



SUPRA SOFT REGULAR OPEN SETS, SUPRA SOFT REGULAR CLOSED SETS AND SUPRA SOFT REGULAR CONTINUITY

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Abstract—In this paper supra soft regular open sets and supra soft regular closed sets are introduced and certain properties are investigated. Further the concept of supra soft regular continuity is introduced and some of their properties are studied in detail.

Keywords—Soft sets, supra soft sets, supra soft topological spaces, supra soft regular open sets, supra soft regular continuity.

I. INTRODUCTION

There are many mathematical problems dealing with uncertainties, where classical mathematical methods are found to be inadequate to solve these problems. Various mathematical set theories such as fuzzy sets introduced by Zadeh [10] and soft sets introduced by Molodstov [6] have been developed to

solve these problems. In 1983, Mashhour et.al [4] introduced the concept of supra topological spaces. In 2011, Shabir and Naz [8] defined soft topological spaces. In 2014 El-sheikh and Abd El-latif [3] introduced the notion of supra soft topological spaces.

In 1937, Stone [9] introduced the notion of regular open sets in topological spaces. In 2016, El-shafei et. al [2] introduced supra regular open sets.

In this paper, the concept of supra soft regular open sets, supra soft regular closed sets and supra soft regular continuity are introduced and studied.

II. Preliminary definitions

Throughout this paper X denotes the initial universe and E denotes the set of parameters for the universe X .

Definition:2.1[6]

Let X be the initial Universe and $P(X)$ denote the power set of X . Let E denote the set of all parameters. Let A be a non-empty subset of E . A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the Universe X . For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set ε -approximate elements of the soft set (F, A) . The set of all soft sets over X is denoted as $S_E(X)$.

Definition 2.2[6]

For two soft sets (F, A) and (G, B) over a common universe X , we say that (F, A) is a soft subset of (G, B) if

$$(1) A \subseteq B$$

(2) For all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \subseteq (G, B)$. (F, A) is said to be a soft super set of (G, B) , if (G, B) is a soft subset of (F, A) . we denote it by $(F, A) \supseteq (G, B)$

Definition 2.3[6]

Two soft sets (F, A) and (G, B) over a common Universe X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

Definition 2.4[5]

For a soft set (F, A) over X , the complement of (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$ for all $\alpha \in A$.

Definition 2.5[6]

The union of two soft sets of (F, A) and (G, B) over the common Universe X is the soft set (H, C) , where $C = A \cup B$ and for all $e \in C$.

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$

Definition 2.6[6]

The intersection (H, C) of two soft sets (F, A) and (G, B) over a common Universe X , denote $(F, A) \cap (G, B)$ is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.7[6]

Let X be a soft topological space over X and (F, A) be a soft set over X . Then the soft closure of (F, A) , denoted by $cl(F, A)$ is the intersection of all soft closed supersets of (F, A) . Clearly (F, A) is the smallest soft closed set over X which contains (F, A) .

Definition 2.8[8]

Let X be a soft topological space over X and (F, A) be a soft set over X . Then the soft interior of (F, A) , denoted by $int(F, A)$ is the Union of all soft open subsets of (F, A) . Clearly (F, A) is the largest soft open set over X which is contained in (F, A)

Definition 2.9[8]

Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X if

$$(1) \emptyset, X \in \tau$$

(2) The union of any number of soft sets in τ belongs to τ

(3) The intersection of any two soft sets in τ belongs to τ

The triplet (X, E, τ) is called a soft topological space over X . the members of τ are said to be soft open sets in X .

Definition 2.10[7]

A subset (F, A) of a soft topological space X is called soft regular closed, if $cl(int(F, A)) = (F, A)$. The complement of soft regular closed set is soft regular open set.

Definition 2.11[1]

Let $S_E(X)$ and $S_K(Y)$ be families of soft sets $u: X \rightarrow Y$ and $p: E \rightarrow K$ be mappings. Therefore $f_{pu}: S_E(X) \rightarrow S_K(Y)$ is called a soft function.

(1) If $(F, A) \in S_E(X)$, then the image of (F, A) under f_{pu} , written as $f_{pu}(F, A)$, is a soft set in $S_K(Y)$ such that

$$f_{pu}(F, A)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F(e)), p^{-1}(k) \neq \emptyset \\ \emptyset, \text{ otherwise} \end{cases}$$

for all $k \in Y$.

(2) If $(G, B) \in S_K(Y)$, then the inverse image of (G, B) under f_{pu} , written as $f_{pu}^{-1}(G, B)$, is a soft set in $S_E(X)$ such that,

$$f_{pu}^{-1}(G, B)(e) = \begin{cases} u^{-1}(G(p(e))), p(e) \in Y \\ \emptyset, \text{ otherwise} \end{cases}$$

For all $e \in E$.

Definition 2.12[5]

Let (X, E, τ_1) and (Y, K, τ_2) be soft topological spaces and $f_{pu}: S_E(X) \rightarrow S_K(Y)$ be a function. Then the function f_{pu} is called soft continuous if $f_{pu}^{-1}(G) \in \tau_1$ for all $G \in \tau_2$.

Definition 2.13[3]

Let τ be a collection of soft sets over a universe X with a fixed set of parameters E , then $\tau^* \subseteq S_E(X)$ is called supra soft topology on X with a fixed set E if

$$(1) \emptyset, X \in \tau^*$$

(2) The soft union of any numbers of soft sets in τ^* belongs to τ^*

The triplet (X, E, τ^*) is called supra soft topological space over X .

Definition 2.14[3]

Let (X, E, τ) be a soft topological space and (X, E, τ^*) be a supra soft topological space. We say that τ^* is a supra soft topology associated with τ if $\tau \subseteq \tau^*$.

Definition 2.15[3]

Let (X, E, τ^*) be a supra soft topological space over X , then the members of τ^* are said to be supra soft open sets in X .

Definition 2.16[3]

Let (X, E, τ^*) be a supra soft topological space over X and $(F, A) \in S_E(X)$. Then the supra soft interior of (F, A) is denoted and defined as,

$$\text{int}^*(F, A) = \bigcup \left\{ (G, B) / (G, B) \text{ is supra soft open set and } (G, B) \subseteq (F, A) \right\}$$

Definition 2.17[3]

Let (X, E, τ^*) be a supra soft topological space over X and $(F, A) \in S_E(X)$. Then the supra soft closure of (F, A) is denoted and defined as,

$$\text{cl}^*(F, A) = \bigcap \left\{ (G, B) / (G, B) \text{ is supra soft closed set and } (F, A) \subseteq (G, B) \right\}$$

Remark 2.18

$$(1) \text{Cl}^*(f^c) = [\text{int}^*(f)]^c$$

$$(2) \text{Int}^*(f^c) = [\text{cl}^*(f)]^c$$

III Supra soft regular open sets and Supra soft

regular closed sets

Definition 3.1

Let (F, A) be a soft set in a supra soft topological space (X, E, τ^*) , then (F, A) is called

(1) a supra soft regular open set if $\text{int}^*(\text{cl}^*(F, A)) = (F, A)$

(2) a supra soft regular closed set if $\text{cl}^*(\text{int}^*(F, A)) = (F, A)$

Theorem 3.2

Let (F, A) be a soft set in a supra soft topological space (X, E, τ^*) , then (F, A) is a supra soft regular open set if and only if $(F, A)^c$ is a supra soft regular closed set.

Proof:

Let (F, A) be a supra soft regular open set. Then,

$$\text{int}^*(\text{cl}^*(F, A)) = (F, A)$$

$$[\text{int}^*(\text{cl}^*(F, A))]^c = (F, A)^c$$

$$\text{Cl}^*[(\text{cl}^*(F, A))^c] = (F, A)^c$$

$$\text{Cl}^*(\text{int}^*[(F, A)^c]) = (F, A)^c$$

$$\text{Cl}^*(\text{int}^*(F, A)^c) = (F, A)^c$$

(F^c, A) is a supra soft regular closed set.

Conversely, $(F, A)^c$ be a supra soft regular closed set. Consider,

$$\text{Cl}^*(\text{int}^*((F, A)^c)) = (F, A)^c$$

$$[\text{cl}^*(\text{int}^*((F, A)^c))]^c = [(F, A)^c]^c$$

$$\text{Int}^*[(\text{int}^*((F, A)^c))^c] = (F, A)$$

$$\text{Int}^*(\text{cl}^*[(F, A)^c]^c) = (F, A)$$

$$\text{Int}^*(\text{cl}^*(F, A)) = (F, A)$$

Theorem: 3.3

Every supra soft regular open set is supra soft open set

Proof:

Let (F, A) be a supra soft regular open set in (X, E, τ^*) . Then $\text{int}^*(\text{cl}^*(F, A)) = (F, A)$. By definition of supra soft interior, $\text{int}^*(F, A)$ is the largest supra soft open set contained in (F, A) . $\text{int}^*(\text{cl}^*(F, A))$ is the largest supra soft open set contained in $\text{cl}^*(F, A)$. (F, A) is a supra soft open set.

Theorem 3.4

Every supra soft regular closed set is supra soft closed set

Proof:

Let (F, A) be a supra soft regular closed set (X, E, τ^*) . $\text{Cl}^*(\text{int}^*(F, A)) = (F, A)$. By the definition of supra soft closure, $\text{cl}^*(F, A)$ is the smallest supra soft closed set containing (F, A) . $\text{cl}^*(\text{int}^*(F, A))$ is the smallest supra soft closed set containing $\text{int}^*(F, A)$. (F, A) is a supra soft closed set.

IV Supra soft Regular Continuity

Definition 4.1

Let (X, E, τ_1^*) and (Y, K, τ_2^*) be two supra soft topological spaces. Let $u: X \rightarrow Y$ and $p: E \rightarrow K$ be two functions. Then the soft mapping $f_{up}: S_E(X) \rightarrow S_K(Y)$ is said to be supra soft regular continuous if $f_{up}^{-1}((G, B))$ is supra soft regular open set over X for every soft open set (G, B) over Y .

Theorem 4.2

Let $S_E(X)$ and $S_K(Y)$ be families of soft sets. Let $f_{up}: S_E(X) \rightarrow S_K(Y)$ be a soft function. Then the following are equivalent:

- (1) f_{up} is supra soft regular continuous.
- (2) The inverse image of each soft closed set over (Y, K) is supra soft regular closed over (X, E) .

Proof: (i) $\Rightarrow$ (ii)

Let (G, B) be a soft closed set over (Y, K) . Then (G^c, B) is a soft open set over (Y, K) . Since f_{up} is a supra soft regular continuous,

$f_{up}^{-1}((G^c, B))$ is supra soft regular open over (X, E) . $(f_{up}^{-1}((G, B)))^c$ is supra soft regular open over (X, E) . Hence $f_{up}^{-1}((G, B))$ is supra soft regular closed over (X, E) .

(ii) $\Rightarrow$ (i)

Let (G, B) be a supra soft regular open set over (Y, K) . Then (G^c, B) is a supra soft regular closed set over (Y, K) . This implies that $f_{up}^{-1}((G^c, B))$ is a supra soft regular closed set over (X, E) . $(f_{up}^{-1}((G, B)))^c$ is supra soft regular closed over (X, E) . Hence $f_{up}^{-1}((G, B))$ is supra soft regular open over (X, E) .

Theorem 4.3

If a function $f_{up}: S_E(X) \rightarrow S_K(Y)$ is supra soft regular continuous iff $f_{up}(\text{reg cl}^*(G, B)) \subseteq \text{cl}^*(f_{up}(G, B))$ for every soft open set (G, B) over (X, E)

Proof:

Let $f_{up}: S_E(X) \rightarrow S_K(Y)$ be supra soft regular continuous. Let (G, B) be a soft open set over (X, E) . Then $\text{cl}^*(f_{up}(G, B))$ is soft closed over (Y, K) . Since f_{up} is supra soft regular continuous,

$f_{up}^{-1}(\text{cl}^*(f_{up}(G, B)))$ is supra soft regular closed over (X, E) .

And $(G, B) \subseteq f_{up}^{-1}(f_{up}(G, B)) \subseteq f_{up}^{-1}(\text{cl}^*(f_{up}(G, B)))$.

As $\text{reg cl}^*(G, B)$ is the smallest supra soft regular closed set containing (G, B) .

$(G, B) \subseteq \text{reg cl}^*(G, B)$

$\text{reg cl}^*(G, B) \subseteq f_{up}^{-1}(\text{cl}^*(f_{up}(G, B)))$.

Hence $f_{up}(\text{reg cl}^*(G, B)) \subseteq \text{cl}^*(f_{up}(G, B))$

Conversely,

Let (G, B) be any soft closed set over (Y, K) . Then $f_{up}^{-1}(G, B) \in S(X, E)$

$f_{up}(\text{reg cl}^*(f_{up}^{-1}(G, B))) \subseteq \text{cl}^*(f_{up}(f_{up}^{-1}(G, B)))$

$f_{up}(\text{reg cl}^*(f_{up}^{-1}(G, B))) \subseteq \text{cl}^*(G, B)$

$\text{reg cl}^*(f_{up}^{-1}(G, B)) \subseteq f_{up}^{-1}(G, B)$

In general $f_{up}^{-1}(G, B) \subseteq \text{reg cl}^*(f_{up}^{-1}(G, B))$.

Thus $f_{up}^{-1}(G, B) = \text{reg cl}^*(f_{up}^{-1}(G, B))$. Hence $f_{up}^{-1}(G, B)$ is supra soft regular closed. Therefore f_{up} is supra soft regular continuous.

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