



Fuzzy Soft K-Hausdorff Spaces

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Abstract: In this paper the definition of the fuzzy separation axiom “Hausdorff” introduced by Katsaras, A [2] is extended to fuzzy soft topological spaces in two different ways. It is shown that these two concepts are hereditary and productive.

Keywords: Fuzzy topology, Fuzzy K-Hausdorff space, Fuzzy soft topology, Fuzzy soft K-Hausdorff space.

I. INTRODUCTION

In 1965, Zadeh [12] introduced the concept of fuzzy set theory which provides us with an intuitively pleasing method of representing one form of uncertainty. In 1968, Chang [1] defined fuzzy topology and later in 1976, Lowen [3] redefined fuzzy topology in a different way. In 1999, Molodstov [6] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. Shabir and Naz [9] defined soft topology. In recent years researchers have contributed a lot towards fuzzification of soft set theory. In 2001, Maji et al. [4] initiated the concept of fuzzy soft set with some properties regarding

union, intersection and complement of a fuzzy soft set, De Morgan’s law etc. In 2011, Tanay and Kandemir [10] initially gave the concept of fuzzy soft topology using fuzzy soft sets and gave the basic notions of it by following Chang. Pazar Varol and Aygun [7] defined fuzzy soft topology in Lowen’s sense.

In section II of this paper, preliminary definitions regarding fuzzy set, soft set and fuzzy soft set are given.

In section III of this paper, the definition of the fuzzy separation axiom “Hausdorff” introduced by Katsaras, A [2] is extended to fuzzy soft topological spaces in two different ways. It is shown that these two concepts are hereditary and productive.

Throughout this paper, X denotes initial universe and E denotes the set of parameters for the universe X .

I. PRELIMINARY DEFINITIONS

Definition: 2.1 [12]

A **fuzzy set** in X is a map $f: X \rightarrow [0, 1] = I$. The family of fuzzy sets in X is denoted by I^X . Following are some basic operations on fuzzy sets. For the fuzzy sets f and g in X ,

- (1) $f = g \Leftrightarrow f(x) = g(x)$ for all $x \in X$.
- (2) $f \leq g \Leftrightarrow f(x) \leq g(x)$ for all $x \in X$.
- (3) $(f \vee g)(x) = \max \{f(x), g(x)\}$ for all $x \in X$.
- (4) $(f \wedge g)(x) = \min \{f(x), g(x)\}$ for all $x \in X$.
- (5) $f^c(x) = 1 - f(x)$ for all $x \in X$. Here f^c denotes the complement of f .
- (6) For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X .
 - (i) $(\bigvee_{\lambda \in \Lambda} f_\lambda)(x) = \bigvee_{\lambda \in \Lambda} f_\lambda(x)$
 - (ii) $(\bigwedge_{\lambda \in \Lambda} f_\lambda)(x) = \bigwedge_{\lambda \in \Lambda} f_\lambda(x)$
- (7) For any $\alpha \in I$, the constant fuzzy set α in X is a fuzzy set in X defined by $\alpha(x) = \alpha$ for all $x \in X$. 0 denotes null fuzzy set in X and 1 denotes universal fuzzy set in X .

Definition: 2.2 [12]

Let f and g be fuzzy sets in X and Y respectively. The **cartesian product** $f \otimes g$ of f and g is a fuzzy set on $X \times Y$ defined by $(f \otimes g)(x, y) = f(x) \wedge g(y)$ for each $(x, y) \in X \times Y$.

Definition: 2.3 [1]

A **fuzzy topological space** is a pair (X, τ) where X is a non empty set and τ is a family of fuzzy sets on X satisfying the following properties:

- (1) The constant functions 0 and 1 belongs to τ .
- (2) $f, g \in \tau$ implies $f \wedge g \in \tau$
- (3) $f_\lambda \in \tau$ for each $\lambda \in \Lambda$ implies $\bigvee_{\lambda \in \Lambda} f_\lambda \in \tau$.

Then τ is called a **fuzzy topology** on X . Every member of τ is called fuzzy open. g is called fuzzy closed in (X, τ) if $g^c \in \tau$.

Definition: 2.4 [11]

$X \times Y$ is the fuzzy topology having the collection $\{f \otimes g / f \in \tau_1, g \in \tau_2\}$ as a basis.

Definition: 2.5 [11]

Let $\{(X_\lambda, \tau_\lambda) / \lambda \in \Lambda\}$ be a family of fuzzy topological spaces and $X = \bigcup_{\lambda \in \Lambda} X_\lambda$. The product topology on X is the one with basic fuzzy open sets of the form $\bigcap_{\lambda \in \Lambda} f_\lambda$ where $f_\lambda \in \tau_\lambda$ and $f_\lambda = 1$ except for finitely many λ 's.

Definition: 2.6 [2]

A fuzzy topological space (X, τ) is said to be **fuzzy Hausdorff** or **fuzzy K - Hausdorff** if $\forall x, y \in X, x \neq y$, there exist $f, g \in \tau$ such that $f(x) > 0, g(y) > 0$ and $f \wedge g = 0$.

Definition: 2.7 [6]

Let $A \subseteq E$. A **soft set** f_A over X is a mapping from E to $P(X)$ i.e., $f_A : E \rightarrow P(X)$ where $P(X)$ is the power set of X .

Definition: 2.8 [4,8]

Let $A \subseteq E$. A **fuzzy soft set** $\tilde{f}_A$ over X is a mapping from E to I^X i.e., $\tilde{f}_A : E \rightarrow I^X$ where $\tilde{f}_A(e) \neq 0$ if $e \in A \subseteq E$ and $\tilde{f}_A(e) = 0$ if $e \notin A$. The family of fuzzy soft sets over X is denoted by $FS(X, E)$.

Definition: 2.9 [8]

The fuzzy soft set $\tilde{f}_A \in FS(X, E)$ is called **null fuzzy soft set** denoted by $\tilde{0}$ if for all $e \in E$, $\tilde{f}_A(e) = 0$.

Definition: 2.10 [8]

The fuzzy soft set $\tilde{f}_A \in FS(X, E)$ is called **universal fuzzy soft set** denoted by $\tilde{1}$ if for all $e \in E$, $\tilde{f}_A(e) = 1$.

Definition: 2.11 [8]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. $\tilde{f}_A$ is called a **fuzzy soft subset** of $\tilde{g}_B$ if $\tilde{f}_A(e) \leq \tilde{g}_B(e)$ for every $e \in E$ and we write $\tilde{f}_A \subseteq \tilde{g}_B$.

Definition: 2.12 [8]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. $\tilde{f}_A$ and $\tilde{g}_B$ are said to be **equal** denoted by $\tilde{f}_A = \tilde{g}_B$ if $\tilde{f}_A \subseteq \tilde{g}_B$ and $\tilde{g}_B \subseteq \tilde{f}_A$.

Definition: 2.13 [8]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. The **union** of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \vee \tilde{g}_B(e)$ for all $e \in E$ where $C = A \cup B$. Here we write $\tilde{h}_C = \tilde{f}_A \cup \tilde{g}_B$.

Definition: 2.14 [8]

Let $\tilde{f}_A, \tilde{g}_B \in FS(X, E)$. The **intersection** of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \wedge \tilde{g}_B(e)$ for all $e \in E$ where $C = A \cap B$. Here we write $\tilde{h}_C = \tilde{f}_A \cap \tilde{g}_B$.

Definition: 2.15 [8]

Let $\tilde{f}_A \in FS(X, E)$. The **complement** of $\tilde{f}_A$ denoted by $\tilde{f}_A^c$ is a fuzzy soft set defined by $\tilde{f}_A^c(e) = 1 - \tilde{f}_A(e)$ for every $e \in E$. Clearly $(\tilde{f}_A^c)^c = \tilde{f}_A$, $1^c = \tilde{0}$ and $\tilde{0}^c = 1$.

Definition: 2.16 [10]

A **fuzzy soft topological space** is a triple

$(X, E, \tilde{\tau})$ where X is a nonempty set, E is a parameter set and $\tilde{\tau}$ is a family of fuzzy soft sets over X satisfying the following properties:

- (1) $\tilde{0}, \tilde{1} \in \tilde{\tau}$
- (2) $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ then $\tilde{f}_A \tilde{\cap} \tilde{g}_B \in \tilde{\tau}$
- (3) If $\tilde{f}_{A_i} \in \tilde{\tau} \forall i \in \Lambda$ then $\bigcup_{i \in \Lambda} \tilde{f}_{A_i} \in \tilde{\tau}$

Then $\tilde{\tau}$ is called a **fuzzy soft topology** over X . Every member of $\tilde{\tau}$ is called fuzzy soft open. $\tilde{g}_B$ is called fuzzy soft closed in $(X, E, \tilde{\tau})$ if $\tilde{g}_B^c \in \tilde{\tau}$.

Definition: 2.17 [14]

Let $(X, E, \tilde{\tau})$ be a fuzzy soft topological space and Y be a nonnull subset of X . Then the **fuzzy soft subset** of $\tilde{f}_A$ over Y denoted by $(\tilde{f}_A)_Y$ is defined as follows: $(\tilde{f}_A)_Y(e) = \mathbf{1} \wedge \tilde{f}_A(e)$, for all $e \in E$.

In otherwords, $(\tilde{f}_A)_Y = \tilde{1} \tilde{\cap} \tilde{f}_A$

Definition: 2.18 [14]

Let $(X, E, \tilde{\tau})$ be a fuzzy soft topological space and Y be a nonnull subset of X . Then $(\tilde{\tau}_Y) = \{(\tilde{f}_A)_Y : \tilde{f}_A \in \tilde{\tau}\}$ is said to be **relative fuzzy soft topology** over Y and $(Y, E, (\tilde{\tau}_Y))$ is called a **fuzzy soft subspace** of $(X, E, \tilde{\tau})$.

Definition: 2.19 [7]

Let $FS(X, E)$ and $FS(Y, E')$ be two families of fuzzy soft sets over X and Y with respect to E and E' respectively. Let $\tilde{f}_A \in FS(X, E)$ and $\tilde{g}_B \in FS(Y, E')$. The **cartesian product** of $\tilde{f}_A$ and $\tilde{g}_B$ denoted by $\tilde{f}_A \tilde{\otimes} \tilde{g}_B$ is a fuzzy soft set over $X \times Y$ with respect to the parameter set $E \times E'$ defined as $\tilde{f}_A \tilde{\otimes} \tilde{g}_B : E \times E' \rightarrow I^X \times I^{Y'}$, $(\tilde{f}_A \tilde{\otimes} \tilde{g}_B)(e, e') = \tilde{f}_A(e) \otimes \tilde{g}_B(e')$ such that $\tilde{f}_A(e) \otimes \tilde{g}_B(e')$ is a fuzzy product of fuzzy sets $\tilde{f}_A(e)$ and $\tilde{g}_B(e')$ where $\tilde{f}_A(e) \otimes \tilde{g}_B(e') : X \times Y \rightarrow I$ and $(\tilde{f}_A(e) \otimes \tilde{g}_B(e'))(x, y) = \tilde{f}_A(e)(x) \wedge (\tilde{g}_B(e')(y))$.

Remark : 2.20 [7]

Let $FS(X, E)$ and $FS(Y, E')$ be two families of fuzzy soft sets over X and Y with respect to E and E' respectively. Let $\tilde{f}_A \in FS(X, E)$ and $\tilde{g}_B \in FS(Y, E')$. Then

- (1) $\tilde{1} \tilde{\otimes} \tilde{1} = \tilde{1}$ denotes the universal fuzzy soft set over $X \times Y$ with respect to $E \times E'$.
- (2) $\tilde{f}_A \tilde{\otimes} \tilde{0} = \tilde{0} \tilde{\otimes} \tilde{g}_B = \tilde{0} = \tilde{0} \tilde{\otimes} \tilde{0}$
- (3) $\forall e \in E, e' \in E'$,

$$((\tilde{f}_A \tilde{\otimes} \tilde{1})(e, e'))(x, y) = (\tilde{f}_A(e))(x),$$

$$((\tilde{1} \tilde{\otimes} \tilde{g}_B)(e, e'))(x, y) = (\tilde{g}_B(e'))(y)$$

where $x \in X$ and $y \in Y$.

Definition: 2.21 [7]

Let $(X_1, E_1, \tilde{\tau}_1)$ and $(X_2, E_2, \tilde{\tau}_2)$ be two fuzzy soft topological spaces. The **fuzzy soft product topology** $\tilde{\tau}_1 \tilde{\otimes} \tilde{\tau}_2$ over $X_1 \times X_2$ with respect to $E_1 \times E_2$ is the fuzzy soft topology having the collection $\{(\tilde{f}_A \tilde{\otimes} \tilde{g}_B) / \tilde{f}_A \in \tilde{\tau}_1, \tilde{g}_B \in \tilde{\tau}_2\}$ as a basis.

II. FUZZY SOFT K-HAUSDORFF SPACES

Definition: 3.1

A fuzzy soft topological space $(X, E, \tilde{\tau})$ is said to be **fuzzy soft K-Hausdorff space of type 1 denoted by $(FK-H)_1$** if for every $e_1, e_2 \in E, e_1 \neq e_2$ there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(e_1) > \mathbf{0}$, $\tilde{g}_B(e_2) > \mathbf{0}$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}$.

Definition: 3.2

For a fixed $e \in E$, a fuzzy soft topological space $(X, E, \tilde{\tau})$ is said to be **fuzzy soft K-Hausdorff space of type 2 denoted by $(FK-H)_2$** if for every $x, y \in X, x \neq y$ there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(e)(x) > 0$, $\tilde{g}_B(e)(y) > 0$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}$.

Theorem: 3.3

- (i) Fuzzy soft subspace of a $(FK-H)_1$ space is $(FK-H)_1$.
- (ii) Fuzzy soft subspace of a $(FK-H)_2$ space is $(FK-H)_2$.

Proof:

- (i) Let $(X, E, \tilde{\tau})$ is a $(FK-H)_1$ space. Let Y be a nonnull subset of X .

Let $(Y, E, (\tilde{\tau})_Y)$ be a fuzzy soft subspace of $(X, E, \tilde{\tau})$ where $(\tilde{\tau})_Y = \{(\tilde{f}_A)_Y : \tilde{f}_A \in \tilde{\tau}\}$ is the relative fuzzy soft topology over Y .

Consider $e_1, e_2 \in E, e_1 \neq e_2$.

Since $(X, E, \tilde{\tau})$ is $(FK-H)_1$, there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(e_1) > \mathbf{0}$, $\tilde{g}_B(e_2) > \mathbf{0}$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}$.

Therefore $(\tilde{f}_A)_Y, (\tilde{g}_B)_Y \in (\tilde{\tau})_Y$

$$\begin{aligned} \text{Also } (\tilde{f}_A)_Y(e_1) &= (\tilde{1} \tilde{\cap} \tilde{f}_A)(e_1) \\ &= \tilde{1}(e_1) \wedge \tilde{f}_A(e_1) \end{aligned}$$

$$\begin{aligned}
&= \tilde{f}_A(e_1) > 0 \\
\therefore (\tilde{f}_A)_Y(e_1) &> 0 \\
\text{Similarly } (\tilde{g}_B)_Y(e_2) &> 0 \\
((\tilde{f}_A)_Y \tilde{\cap} (\tilde{g}_B)_Y)(e)(x) \\
&= ((\tilde{1} \tilde{\cap} \tilde{f}_A) \tilde{\cap} (\tilde{1} \tilde{\cap} \tilde{g}_B))(e)(x) \\
&= (\tilde{1} \tilde{\cap} \tilde{f}_A)(e)(x) \wedge (\tilde{1} \tilde{\cap} \tilde{g}_B)(e)(x) \\
&= (\tilde{1}(e)(x) \wedge f_A(e)(x)) \wedge (\tilde{1}(e)(x) \wedge \tilde{g}_B(e)(x)) \\
&= f_A(e)(x) \wedge \tilde{g}_B(e)(x) \\
&= (\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e)(x) \\
&= \tilde{0}(e)(x) \\
\therefore (\tilde{f}_A)_Y \tilde{\cap} (\tilde{g}_B)_Y &= \tilde{0} \\
\text{Hence } (Y, E, (\tilde{\tau})_Y) &\text{ is } (FK-H)_1. \\
\text{(ii) Let } (X, E, \tilde{\tau}) &\text{ is a } (FK-H)_2 \text{ space. Let } Y \text{ be a} \\
&\text{nonnull subset of } X. \\
\text{Let } (Y, E, (\tilde{\tau})_Y) &\text{ be a fuzzy soft subspace of } (X, E, \tilde{\tau}). \\
\text{Fix } e \in E. \text{ Consider } x, y \in Y, x \neq y \\
\text{Therefore } x, y \in X. \\
\text{Since } (X, E, \tilde{\tau}) &\text{ is } (FK-H)_2, \text{ there exist } \tilde{f}_A, \tilde{g}_B \in \tilde{\tau} \\
&\text{such that } \tilde{f}_A(e)(x) > 0, \tilde{g}_B(e)(x) > 0 \text{ and} \\
&\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}. \\
\text{Therefore } (\tilde{f}_A)_Y, (\tilde{g}_B)_Y &\in (\tilde{\tau})_Y \\
\text{Also } (\tilde{f}_A)_Y(e)(x) &= (\tilde{1} \tilde{\cap} \tilde{f}_A)(e)(x) \\
&= \tilde{1}(e)(x) \wedge \tilde{f}_A(e)(x) \\
&= \tilde{f}_A(e)(x) > 0 \\
\therefore (\tilde{f}_A)_Y(e)(x) &> 0 \\
\text{Similarly } (\tilde{g}_B)_Y(e)(x) &> 0 \\
((\tilde{f}_A)_Y \tilde{\cap} (\tilde{g}_B)_Y)(e)(x) \\
&= ((\tilde{1} \tilde{\cap} \tilde{f}_A) \tilde{\cap} (\tilde{1} \tilde{\cap} \tilde{g}_B))(e)(x) \\
&= (\tilde{1} \tilde{\cap} \tilde{f}_A)(e)(x) \wedge (\tilde{1} \tilde{\cap} \tilde{g}_B)(e)(x) \\
&= (\tilde{1}(e)(x) \wedge f_A(e)(x)) \wedge (\tilde{1}(e)(x) \wedge \tilde{g}_B(e)(x)) \\
&= \tilde{f}_A(e)(x) \wedge \tilde{g}_B(e)(x) \\
&= (\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e)(x) \\
&= \tilde{0}(e)(x) \\
\therefore (\tilde{f}_A)_Y \tilde{\cap} (\tilde{g}_B)_Y &= \tilde{0} \\
\text{Hence } (Y, E, (\tilde{\tau})_Y) &\text{ is } (FK-H)_2.
\end{aligned}$$

Definition: 3.4

Let $(X, E, \tilde{\tau})$ be a fuzzy soft topological space and $G \subseteq E$. Then $(X, G, \tilde{\tau}_G)$ is called a **fuzzy soft p - subspace** of $(X, E, \tilde{\tau})$ relative to the parameter set G where $\tilde{\tau}_G = \{(\tilde{f}_A)/G : \tilde{f}_A \in \tilde{\tau}\}$ and $(\tilde{f}_A)/G$ is the restriction map on G .

Theorem: 3.5

- (i) Fuzzy soft p - subspace of a $(FK-H)_1$ space is $(FK-H)_1$.
- (ii) Fuzzy soft p - subspace of a $(FK-H)_2$ space is $(FK-H)_2$.

Proof:

- (i) Let $(X, E, \tilde{\tau})$ is a $(FK-H)_1$ space.

Let $G \subseteq E$.

Let $(X, G, (\tilde{\tau})_G)$ be a fuzzy soft subspace of

$(X, E, \tilde{\tau})$ where $(\tilde{\tau})_G = \{(\tilde{f}_A)/G : \tilde{f}_A \in \tilde{\tau}\}$ relative to the parameter set G .

Consider $g_1, g_2 \in G$ such that $g_1 \neq g_2$ then $g_1, g_2 \in E$ there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(g_1) > 0$, $\tilde{g}_B(g_2) > 0$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}$

Therefore $(\tilde{f}_A)/G, (\tilde{g}_B)/G \in (\tilde{\tau})_G$

Also $((\tilde{f}_A)/G)(g_1) = \tilde{f}_A(g_1) > 0$

$((\tilde{g}_B)/G)(g_2) = \tilde{g}_B(g_2) > 0$

and $((\tilde{f}_A)/G) \tilde{\cap} ((\tilde{g}_B)/G)(g)(x)$

$= ((\tilde{f}_A)/G)(g)(x) \wedge ((\tilde{g}_B)/G)(g)(x)$

$= (\tilde{f}_A)(g)(x) \wedge (\tilde{g}_B)(g)(x)$

$= (\tilde{f}_A \tilde{\cap} \tilde{g}_B)(g)(x)$

$= \tilde{0}(g)(x)$

$\therefore ((\tilde{f}_A)/G) \tilde{\cap} ((\tilde{g}_B)/G) = \tilde{0}$

Hence $(X, G, (\tilde{\tau})_G)$ is $(FK-H)_1$.

- (ii) Let $(X, E, \tilde{\tau})$ is a $(FK-H)_2$ space.

Let $G \subseteq E$. Let $(X, G, (\tilde{\tau})_G)$ be a fuzzy soft p - subspace of $(X, E, \tilde{\tau})$

Fix $g \in G \Rightarrow g \in E$.

Consider $x, y \in X$ such that $x \neq y$, then there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(g)(x) > 0$, $\tilde{g}_B(g)(y) > 0$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}$

Therefore $(\tilde{f}_A)/G, (\tilde{g}_B)/G \in (\tilde{\tau})_G$

Also $((\tilde{f}_A)/G)(g)(x) = \tilde{f}_A(g)(x) > 0$

$((\tilde{g}_B)/G)(g)(x) = \tilde{g}_B(g)(x) > 0$

and $((\tilde{f}_A)/G) \tilde{\cap} ((\tilde{g}_B)/G)(g)(x)$

$= ((\tilde{f}_A)/G)(g)(x) \wedge ((\tilde{g}_B)/G)(g)(x)$

$= (\tilde{f}_A)(g)(x) \wedge (\tilde{g}_B)(g)(x)$

$= (\tilde{f}_A \tilde{\cap} \tilde{g}_B)(g)(x)$

$= \tilde{0}(g)(x)$

$\therefore ((\tilde{f}_A)/G) \tilde{\cap} ((\tilde{g}_B)/G) = \tilde{0}$

Hence $(X, G, \tilde{\tau}_G)$ is $(FK-H)_2$.

Theorem: 3.6

- (i) Product of two $(FK-H)_1$ spaces is $(FK-H)_1$.

(ii) Product of two (FK-H)₂ spaces is (FK-H)₂.

Proof:

(i) Let $(X_1, E_1, \tilde{\tau}_1)$ and $(X_2, E_2, \tilde{\tau}_2)$ be two (FK-H)₁ spaces.

Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}_1$ such that $\tilde{f}_A(e_1) > 0, \tilde{g}_B(e_2) > 0$ and $\tilde{f}_A \cap \tilde{g}_B = \tilde{0}$

Therefore $\tilde{f}_A \otimes \tilde{1}, \tilde{g}_B \otimes \tilde{1} \in \tilde{\tau}_1 \otimes \tilde{\tau}_2$.

$(\tilde{f}_A \otimes \tilde{1})(e_1, k_1) = \tilde{f}_A(e_1) \otimes \tilde{1}(k_1) > 0$.

$(\tilde{g}_B \otimes \tilde{1})(e_2, k_2) = \tilde{g}_B(e_2) \otimes \tilde{1}(k_2) > 0$.

$\tilde{f}_A \cap \tilde{g}_B = \tilde{0}$

$\Rightarrow (\tilde{f}_A \cap \tilde{g}_B)(e)(x) = 0$ for all $e \in E$, for all $x \in X$

$\Rightarrow \tilde{f}_A(e)(x) \wedge \tilde{g}_B(e)(x) = 0$ for all $e \in E$
for all $x \in X$

$\Rightarrow$ either $\tilde{f}_A(e)(x) = 0$ or $\tilde{g}_B(e)(x) = 0$
for all $e \in E$, for all $x \in X$

$\Rightarrow$ either $\tilde{f}_A(e)(x) \wedge \tilde{1}(k)(y) \neq 0$ or

$\tilde{g}_B(e)(x) \wedge \tilde{1}(k)(y) = 0$ for all $e \in E$, for all $k \in K$,
for all $x \in X$ and for all $y \in Y$

$\Rightarrow$ either $(\tilde{f}_A \otimes \tilde{1})(e, k)(x, y) = 0$ or

$(\tilde{g}_B \otimes \tilde{1})(e, k)(x, y) = 0$ for all $(e, k) \in E \times K$
for all $(x, y) \in X \times Y$.

$\Rightarrow (\tilde{f}_A \otimes \tilde{1}) \cap (\tilde{g}_B \otimes \tilde{1}) = \tilde{0}$

The proof in the other case is similar.

Hence $(X \times Y, E \times K, \tilde{\tau}_1 \otimes \tilde{\tau}_2)$ is a (FK-H)₁.

(ii) Let $(X_1, E_1, \tilde{\tau}_1)$ and $(X_2, E_2, \tilde{\tau}_2)$ be two (FK-H)₂ spaces.

Fix $(e, k) \in E \times K$ where $e \in E, k \in K$.

Consider two distinct points $(x_1, y_1), (x_2, y_2) \in X \times Y$. Either $x_1 \neq x_2$ or $y_1 \neq y_2$.

Assume $x_1 \neq x_2$. Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that

$\tilde{f}_A(e)(x) > 0, \tilde{g}_B(e)(x) > 0$ and $\tilde{f}_A \cap \tilde{g}_B = \tilde{0}$

Therefore $\tilde{f}_A \otimes \tilde{1}, \tilde{g}_B \otimes \tilde{1} \in \tilde{\tau}_1 \otimes \tilde{\tau}_2$.

$(\tilde{f}_A \otimes \tilde{1})(e, k)(x_1, y_1) = \tilde{f}_A(e)(x_1) \wedge \tilde{1}(k)(y_1)$
 $= \tilde{f}_A(e)(x_1) > 0$

$(\tilde{g}_B \otimes \tilde{1})(e, k)(x_2, y_2) = \tilde{g}_B(e)(x_2) \wedge \tilde{1}(k)(y_2)$
 $= \tilde{g}_B(e)(x_2) > 0$

$\tilde{f}_A \cap \tilde{g}_B = \tilde{0}$

$\Rightarrow (\tilde{f}_A \cap \tilde{g}_B)(e)(x) = 0$ for all $e \in E$, for all $x \in X$

$\Rightarrow \tilde{f}_A(e)(x) \wedge \tilde{g}_B(e)(x) = 0$ for all $e \in E$
for all $x \in X$

$\Rightarrow$ either $\tilde{f}_A(e)(x) = 0$ or $\tilde{g}_B(e)(x) = 0$

for all $e \in E$, for all $x \in X$

$\Rightarrow$ either $\tilde{f}_A(e)(x) \wedge \tilde{1}(k)(y) \neq 0$ or

$\tilde{g}_B(e)(x) \wedge \tilde{1}(k)(y) = 0$ for all $e \in E$, for all $k \in K$,
for all $x \in X$ and for all $y \in Y$

$\Rightarrow$ either $(\tilde{f}_A \otimes \tilde{1})(e, k)(x, y) = 0$ or

$(\tilde{g}_B \otimes \tilde{1})(e, k)(x, y) = 0$ for all $(e, k) \in E \times K$
for all $(x, y) \in X \times Y$

$\Rightarrow (\tilde{f}_A \otimes \tilde{1}) \cap (\tilde{g}_B \otimes \tilde{1}) = \tilde{0}$

The proof in the other case is similar.

Hence $(X \times Y, E \times K, \tilde{\tau}_1 \otimes \tilde{\tau}_2)$ is a (FK-H)₂ space.

Definition: 3.7

Let $\{(X_\lambda, E_\lambda, \tilde{\tau}_\lambda) / \lambda \in \Lambda\}$ be families of fuzzy soft topological spaces. Let $X = \prod_{\lambda \in \Lambda} X_\lambda$ and

$E = \prod_{\lambda \in \Lambda} E_\lambda$. The fuzzy soft product topology $\tilde{\tau}$ over (X, E) is the one with basic fuzzy soft open sets of the form $\prod_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}$ where $\tilde{f}_{A_\lambda} \in \tilde{\tau}_\lambda$ and $\tilde{f}_{A_\lambda} = \tilde{1}$ except for finitely many λ 's. Here

$((\prod_{\lambda \in \Lambda} \tilde{f}_{A_\lambda})(e_\lambda)(x_\lambda))_{\lambda \in \Lambda} = \bigwedge_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}(e_\lambda)(x_\lambda)$

for every $(x_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} X_\lambda$ and

for every $(e_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} E_\lambda$.

Theorem: 3.8

- (i) Arbitrary product of (FK-H)₁ spaces is (FK-H)₁.
- (ii) Arbitrary product of (FK-H)₂ spaces is (FK-H)₂.

Proof:

(i) Let $(X_\lambda, E_\lambda, \tilde{\tau}_\lambda) / \lambda \in \Lambda$ be a collection of (FK-H)₁ spaces.

Let $X = \prod_{\lambda \in \Lambda} X_\lambda$ and $E = \prod_{\lambda \in \Lambda} E_\lambda$ and $\tilde{\tau} = \prod_{\lambda \in \Lambda} \tilde{\tau}_\lambda$.

Consider two distinct points

$(e_\lambda)_{\lambda \in \Lambda}, (k_\lambda)_{\lambda \in \Lambda} \in \prod_{\lambda \in \Lambda} E_\lambda$.

Assume $e_\mu \neq k_\mu$ for some $\mu \in \Lambda$, there exist

$\tilde{f}_{A_\mu}, \tilde{g}_{B_\mu} \in \tilde{\tau}_\mu$ such that $\tilde{f}_{A_\mu}(e_\mu) > 0, \tilde{g}_{B_\mu}(k_\mu) > 0$

and $\tilde{f}_{A_\mu} \cap \tilde{g}_{B_\mu} = \tilde{0}$.

Let $\tilde{f}_A = \prod_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}$ where $\tilde{f}_{A_\lambda} = \tilde{1}$ for $\lambda \neq \mu$ and

$\tilde{g}_B = \prod_{\lambda \in \Lambda} \tilde{g}_{B_\lambda}$ where $\tilde{g}_{B_\lambda} = \tilde{1}$ for $\lambda \neq \mu$.

Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau} = \prod_{\lambda \in \Lambda} \tilde{\tau}_\lambda$.

Also

$\tilde{f}_A((e_\lambda)_{\lambda \in \Lambda}) = (\prod_{\lambda \in \Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda \in \Lambda}$

$= \bigwedge_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}(e_\lambda)$

$= \tilde{f}_{A_\mu}(e_\mu) > 0$

$$\therefore \tilde{f}_A((e_\lambda)_{\lambda \in \Lambda}) > 0$$

$$\text{Similarly } \tilde{g}_B((e_\lambda)_{\lambda \in \Lambda}) > 0$$

$$\begin{aligned} & ((\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \\ &= ((\Pi_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}) \tilde{\cap} (\Pi_{\lambda \in \Lambda} \tilde{g}_{B_\lambda}))(e_\lambda)_{\lambda \in \Lambda}(x_\lambda)_{\lambda \in \Lambda} \\ &= ((\Pi_{\lambda \in \Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \wedge ((\Pi_{\lambda \in \Lambda} \tilde{g}_{B_\lambda})(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \\ &= \tilde{f}_{A_\mu}(e_\mu)(x_\mu) \wedge \tilde{g}_{B_\mu}(e_\mu)(x_\mu) \\ &= ((\tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B_\mu})(e_\mu))(x_\mu) \\ &= (\tilde{0}(e_\mu))(x_\mu) \\ &= (\Pi_{\lambda \in \Lambda} \tilde{0})(e_\lambda)_{\lambda \in \Lambda}(x_\lambda)_{\lambda \in \Lambda} \\ &= \tilde{0}(e_\lambda)_{\lambda \in \Lambda}(x_\lambda)_{\lambda \in \Lambda} \\ \therefore \tilde{f}_A \tilde{\cap} \tilde{g}_B &= \tilde{0} \end{aligned}$$

Hence Arbitrary product of (FK-H)₁ spaces is (FK-H)₁.

(ii) Let $\{(X_\lambda, E_\lambda, \tilde{\tau}_\lambda) / \lambda \in \Lambda\}$ be a collection of (FK-H)₂ spaces.

Let $X = \Pi_{\lambda \in \Lambda} X_\lambda$, $E = \Pi_{\lambda \in \Lambda} E_\lambda$ and $\tilde{\tau} = \Pi_{\lambda \in \Lambda} \tilde{\tau}_\lambda$.

Fix $(e_\lambda)_{\lambda \in \Lambda} \in E = \Pi_{\lambda \in \Lambda} E_\lambda$ where $e_\lambda \in E_\lambda$.

Consider two distinct points

$$(x_\lambda)_{\lambda \in \Lambda}, (y_\lambda)_{\lambda \in \Lambda} \in X = \Pi_{\lambda \in \Lambda} X_\lambda.$$

Assume $X_\mu \neq y_\mu$ for some $\mu \in \Lambda$, there exist

$$\tilde{f}_{A_\mu}, \tilde{g}_{B_\mu} \in \tilde{\tau}_\mu \text{ such that } \tilde{f}_{A_\mu}(e_\mu)(x_\mu) > 0,$$

$$\tilde{g}_{B_\mu}(e_\mu)(y_\mu) > 0 \text{ and } \tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B_\mu} = \tilde{0}.$$

Let $\tilde{f}_A = \Pi_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}$ where $\tilde{f}_{A_\lambda} = \tilde{1}$ for $\lambda \neq \mu$ and

$$\tilde{g}_B = \Pi_{\lambda \in \Lambda} \tilde{g}_{B_\lambda} \text{ where } \tilde{g}_{B_\lambda} = \tilde{1} \text{ for } \lambda \neq \mu.$$

Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau} = \Pi_{\lambda \in \Lambda} \tilde{\tau}_\lambda$. Also

$$\begin{aligned} (\tilde{f}_A(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} &= ((\Pi_{\lambda \in \Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \\ &= \bigwedge_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}(e_\lambda)(x_\lambda) \\ &= \tilde{f}_{A_\mu}(e_\mu)(x_\mu) > 0 \end{aligned}$$

$$\begin{aligned} (\tilde{g}_B(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} &= ((\Pi_{\lambda \in \Lambda} \tilde{g}_{B_\lambda})(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \\ &= \bigwedge_{\lambda \in \Lambda} \tilde{g}_{B_\lambda}(e_\lambda)(x_\lambda) \\ &= \tilde{g}_{B_\mu}(e_\mu)(x_\mu) > 0 \end{aligned}$$

$$\begin{aligned} & ((\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \\ &= ((\Pi_{\lambda \in \Lambda} \tilde{f}_{A_\lambda}) \tilde{\cap} (\Pi_{\lambda \in \Lambda} \tilde{g}_{B_\lambda}))(e_\lambda)_{\lambda \in \Lambda}(x_\lambda)_{\lambda \in \Lambda} \\ &= ((\Pi_{\lambda \in \Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \wedge ((\Pi_{\lambda \in \Lambda} \tilde{g}_{B_\lambda})(e_\lambda)_{\lambda \in \Lambda})(x_\lambda)_{\lambda \in \Lambda} \\ &= \tilde{f}_{A_\mu}(e_\mu)(x_\mu) \wedge \tilde{g}_{B_\mu}(e_\mu)(x_\mu) \\ &= ((\tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B_\mu})(e_\mu))(x_\mu) \\ &= (\tilde{0}(e_\mu))(x_\mu) \\ &= (\Pi_{\lambda \in \Lambda} \tilde{0})(e_\lambda)_{\lambda \in \Lambda}(x_\lambda)_{\lambda \in \Lambda} \end{aligned}$$

$$= \tilde{0}(e_\lambda)_{\lambda \in \Lambda}(x_\lambda)_{\lambda \in \Lambda}$$

$$\therefore \tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}$$

Hence $(X, E, \tilde{\tau})$ is (FK-H)₂ space.

IV. CONCLUSIONS

In this paper, the concept fuzzy soft K - Hausdorff is introduced and some basic properties regarding this concept are proved.

REFERENCES

- [1] C. L. Chang, "Fuzzy Topological Spaces", *Journal of Mathematical Analysis and Application*, vol. 24, no. 1, pp. 182-190, 1968.
- [2] A. K. Katsaras, "Ordered Fuzzy Topological Spaces", *Journal of Mathematical Analysis and Applications*, vol. 84, pp. 44 -58, 1981.
- [3] R. Lowen, "Fuzzy Topological Spaces and Fuzzy Compactness", *Journal of Mathematical Analysis and Applications*, vol. 56, pp. 621-633, 1976.
- [4] P. K. Maji, R. Biswas and A. R. Roy, "Fuzzy Soft Sets", *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589-602, 2001.
- [5] S. Abinaya 2G.Arulkumaran, "Detecting Black Hole Attack Using Fuzzy Trust Approach in MANET", *International Journal of Innovations in Scientific and Engineering Research (IJISER)*, Vol.4, No.3, pp.102-108, 2017.
- [6] P. K. Maji, R. Biswas and A. R. Roy, "Soft Set Theory", *Computers and Mathematics with Applications*, vol. 45, no. 4-5, pp. 555-562, 2003.
- [7] D. Molodstov, "Soft Set Theory – First Results", *Computers and Mathematics with Applications*, vol. 37, no. 4-5, pp. 19-31, 1999.
- [8] B. Pazar Varol and H. Aygun, "Fuzzy Soft Topology", *Hacettepe Journal of Mathematics and Statistics*, vol. 41, no. 3, pp. 407-419, 2012.
- [9] S. Roy and T. K. Samanta, "A Note on Fuzzy Soft Topological Spaces", *Annals of Fuzzy Mathematics and Informatics*, vol. 3, no. 2, pp. 305-311, 2012.
- [10] MAI Navid, NH Niloy, "Data Mining Algorithm on Fuzzy Weighted Association Rules", *International Research Journal of Multidisciplinary Science & Technology (irjmrs)*, Vol.1, No.7, pp.480-488, 2016.
- [11] M. Shabir and M. Naz, "On Soft Topological Spaces", *Computers and Mathematics with Applications*, vol. 61, no. 7, pp. 1786-1799, 2011.
- [12] B. Tanay and M. B. Kandemir, "Topological Structure of Fuzzy Soft Sets", *Computers and Mathematics with Applications*, vol. 61, no. 10, pp. 2952-2957, 2011.
- [13] C. K. Wong, "Fuzzy Topology : Product and Quotient Theorems", *Journal of Mathematical Analysis and Applications*, vol. 45, no. 2, pp. 512-521, 1974.
- [14] L. A. Zadeh, "Fuzzy Sets", *Information and Control*, vol. 8, pp. 338-353, 1965.
- [15] A. Zahedi Khameneh, A. Kilieman and A. R. Salleh, "Fuzzy Soft Product Topology", *Annals of Fuzzy Mathematics and Informatics*, vol. 10, no. 10, pp. 1-20, 2013.
- [16] A.Kharal and B.Ahmad, "Mappings on Fuzzy Soft Classes", *Advances in Fuzzy Systems*, vol.2009, Article ID 407890, 6 pages, 2009.

