that under the influence of difference of pressure at the ends of the cylinder, the central portion of the fluid has an approximately constant velocity across the section but increasing with time, whereas at the cylindrical boundary friction retards the motion of the fluid. The periodic motion of a homogeneous viscous fluid in a long cylinder under the influence of periodic pressure gradient was investigated experimentally by Richardson and Tyler [13] and theoretically by Sexl [14]. In this case also the velocity distribution is of parabolic form.

Forced Oscillation in an inviscous heterogeneous fluid have been discussed by many authors Krishna and Sharma [6], Hendershott [4], Rao and Rao [12], Sharma and Naidu [15], Naidu [5] and Channabassappa and Ranganna [1]. But there is wide scope of research on the oscillations in heterogeneous viscous fluid. Gupta and Goyal [3] & Kumar et al [7] studied the flow of MHD stratified flow through porous medium. Dass et al [2] analysed the influences of thermal source and variable magnetic field on unsteady hydromagnetic flow of a inviscid heterogeneous fluid past a flat porous moving plate in slip flow regime. Khandelwal and Jain [10] considered unsteady MHD flow of heterogeneous fluid through pervious medium over moving plate in slip flow regime.

By assuming velocity profiles as function of t and y , deriving similarity solution to velocity profiles Prakash [11] analysed heat transfer in MHD flow of dusty viscoelastic stratified fluid in porous medium under variable viscosity. In this paper we analyse the flow of density stratified fluid flow through rectangular channels due to periodic suction and injection through the porous wall. The mathematical modelling of the problem is presented by considering the stratification in the unperturbed state as a linear function of y only. The theoretical description and mathematical modelling of stratification along with explanation to Brunt Vaisala frequency are presented in the following section.

2. FORMULATION OF MATHEMATICAL MODEL

The density differences will have a strong effect on the dynamics and mixing of heterogeneous fluids due to the presence of gravity. The atmosphere and sea water are continuously heterogeneous due to temperature changes, composition and pressure. These changes in ocean and atmosphere can lead to significant variations of density of the fluid in the vertical direction. As an example fresh water from river can rest on top of the sea water, and due to the small diffusibility, the density contrast remains for a long time. The oscillations in the fluid happens by the presence of density stratifications. The buoyancy forces are the forces that are responsible for the restoration of the fluid structure.

The variations in density along with gravitational field g result in variable buoyancy forces all over the fluid and a significant effect to the dynamical equations. This really means that light fluid will tend to move above heavy fluid. In other words, in the presence of gravity, a density stratified fluid has a preferred organization. As Yih (1980) pointed out that without gravity the stratification of a fluid have only minor effects on its behaviour. The stability of fluid at rest is basically the alignment of heavy fluid below light fluid in the presence of gravity. The density structure of stratified fluid can lead to the following three possible stability states.

(i) Stable state when $\frac{\partial \rho}{\partial y} < 0$. (ii) Neutral state when $\frac{\partial \rho}{\partial y} = 0$. (iii) Unstable state when $\frac{\partial \rho}{\partial y} > 0$. Stably stratified fluids tend to maintain their structure, and the stabilizing forces that enforce this

preferred organization are the main topics of the dynamics of stratified fluids. Neutrally stable fluids have no preferred structure and fluid particles exhibits similar resistance to motion in all directions. When heavy fluid overlays light fluid, unstable fluid structure starts. Consequently it will tend towards the stable state. Small disturbances are hard to prevent; thus, for our purposes we will consider stratification disturbances where partial derivative of ρ with respect to y is greater than zero unstable.

In the present research discussion we assume the stratification as a function of x , y and t as the flow considered are axisymmetric two dimensional flow. The equation of stratification is divided into two parts namely (i) density in the perturbed state, (ii) density in the unperturbed state. Density in the unperturbed state is always assumed to a linear function of y as the stratification is taken in y -direction. In the perturbed state it is a function of x , y and t . The equations are as follows,

$$\rho = \rho_0(y) + \rho^*(x, y, t) \quad (1)$$

$$\rho_0(y) = \rho_0'(1 - \beta y) \quad (2)$$

where ρ_0' is the mean density and β is the stratification parameter. The equation of incompressibility for stratified fluid is given by

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = 0 \quad (3)$$

Applying the Equations (1) and (2) in equation (3) and neglecting terms of order two, we get

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho^*}{\partial t} \quad (4)$$

$$\frac{\partial \rho}{\partial t} = \rho_0' \beta v \quad (5)$$

We use the above set of equations in the following chapters as the main equations for stratification. For an incompressible stratified stokes flow, the fluid particle has to satisfy the following continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

The momentum equations are

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \nabla^2 u \quad (7)$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \nabla^2 v - \rho g \quad (8)$$

When the motion has vertical small scale compared with the scale of background density we can apply Boussinesq's approximation which means taking density as constant in computing rates of change of momentum from acceleration but taking full account of the density variation when they give rise to buoyancy forces i.e. when there is a multiplying factor g in the vertical component of the momentum equation. By applying Boussinesq's approximation momentum equations becomes

$$\rho_0' \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \nabla^2 u \quad (9)$$

$$\rho_0' \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \nabla^2 v - \rho^* g \quad (10)$$

Equations (6), (3), (9), (10), (1) and (2) comprises the equation of a vertically density stratified viscous incompressible fluid.

3. BRUNT VAISALA FREQUENCY

Stratified fluid motions are often described in terms of a few dominant parameters and non-dimensional numbers. The most common of these is the buoyancy frequency. When the fluid reaches the new location say above the starting point, it is heavier than the surrounding fluid and experiences a resorting force diverted back to starting position. If we use Newton's second law to estimate the acceleration of the particle, we have $F = ma$

$$\Rightarrow -(\rho_0' \beta y) g \delta v = (\rho_0' \delta v) \frac{d^2 y}{dt^2} \Rightarrow \frac{d^2 y}{dt^2} + \beta g v = 0$$

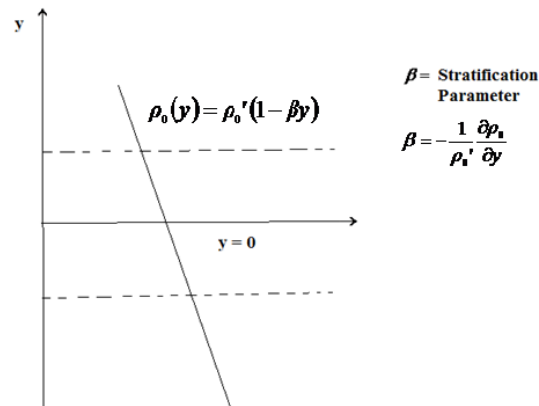


Figure 1 Natural frequency of linear stratification profile

where
$$\beta = -\frac{1}{\rho_0'} \frac{\partial \rho_0}{\partial y}.$$

This equation has the same form as the equation that describes one dimensional simple harmonic motion. The co-efficient βg gives square of the eigen frequency of the motion. We call the eigen frequency as buoyancy or Brunt-Vaisala frequency.

$$N = \left(-\frac{g}{\rho_0'} \frac{\partial \rho_0}{\partial y} \right)^{1/2}$$

$$\Rightarrow N = \sqrt{\beta g}$$

The packet of fluid moves back to its original location, but because of its acceleration under gravity it overshoots $y=0$ and starts to decelerate as it enters heavier fluid. Ultimately it swings back and in the absence of viscous damping the harmonic motion would continue with frequency N . The value of $\frac{\partial \rho_0}{\partial y}$ is always negative as we consider stratified fluid (in the vertical direction) with decreasing density in the vertical direction with gravitational force field in the opposite direction (vertically downwards). Therefore stability case (i) only is applicable to our problem and case (ii) and (iii) are not applicable in our present situation. These situations can be applied to various practical problems that arises in environmental flows and the flow involving density stratification at chemical industries and production industries. Also various parameters including magnetohydrodynamics and lubrication approximations and variable suction and injection velocities.

4. RESULTS AND DISCUSSIONS

By solving equations (6), (3), (9), (10), (1) and (2) by eliminating pressure terms and by introducing stream function after converting it into an ordinary differential equation with complex constant coefficient we get the graphical representations of transverse velocity profiles for various cases of stratification, MHD values, narrow channel approximation and variable suction and injection velocity are presented below.

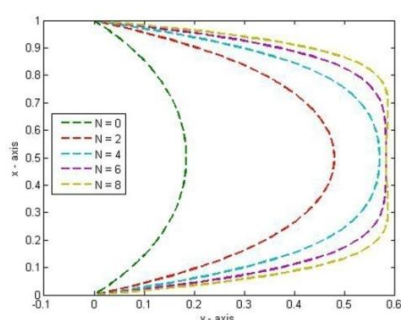


Figure 2: velocity Profile for varying stratification

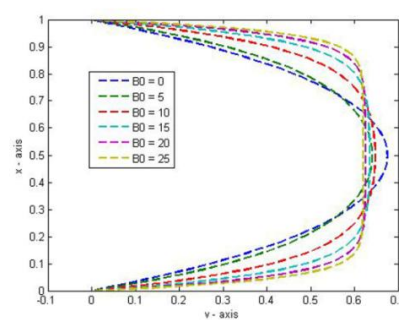


Figure 3: velocity Profile for varying MHD values

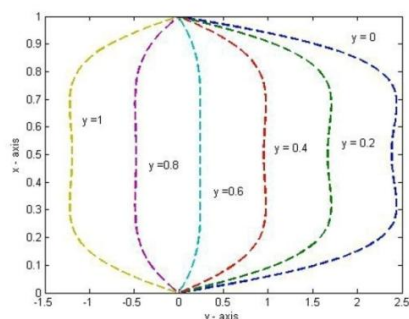


Figure 4: velocity Profile for varying stratification for narrow channels

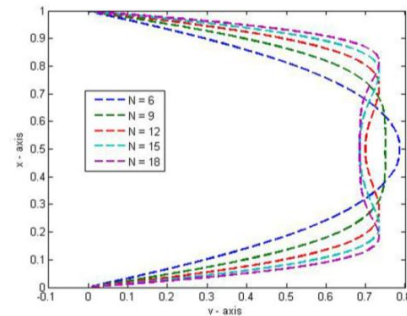


Figure 5: velocity Profile for varying stratification with variable suction and injection

Figure 2 represents the velocity profiles varying stratification parameter values in case of viscous oscillatory flow of density stratified flow between parallel vertical plates with a porous wall. Figure 3 depicts the variations in velocity profiles due to various MHD values keeping stratification parameter as constant. Figure 4 shows the changes in transverse velocity profile due to the introduction of lubrication approximation in the geometry of thee problem for varying stratification values. Figure 5 depicts the effect of variable suction and injection velocity on stratification and transverse velocity profile. From all of above cases we find the parabolic nature of the velocity profiles and the effect of stratification reversing the flow at the centre of the channel.

5. CONCLUSION

In this paper we discussed the effects of stratified fluid flow through rectangular channels under various conditions (i) with constant suction and injection velocity, (ii) under influence of Magnetohydrodynamic flow, (iii) with Lubrication approximation and (iv) with variable suction and injection velocity. The graphical representations of the velocity profiles shows that the effect of stratifications are found in general at the centre of the channel whereas no effects of it are found at the boundary. The stratification opposes the fluid flow in the direction of gravity and reverses the flow exactly nearer to centre of the channel.

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