



A STUDY ON SPECTRAL LEAKAGE AND ITS REDUCTION WITH THE UTILITY OF WINDOWS REVISITED

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Abstract: In general, the discrete Fourier transform (DFT) contains two error sources, namely picket-fence effect and spectral leakage. The first one, i.e. the picket-fence impact can be minimized by increasing the number of frequency points. This study mainly focuses on the spectral leakage effect and its reduction by using various windows. For the case of without leakage, a sinusoid signal with five cycles per second portion having 512 points is considered; for the spectral leakage example, a sinusoid signal with 4.75 cycles per second portion having 485 points is considered. Simulation results are obtained and compared with different windows such as Hamming, Hanning, Blackman, etc. using a Matlab software tool.

Keywords: CMOS VLSI, fault-tolerant, majority voter, reliability, TMR.

1. Introduction

The discrete time Fourier transform (DTFT) is not suitable in digital computers either for the computation of the response of the system or the frequency analysis of the signal. Hence the DFT, especially the fast Fourier transform (FFT) – the sampled version of the DTFT – is the mostly used technique for the same in digital computers. But the two error sources such as picket-fence effect and the spectral leakage play a crucial role in determining or obtaining the accurate spectrum results [1–3]. The approximation of the continuous DTFT spectrum with the discrete frequency points causes the picket-fence effect as illustrated in Figure 1. This is also called as resolution bias error since it poorly interprets the signal peaks and valleys. This effect can be reduced by using more number of frequency points during the FFT computation or by zero padding interpolation method [4–9].

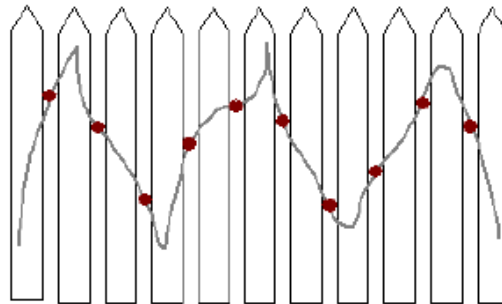


Figure 1. The picket-fence effect.

The spectral leakage occurs mainly due to the truncation of the infinitely long signal to a finite one. During the truncation or windowing, if the transition (from the end of the one window to the beginning of the next window) is too abrupt, the results of DFT will be poor results in spectral leakage. If the edge portions of the DFT record length taper, then the effects of the spectral leakage will be reduced [10–12]. This can be achieved by deploying appropriate windows, such as Hamming, Hanning, Blackmann, etc.

2. Materials and Methods

The various windows used in this study are outlined here. The signals are chosen in such a way that the spectral leakage occurs and does not occur respectively. Finally, the spectral and SNR analysis is performed with various windows and the results are analyzed for a relative study in the next section.

2.1. Rectangular (Dirichlet) window

Here the sampled portion of the signal is considered without any change. Irrespective of the window length, this window has the stopband attenuation around 21 dB. In common, for all the windows, the transition width can be decreased with more window, not the stopband attenuation. In general, rectangular window has the poor stopband attenuation than other windows [12,13]. The spectral characteristics of this window are shown in Figure 1 with the lengths 11, 31 and 51. This reveals that irrespective of the length, the window has the stopband attenuation around 21 dB. This window is expressed as,

$$W_{\text{rect}}(n)=1, \quad 0 \leq n \leq N-1 \quad (1)$$

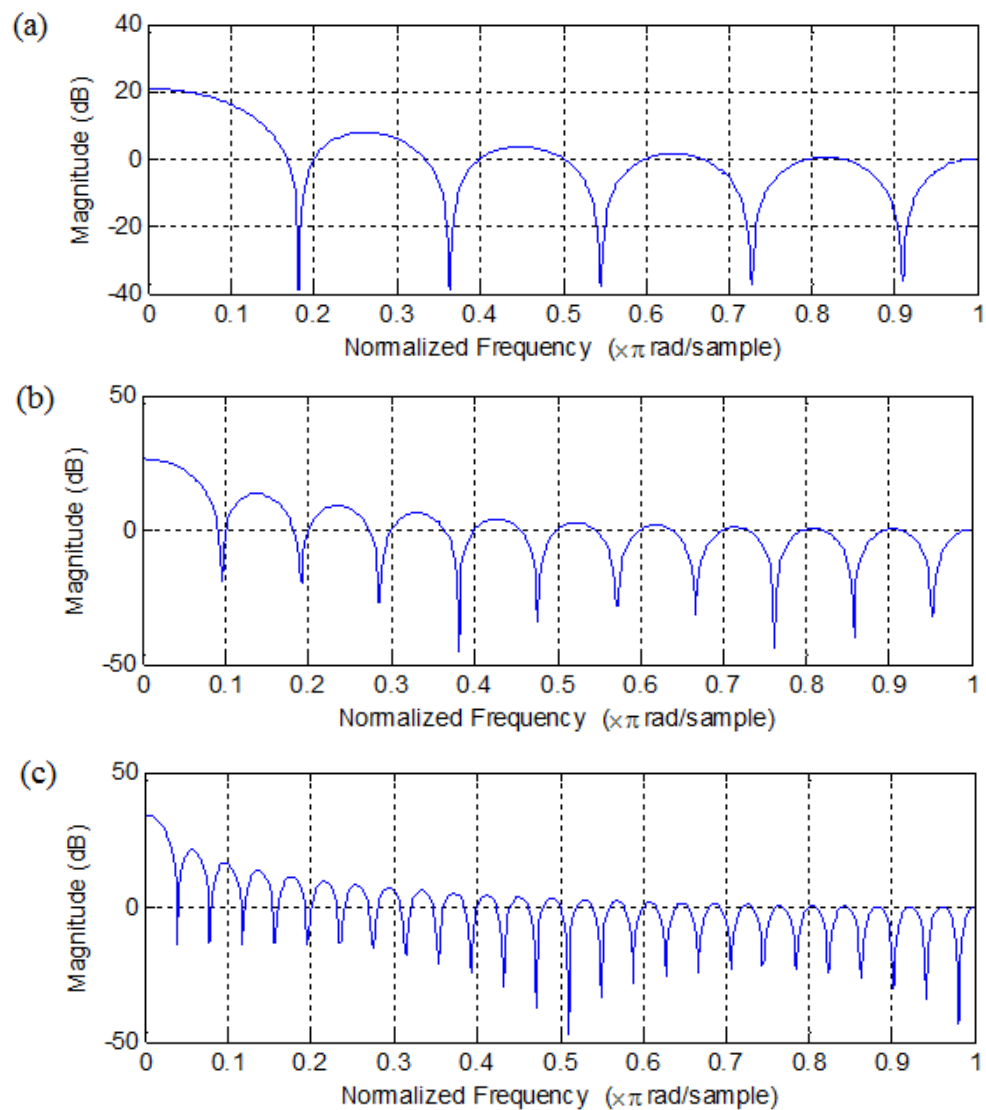


Figure 1. Spectral characteristics of a rectangular window of length: (a) 11; (b) 31; (c) 51.

2.2. Hanning window

The cosine bell, raised cosine and the von Hann are the other names for the Hanning window. The stopband attenuation is around 30 dB for this window, which is better than the rectangular window. But the transition width happens to be three times higher than the rectangular window. These features are illustrated in Figure 2 with window lengths of 11 and 31. The equation for this window is expressed as,

$$W_{hann}(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \leq n \leq N-1 \quad (2)$$

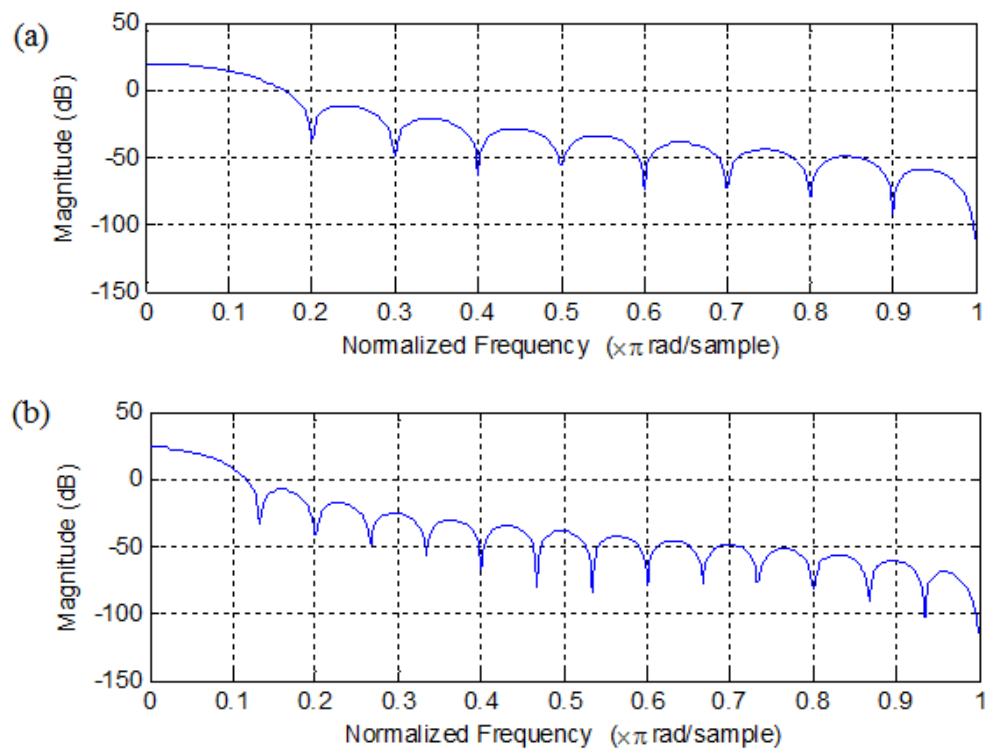


Figure 2. Spectral characteristics of a Hanning window of length: (a) 11; (b) 31.

2.3 Hamming window

This window provides much better stopband attenuation around 40 dB, i.e., 10 dB higher than the Hanning window but at the cost of broader transition width. The equation of Hamming window is [12,13]:

$$W_{hamm}(n) = 0.5 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \leq n \leq N-1 \quad (3)$$

2.4 Blackman window

The tradeoff between a stopband attenuation and the transition width can be achieved using Blackman window. That is, using this window, 74 dB stopband attenuation can be made but at the cost of more transition width – six times higher than the rectangular window. This window function is expressed as [12,13],

$$W_{blackman}(n) = 0.42 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos\left(\frac{4\pi n}{N-1}\right), \quad n \leq \left\lfloor \frac{N-1}{2} \right\rfloor \quad (4)$$

2.5 Bartlett window

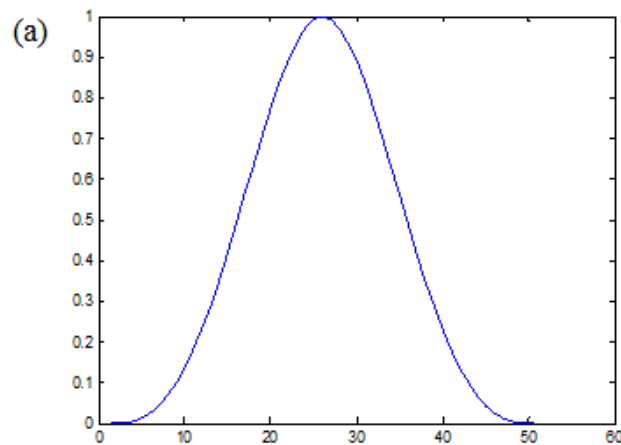
The peak of the first side lobe of this window is around 26 dB. The straight lines are used for tapering since this is a kind of triangle. Conceptually, this window is the convolution of the two rectangular windows [12,13]. The functionality of this window looks similar to the triangular window due to zeros inclusion at the end of tapering. The pattern which includes zeros at the edges is referred to as “triangular window”; otherwise remains “Bartlett window”. The Bartlett window is expressed as [12,13],

$$(5)$$

2.6 Bohman window

The Bohman window of length – 101 and its Amplitude spectrum are shown in Figure 3 with 46 dB – the peak of the first side lobe. The expression for this window is expressed as [12,13],

$$W_{bohman}(n) = \left(1 - \frac{|n|}{N/2}\right) \cos\left(\pi \frac{|n|}{N/2}\right) + \frac{1}{\pi} \sin\left(\pi \frac{|n|}{N/2}\right), \quad 0 \leq n \leq \frac{N}{2} \quad (6)$$



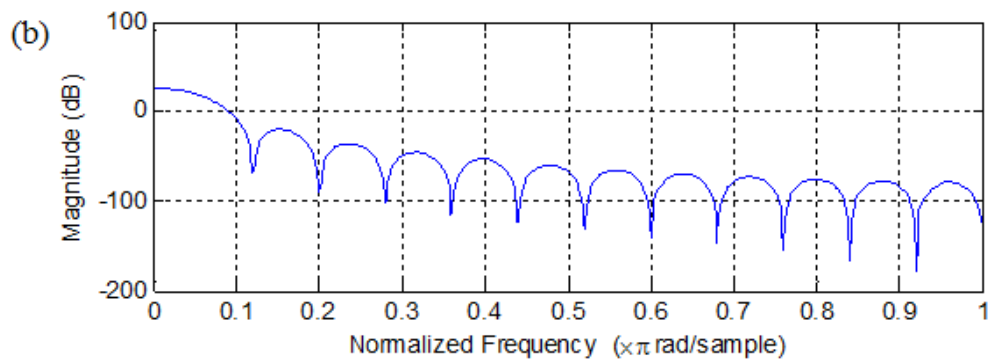


Fig. 3: Bohman window of length 51. (a) coefficients; (b) spectrum

2.7 Parabolic and Riemann windows

The equation of the parabolic window is expressed as [13],

$$W_{\text{parabolic}}(n) = 1 - \left(\frac{n - \frac{N}{2}}{N/2} \right)^2, \quad 0 \leq n \leq N-1 \quad (7)$$

and the Riemann window is expressed as [27]:

$$W_{\text{Riemann}}(n) = \frac{\sin \frac{2\pi n}{N}}{\frac{2\pi n}{N}}, \quad 0 \leq |n| \leq \frac{N}{2} \quad (8)$$

2.8 Parzen and Tukey windows

The Parzen window is expressed as [13],

$$W(n) = \begin{cases} 1 - 6 \left[\frac{n}{N/2} \right]^2 \left[1 - \frac{|n|}{N/2} \right], & 0 \leq |n| \leq \frac{N}{4} \\ 2 \left[1 - \frac{|n|}{N/2} \right]^3, & \frac{N}{4} \leq |n| \leq \frac{N}{2} \end{cases} \quad (9)$$

and the equation of the Cosine Taper (Tukey) window is [27]:

$$W(n) = \begin{cases} 1, & 0 \leq |n| \leq \alpha \frac{N}{2} \\ 0.5 \left[1 + \cos \left[\pi \frac{n - \alpha(N/2)}{(1-\alpha)(N/2)} \right] \right], & \alpha \frac{N}{2} \leq |n| \leq \frac{N}{2} \end{cases} \quad (10)$$

2.9 Hann-Poisson and Gaussian windows

The Hann-Poisson window expressed as [13],

$$W(n) = 0.5 \left[1 + \cos \left(\pi \frac{n}{N/2} \right) \right] \exp \left(-\alpha \frac{|n|}{N/2} \right), \quad 0 \leq |n| \leq \frac{N}{2} \quad (11)$$

and the equation of the Gaussian window is [13]:

$$W(n) = \exp \left[-\frac{1}{2} \left(\alpha \frac{n}{N/2} \right)^2 \right], \quad 0 \leq |n| \leq \frac{N}{2} \quad (12)$$

3. Simulation Results and Discussion

Figure 4 shows the DFT leakage effect with Hanning window. The simulation results are obtained for two cases. In the first case, the signal with five cycles per second having 512 frequency points is considered as the first cases. The spectral leakage occurs here due to the windowing by Hanning. If the truncation uses the rectangular window, the leakage will not occur. In the second case, the 4.75 cycles per second portion with 484 points is considered. Here the more leakage occurs during the truncation with a rectangular window and the impact of leakage can be minimized using different windows.

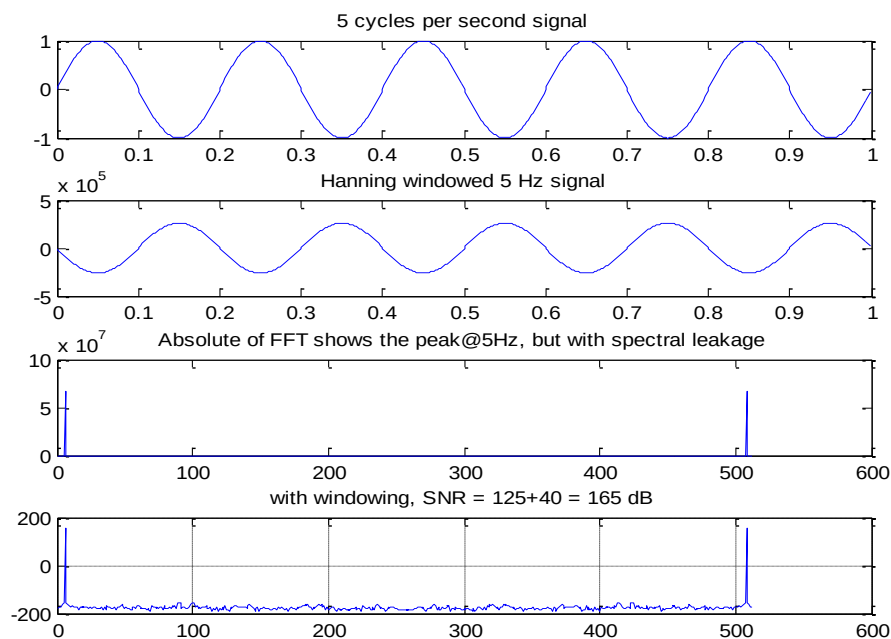


Fig. 4: A 5 cps signal windowed using Hanning – DFT leakage effect.

The SNR results using different windows for both the cases – with and without DFT leakage – are tabulated in Table 1. The better SNR results can be obtained using windows during the leakage scenario as in Table. For example, during the first case, the Hanning window obtains 166.96 dB while the same window produces better SNR (172.48 dB) during the second case (due to leakage).

Table 1. SNR results using different windows

Window	SNR (dB) (First case)	SNR (dB) (Second case)
Rectangular	353.35	52.96
Hanning	166.96	172.48
Hamming	168.25	170.15
Blackman	174.22	180.45
Bartlett	353.35	101.33
Triangular	356.81	138.00
Bohman	213.18	191.26
Parabolic	352.25	134.18
Riemann	352.38	226.12
Parzenwin	208.38	223.59
Tuekywin	177.40	151.46
Hann-Poisson, $\alpha = 0.5$	131.42	156.22
Gaussian, $\alpha = 3$	145.11	85.39
Poisson, $\alpha = 0.5$	363.55	277.11

4. Conclusion

The primary objective of this study paper is to interpret the DFT leakage and the usage of different windows to minimize the spectral leakage effects. Simulation results are obtained using Matlab for both the cases, i.e. with and without spectral leakage using different windows.

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