

Dramatizing of graphs for complementary perfect domination with vertex coloring

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Abstract

The concept of complementary perfect domination number of a graph was introduced by G. Mahadevan in [3]. Subsequently the authors obtained a nice characterization of sum of this number with chromatic number of order up to $2p-6$. In this paper, we dramatizing of graphs whose sum of complementary perfect domination number with chromatic number is equal to $2p-7$ for any $p > 3$

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Key Words and Phrases: Domination number, complementary perfect domination number, chromatic number.

1. Introduction

By a graph $G = (V, E)$ simple undirected connected graph. A subset $S \subseteq V$ is called a complementary perfect dominating set in G if $\langle S \rangle$ is connected and $\langle V - S \rangle$ has a perfect Matching. The minimum cardinality taken over all complementary perfect dominating set in G is called the complementary perfect domination number of G and is denoted by $\gamma_{cp}(G)$. The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour and is denoted by $\chi(G)$. In [3] Mahadevan G and A. Ayisha characterized the classes of graphs for which the complementary perfect domination number and chromatic number of order equals to $2p-6$ for any $p > 3$. In this paper, we characterize the extremal graphs whose chromatic number with complementary perfect domination number equals to $2p-7$ for any $p > 3$.

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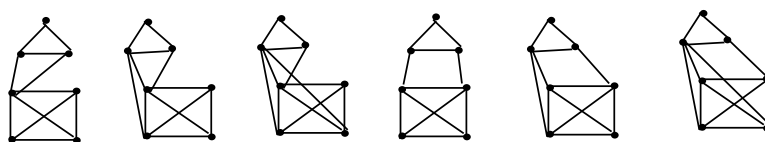
2. Preliminary Results

Theorem 1 [6] For any graph G , $\gamma_{cp}(G) \leq p-2$

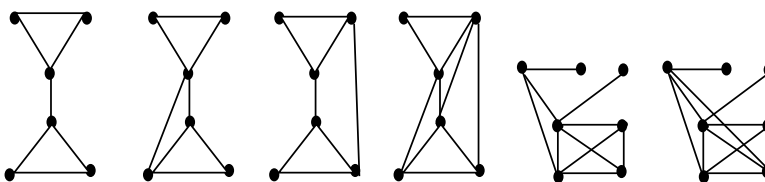
Theorem 2 [6] For any graph G , $\gamma_{cp}(G) = p$ if and only if G is a star.

Theorem 3 [4] Let G be a connected graph with $\gamma_{cp}(G) = p-4$ and $\chi(G) = p-3$. Then $\gamma_{cp}(G) + \chi(G) = 2p-7$ if and only if G is isomorphic to the family of $G \in \mathcal{G}_1 = \{ K_6(3,0,0,0,0,0), K_6(2,1,0,0,0,0), K_6(1,1,1,0,0,0), K_5(3,0,0,0,0), K_5(2,1,0,0,0), K_5(1,1,1,0,0), K_4(P_4), K_4(P_3, P_2, 0, 0), K_5(u(P_2, P_2), 0, 0, 0, 0), K_5(P_3, P_2, 0, 0, 0), K_3(P_4) \text{ and the following graphs in figure 1.1} \}$

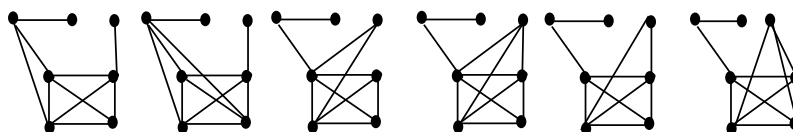
Graphs J_i , where $1 \leq i \leq 42$



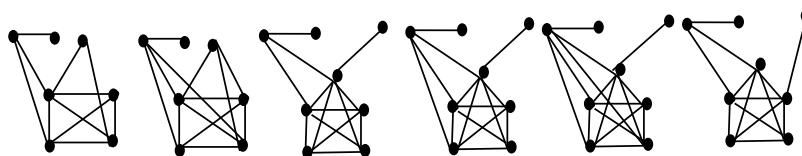
J_7 J_8 J_9 J_{10} J_{11} J_{12}



J_{13} J_{14} J_{15} J_{16} J_{17} J_{18}



J_{19} J_{20} J_{21} J_{22} J_{23} J_{24}



J_{25} J_{26} J_{27} J_{28} J_{29} J_{30}

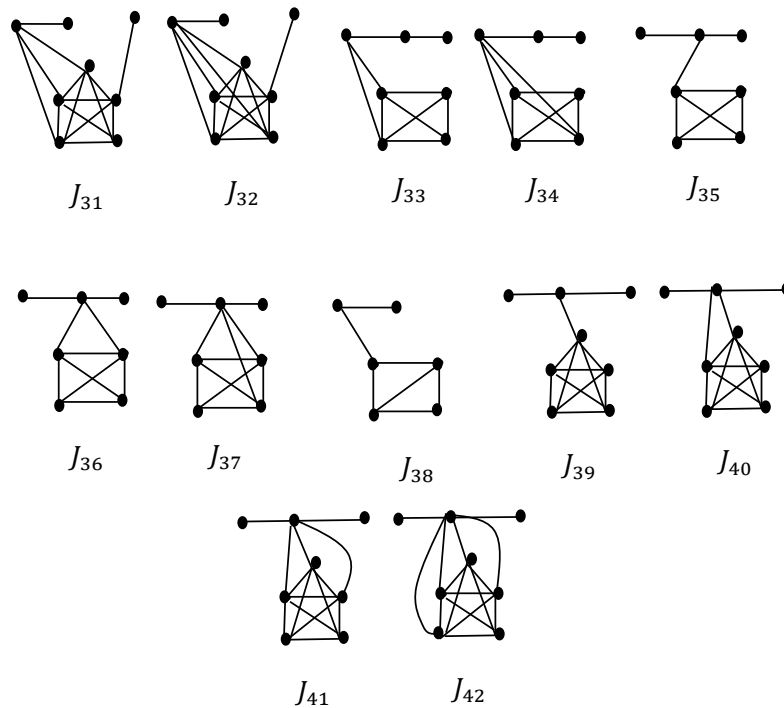


Figure 1

Notations

1. $K_p(m_1, m_2, \dots, m_n)$ is the graph obtained from K_p by pasting m_i edges to the vertex u_i of K_p , $1 \leq i \leq n$.
2. $K_p(sP_k, \cdot)$ is the graph obtained from K_p by attaching an end vertex of P_k to a vertex of K_p s times.
3. $C_p(sP_k, \cdot)$ is the graph obtained from C_p by attaching an end vertex of P_k to a vertex of C_p , s times.
4. $P_p(sP_k, 0)$ is the graph obtained from P_p by attaching an end vertex of P_k to a vertex of P_p , s times.
5. $P_p(u(P_n, P_m), mP_k, K)$ is the graph obtained from P_p by attaching the end vertices of P_n and P_m to any one vertex u of P_p and attaching the m times of the end vertices of P_k to other

vertex of P_p and so on.

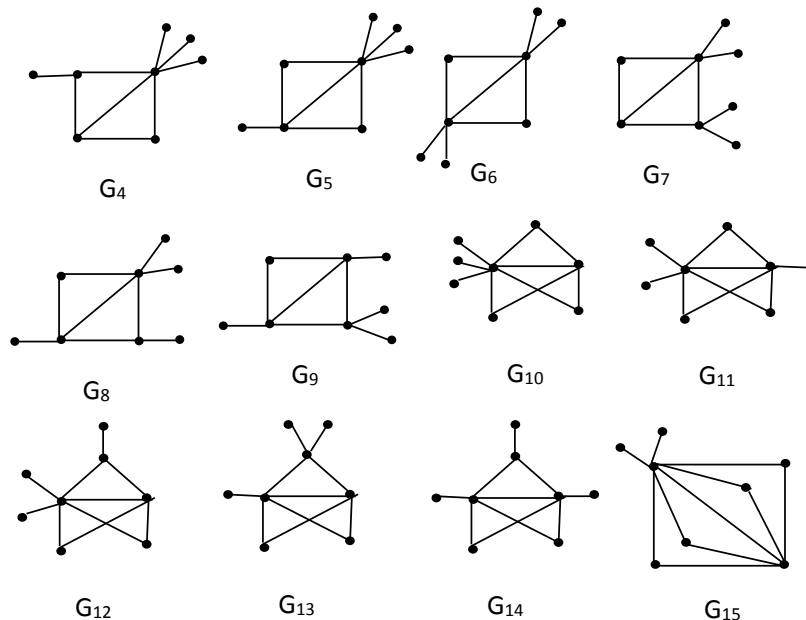
6. $C_p(u(P_n, P_m), mP_k, K)$ is the graph obtained from C_p by attaching the end vertices of P_n and P_m to any one vertex u of C_p and attaching the m times of the end vertices of P_k to other vertex of C_p and so on.

7. $C_3(m_1, m_2, 0)$ is the graph obtained from C_3 by identifying the centers of stars K_{1,m_2} and K_{1,m_1} at any two vertices C_3 respectively.

3. MAIN RESULT

Proposition. 3.1 Let G be a connected graph with $\gamma_{cp}(G) = p-2$ and $\chi(G) = p-5$. Then $\gamma_{cp}(G) + \chi(G) = 2p-7$ if and only if G is isomorphic to the family of $G \in \mathcal{G}_2 = \{ K_4(5,0,0,0), K_4(4,1,0,0), K_4(3,2,0,0), K_4(3,1,1,0), K_4(2,2,1,0), K_3(5,0,0), K_3(4,1,0), K_3(3,2,0), K_3(3,1,1), K_3(2,2,1), P_4(3P_2), C_4(P_2,0,P_2,P_2), C_4(P_2,2P_2,0,0), P_3(4P_2), P_4(3P_2,0), C_3(u(2P_3,P_2),0,0), P_3(u(P_3,2P_2),0), P_2(u(P_2,P_3),2P_2), P_3(u(P_2,P_4),0), P_4(u(P_3,P_2),0), P_3(5P_2,0), P_2(3P_2,2P_2), P_3(2P_2,2P_2), P_4(3P_2,0), P_5(2P_2,0), C_5(P_3), C_4(P_4), C_3(P_2,P_4,0), C_3(u(P_2,P_4),0,0), C_3(u(P_2,P_3)2P_2,0), C_4(u(P_3,P_2),0,0,0), C_3(2P_3,0,0), C_3(P_3,P_2,P_2)$ and $C_3(u(P_3,P_2),P_2,0), S(K_{1,3}), C_4(0.P_2,0,P_3)$, and the following figure 2 }

Graphs G_i , where $1 \leq i \leq 35$



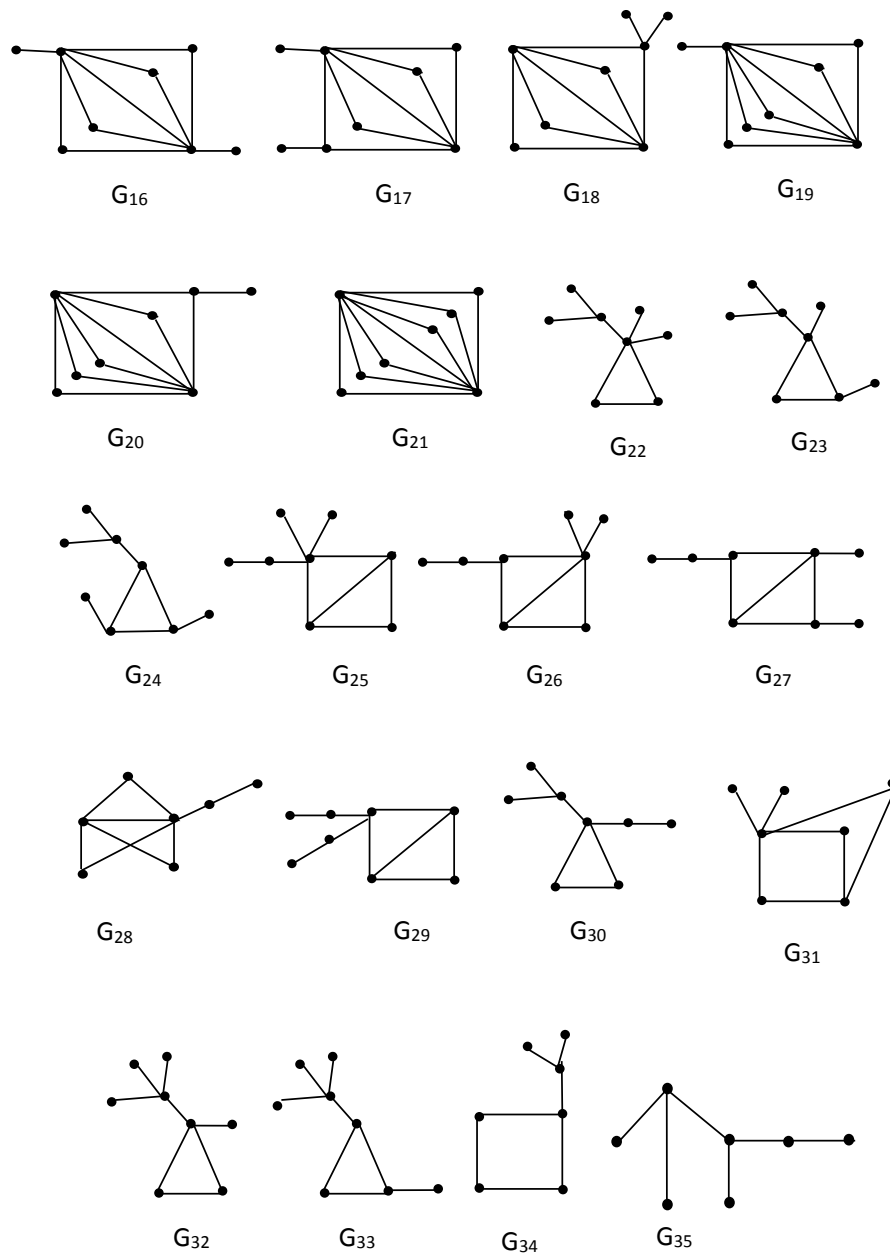


Figure 2

Proof. Since $\chi(G) = p-5$, G contains a clique K on $p-5$ vertices or does not contain a clique K on $p-5$ vertices. Let G contains a clique K on $p-5$ vertices and let $S = \{v_1, v_2, v_3, v_4, v_5\} \in V(G) - V(K)$. Then the possibilities of the induced sub graph are $\langle S \rangle =$

$\bar{K}_5$, P_5 , $P_4 \cup K_1$, $P_3 \cup \bar{K}_2$, $P_2 \cup \bar{K}_3$, $P_2 \cup P_2 \cup K_1$, $P_3 \cup K_2$, $K_{1,4}$, $K_{1,3} \cup K_1$, C_5 , $C_4 \cup K_1$, $C_4(P_2)$, K_5 , W_5 , F_2 , $K_5 - \{e\}$, $K_4 \cup K_1$, $K_4(P_2)$, $K_3 \cup \bar{K}_2$, $C_3(P_2) \cup K_1$, $K_3 \cup K_2$, $C_3(P_3)$, $C_3(2P_2)$, $C_3(P_2, P_2, 0)$, $B(2, 1)$, $K_4 - \{e\} \cup K_1$ and the following graphs in Figure 3.

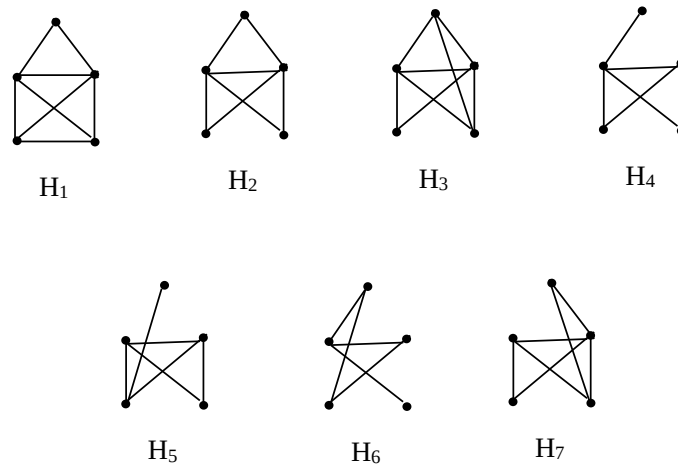


Figure 3

Case (1) $\langle S \rangle = \bar{K}_5 = (v_1, v_2, v_3, v_4, v_5)$

(i) Let all the vertices of $\bar{K}_5$ are adjacent to u_i in K . (ii) Let four vertices of $\bar{K}_5$ are adjacent to u_i and one vertex is adjacent to $u_j \in K$. (iii) Let three vertices of $\bar{K}_5$ are adjacent to u_i and two vertices are adjacent to $u_j \in K$. (iv) Let two vertices of $\bar{K}_5$ are adjacent to u_i and two vertices are adjacent to $u_j \in K$ and one vertex is adjacent to $u_k \in K$.

Sub case (a) $|V(K)|$ is even. In (i),(ii),(iii),(iv), $\{v_1, v_2, v_3, v_4, v_5, u_i, u_j\}$ for $j \neq i$ in K forms a CPD-set of G . Hence $\gamma_{cp} = 7$ so that $p = 9$. Then K is K_4 . If $d(v_i) = 1$ for all i , then $G \cong K_4(5, 0, 0, 0)$ or $K_4(4, 1, 0, 0)$ or $K_4(3, 2, 0, 0)$ or $K_4(3, 1, 0, 0)$ or $K_4(2, 2, 1, 0)$. If $d(v_i) > 1$, for some i , then we get a contradiction.

Sub case (b) $|V(K)|$ is odd. In (i),(ii),(iii),(iv), $\{v_1, v_2, v_3, v_4, v_5, u_i\}$ forms a CPD-set of G . Hence $\gamma_{cp} = 7$ so that $p = 9$. Then $K = K_3$. If $d(v_i) = 1$, then $G \cong K_3(3, 2, 0)$ or $K_3(3, 1, 1)$ or $K_3(2, 2, 1)$. If $d(v_1) = 2$ and $d(v_2) = d(v_3) = d(v_4) = d(v_5) = 1$, then $G \cong G_i, 1 \leq i \leq 9$. If $d(v_1) = d(v_2) = 2$ and $d(v_3) = d(v_4) = d(v_5) = 1$ then $G \cong G_i, 10 \leq i \leq 14$. If $d(v_1) = d(v_2) = d(v_3) = 2$ and $d(v_4) = d(v_5) = 1$ then $G \cong G_i, 15 \leq i \leq 18$. If $d(v_1) = d(v_2) = d(v_3) = d(v_4) = 2$ and $d(v_5) = 1$, then $G \cong G_i, 19 \leq i \leq 21$.

Case (2) $\langle S \rangle = P_5 = (v_1, v_2, v_3, v_4, v_5)$

Sub case (a) $|V(K)|$ is even. Let v_1 (or) v_5 be adjacent to some $u_i \in K$. Then $\{v_5, v_2, u_j\}$ for $j \neq i$ in K forms a CPD - set of G . Hence $\gamma_{cp} = 3$ so that $p = 5$ which is a contradiction. Let v_2 (or equivalently v_3, v_4) be adjacent to some u_i in K . Then $\{v_1, v_4, v_5, u_i\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 4$ so that $p = 6$, which is a contradiction.

Sub case (b) $|V(K)|$ is odd. Let v_1 or v_5 be adjacent to some $u_i \in K$. Then $\{v_5, v_2, u_j, u_k\}$ for $j \neq k \neq i$ in K forms a CPD - set of G . Hence $\gamma_{cp} = 4$ so that $p = 6$, which is a contradiction. Let v_2 (or equivalently v_3, v_4) be adjacent to some u_i in K . Then $\{v_1, v_4, v_5, u_i\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 4$ so that $p = 6$, which is a contradiction.

Case (3) $\langle S \rangle = P_4 \cup K_1$

Let v_1, v_2, v_3, v_4 are the vertices of P_4 and v_5 be the isolated vertex.

Sub case (a) $|V(K)|$ is even. (i) Let v_1 (or equivalently v_4) and v_5 are adjacent to some $u_i \in K$. Then $\{v_3, v_4, v_5, u_i, u_j\}$ for $j \neq i$ in K forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$. In this case the possible graphs are $P_5(2P_2)$, $C_4(P_2, 2P_2, 0, 0)$, $C_4(P_3, 0, P_2, 0)$. Let v_1 (or equivalently v_4) be adjacent to some $u_i \in K$ and v_5 is adjacent to some u_j in K for $j \neq i$. Then $\{v_1, v_4, v_5\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 3$ so that $p = 5$ which is a contradiction. (ii) Let v_2 (or equivalently v_3) and v_5 be adjacent to some $u_i \in K$. Then $\{v_1, v_4, v_5, u_i, u_j\}$ for $j \neq i$ in K forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$. In this case the possible graphs are $P_2(u(P_2, P_3), 2P_2)$, $C_4(P_2, 0, P_2, P_2)$, $C_4(P_2, 2P_2, 0, 0)$. Let v_2 (or equivalently v_3) be adjacent to some u_i in K and v_5 is adjacent to some u_j in K for $j \neq i$. Then $\{v_1, v_4, v_5, u_i, u_j\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$. In this case the possible graph is $P_4(u(P_2, P_3), 0)$.

Sub case (b) $|V(K)|$ is odd. (i) Let v_1 (or equivalently v_4) and v_5 are adjacent to some $u_i \in K$. Then $\{v_1, v_4, v_5, u_i\}$ for some in K forms a CPD - set of G . Hence $\gamma_{cp} = 4$ so that $p = 6$, which is a contradiction. (ii) Let v_2 (or equivalently v_3) and v_5 are adjacent to some $u_i \in K$. Let v_2 (or equivalently v_3) be adjacent to some u_i in K_4 and v_5 is adjacent to some u_j in K for $j \neq i$. Then $\{v_1, v_4, v_5, u_i\}$ in K forms a CPD - set of G . Hence $\gamma_{cp} = 4$ and $p = 6$, which is a contradiction. Hence no graph exists.

Case (4) $\langle S \rangle = P_3 \cup \bar{K}_2$

Let v_1, v_2, v_3 are the vertices of P_3 and v_4, v_5 be the isolated vertices.

Sub case (a) $|V(K)|$ is even. If at least one u_i is adjacent to v_1 (or equivalently v_3) and v_4, v_5 , then $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$. In this case the possible graphs are $P_4(3P_2)$, $C_4(P_2, 2P_2, 0, 0)$. If u_i is adjacent to v_1 (or equivalently v_3) and u_j is adjacent to v_4, v_5 for $i \neq j$, then $\{v_1, v_4, v_5\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 3$, which is a contradiction. If u_i are adjacent to v_2, v_4, v_5 , then $\{v_1, v_4, v_5\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$. In this case the possible graphs are $P_2(u(P_2, P_3), 2P_2)$, $C_4(P_2, 0, P_2, P_2)$. If

u_i is adjacent to v_2 and u_j is adjacent to $\{v_4, v_5\}$ for $i \neq j$. In this case the possible graph is $P_4(u(P_2, P_3), 0)$.

Sub case (b) $|V(K)|$ is odd. If at least one u_i is adjacent to v_1 (or equivalently v_3) and v_4, v_5 , then $\{v_1, v_4, v_5, u_i\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 4$ so that $p = 6$, which is a contradiction. If u_i is adjacent to v_1 (or equivalently v_3) and u_j is adjacent to v_4, v_5 for $i \neq j$, then $\{v_1, v_4, v_5, u_i\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 4$ so that $p = 6$, which is a contradiction. If u_i is adjacent to v_2, v_4, v_5 , then $\{v_1, v_3, v_4, v_5, u_i, u_j\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 6$ so that $p = 8$. If v_2, v_4, v_5 are adjacent to u_1 and $d(v_2) = 4$, then $G \cong G_7$ and $G_i, 22 \leq i \leq 24$.

Case (5) $\langle S \rangle = P_2 \cup \bar{K}_3$

Let v_1, v_2 are the vertices of P_2 and v_3, v_4, v_5 are the vertices of $\bar{K}_3$

(i) Let v_1 (or equivalently v_2) and v_3, v_4, v_5 are adjacent to u_i or (ii) Let v_1 (or equivalently v_2) be adjacent to u_i and v_3, v_4, v_5 are adjacent to u_j for $i \neq j$ or (iii) Let all the vertices are adjacent to different vertices of K_{p-5} .

Sub case (a) $|V(K)|$ is even. In (i), (ii), (iii), $\{v_2, v_3, v_4, v_5, u_i\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$. In this case the possible graphs are $P_3(4P_2), P_4(3P_2), C_4(3P_2), C_4(2P_2, P_2, 0, 0), C_4(P_2, P_2, P_2, 0)$.

Sub case (b) $|V(K)|$ is odd. In the above (i), (ii), $\{v_2, v_3, v_4, v_5, u_j, u_k\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 6$ so that $p = 8$. In this case the possible graphs are $C_3(u(P_3, 3P_2), 0, 0), C_3(u(P_3, P_2), 2P_2, 0), C_3(P_3, 2P_2, 0)$. Let v_1 (or equivalently v_2) and v_3, v_4, v_5 are adjacent to u_1 and $d(v_1) = 3$ then $G \cong G_8$. Let all the vertices of v_i are adjacent to different vertices of K , then $G \cong G_6, G_7$ and $G_i, 25 \leq i \leq 28$. In (iii) $\{v_2, v_3, v_4, v_5\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 4$, which is a contradiction.

Case (6) $\langle S \rangle = 2P_2 \cup K_1$

Let v_1, v_2 are the vertices of one P_2 and v_3, v_4 are the vertices of another P_2 , and v_5 be the isolated vertices.

(i) Let v_1 (or equivalently v_2) and v_3 (or equivalently v_4) and v_5 are adjacent to u_i in K . (ii) Let v_1 (or equivalently v_2) and v_3 (or equivalently v_4) be adjacent to u_i in K and v_5 be adjacent to u_j in K for some $i \neq j$. (iii) Let v_1 (or equivalently v_2) be adjacent to u_i in K and v_3 (or equivalently v_4) and v_5 are adjacent to u_j in K for some $i \neq j$. (iv) Let v_1 (or equivalently v_2) be adjacent to u_i and v_3 (or equivalently v_4) be adjacent to u_j in K and v_5 be adjacent to u_k in K for some $i \neq j \neq k$.

Sub case (a) $|V(K)|$ is even. In (i), (ii), (iii) $\{v_2, v_3, v_4, v_5, u_j\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$. If u_1 is adjacent to v_1, v_3, v_5 , then $G \cong P_3(u(P_3, 2P_2), 0, 0)$. If u_1 is adjacent to v_1, v_3, v_5 and if u_2 is adjacent to v_2 and $d(v_2) = 2$,

then $G \cong C_4(u(P_3, P_2), 0, 0, 0)$. If u_1 is adjacent to v_1, v_3 and u_2 is adjacent to v_5 , then $G \cong P_3(2P_3, 0, 0)$. If u_1 is adjacent to v_1 and u_2 is adjacent to v_4, v_5 , then $G \cong P_4(u(P_3, P_2), 0)$. If u_1 is adjacent to v_1 and u_2 is adjacent to v_4, v_5 and $d(v_2) = 2$, then $G \cong C_4(u(P_3, P_2), 0, 0, 0)$.

Sub case (b) $|V(K)|$ is odd. In (i), (ii), (iii) $\{v_2, v_3, v_4, v_5, u_j, u_k\}$ forms a CPD - set of G . Hence $\gamma_{cp} = 6$ and hence $p = 8$ so that $K = K_3$. If u_1 is adjacent to v_1, v_3, v_5 , then $G \cong C_3(u(P_3, P_3, P_2), 0, 0)$. If u_1 is adjacent to v_1, v_3, v_5 and $d(v_5) = 2$, then $G \cong G_{29}$. If u_1 is adjacent to v_1, v_3 and u_2 is adjacent to v_5 , then $G \cong C_3(2P_3, P_2, 0)$. In all the remaining cases $\gamma_{cp} = 4$, which is a contradiction.

Case (7) $\langle S \rangle = P_3 \cup P_2$

Let v_1, v_2, v_3 are the vertices of P_3 and v_4, v_5 are the vertices of P_2 .

- (i) Let v_1 (or equivalently v_3) and v_4 (or equivalently v_5) are adjacent to u_i in K , then $\{v_2, v_3, v_4, v_5, u_j\}$ forms a CPD- set of G hence $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$.
- (ii) Let v_2 be adjacent to u_i and v_4 (or equivalently v_5) be adjacent to u_i , then $\{v_1, v_3, v_4, v_5, u_j\}$ forms a CPD- set of G hence $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$.
- (iii) Let v_2 be adjacent to u_i and v_5 be adjacent to u_j for $i \neq j$. Then $\{v_1, v_3, v_4, v_5, u_j\}$ forms a CPD- set of G hence $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$.
- (iv) Let v_1 be adjacent to u_i and v_4 (or equivalently v_5) be adjacent to u_j for some $i \neq j$. Then $\{v_3, v_5, u_i\}$ forms a CPD- set of G . Hence $\gamma_{cp} = 3$, which is a contradiction.

Sub case (a) $|V(K)|$ is even. In the above (i), (ii), (iii) $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$. If v_1 and v_4 are adjacent to u_1 , then $G \cong C_4(0, P_2, 0, P_3)$. If v_1 and v_4 are adjacent to u_1 and $d(v_2) = 3$, then $G \cong C_4(u(P_3, P_2), 0, 0, 0)$. If v_2 and v_4 are adjacent to u_1 , then $G \cong P_2(u(P_3, P_2), 2P_2)$.

Sub case (b) $|V(K)|$ is odd. In (i), (iii), (iv) $\gamma_{cp} = 4$, which is a contradiction. In (ii) Let v_2 and v_4 are adjacent to u_1 . Then $\{v_1, v_3, v_5, u_j, u_k\}$ for $i \neq j \neq k$ forms a CPD- set of G hence $\gamma_{cp} = 6$ so that $p = 8$ and hence $K = K_3$, then $G \cong G_{30}$.

Case (8) $\langle S \rangle = K_{1,4}$

Let v_1 be the root vertex and v_2, v_3, v_4, v_5 are adjacent to v_1 .

- (i) Let v_1 be adjacent to u_i in K . or
- (ii) Let v_2 or any one of $\{v_3, v_4, v_5\}$ be adjacent to u_i in K . or
- (iii) Let v_1 and v_2 (or any one of v_3, v_4, v_5) are adjacent to u_i or
- (iv) Let v_1 be adjacent to u_i and v_2 (or any one of v_3, v_4, v_5) be adjacent to u_j for some $i \neq j$

Sub case (a) $|V(K)|$ is even. In (i), (ii), (iii), (iv) $\gamma_{cp} = 5$ and hence $p = 7$ so that $K = K_2$. If u_1 is adjacent to v_1 , then $G \cong P_3(4P_2)$. In this graph if $d(v_1) > 5$, then we get a contradiction.

If u_1 is adjacent to v_2 , then $G \cong P_4(3P_2)$. In this graph $d(v_2) > 2$, we get a contradiction. If u_1 is adjacent to v_1 and u_2 is adjacent to v_2 , then $G \cong C_4(3P_2, 0, 0, 0)$. In this graph if $d(v_3) = 2$, then $G \cong G_{31}$.

Sub case (b) $|V(K)|$ is odd. In (ii), (iii), (iv) $\gamma_{cp} = 4$, which is a contradiction. In case (i) $\{v_2, v_3, v_5, u_j, u_k\}$ for $i \neq j \neq k$ forms a CPD- set of G hence $\gamma_{cp} = 6$ so that $p = 8$ hence $K = K_3$. Let v_1 be adjacent to u_1 in K . Then $G \cong P_2(4P_2, C_3)$ to u_1 . In this graph if $d(v_1) = 6$, then $G \cong G_1$. If $d(v_1) > 6$, we get a contradiction.

Case (9) $\langle S \rangle = K_{1,3} \cup K_1$

Let v_1 be the root vertex and v_2, v_3, v_4 are adjacent to v_1 and v_5 be the isolated vertex.

(i) Let v_1 and v_5 be adjacent to u_1 . Then $\{v_2, v_3, v_4, v_5, u_j\}$ for $i \neq j$ or

(ii) Let v_2 (or any one vertex of v_3, v_4) be adjacent to u_1 and v_5 be adjacent to u_1 . Then $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD -set of G or

(iii) Let v_1 be adjacent to u_i and v_5 be adjacent to u_j for $i \neq j$. Then $\{v_2, v_3, v_4, v_5, u_j\}$ forms a CPD- set of G or

(iv) Let v_2 (or any one vertex of v_3, v_4) be adjacent to u_i and v_5 be adjacent to u_j for $i \neq j$. Then $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD- set of G

Sub case (a) $|V(K)|$ is even. In (i), (ii), (iii), (iv) $\gamma_{cp} = 5$ and hence $p = 7$ so that $K = K_2$. If v_1 and v_5 are adjacent to u_1 , then $G \cong P_2(3P_2, 2P_2)$. In this graph if $d(v_4) = 2$, then $G \cong C_4(2P_2, P_2, 0, 0)$. If $d(v_1) > 4$ we get a contradiction. Let v_1 be adjacent to u_1 and v_5 be adjacent to u_2 , then $G \cong P_4(3P_2)$. In this graph if $d(v_4) = 2$, then $G \cong C_4(2P_2, 0, P_2, 0)$. Let v_2 be adjacent to u_1 and v_5 be adjacent to u_1 . Then $G \cong P_2(2P_2, 2P_2)$. In this graph $d(v_3) = 2$, then $G \cong C_4(P_2, 0, 0, P_2)$. Let v_2 be adjacent to u_1 and v_5 be adjacent to u_2 . Then $G \cong P_5(2P_2)$. In this graph if $d(v_3) = 2$. Then $G \cong C_4(P_2, 0, P_3, 0)$. Let v_2 be adjacent to u_1 and v_5 adjacent to u_2 , and $d(v_4) = 2$ then $G \cong C_4(P_2, 0, P_3, 0)$.

Sub case (b) $|V(K)|$ is odd. In (ii) and (iv) $\{v_3, v_4, v_5, u_i\}$ forms a CPD- set of G $\gamma_{cp} = 4$, which is a contradiction. In (i) and (iii) $\{v_2, v_3, v_4, v_5, u_j, u_k\}$ for $j \neq k \neq i$ forms a CPD- set of G hence $\gamma_{cp} = 6$ so that $p = 8$ hence $K = K_3$. Let v_1 and v_5 are adjacent to u_1 and $d(v_1) = 4$. Then $G \cong G_{32}$. Let v_1 and v_5 are adjacent to u_1 . If $d(v_1) = 5$, then $G \cong G_3$. Let v_1 be adjacent to u_1 and v_5 be adjacent to u_2 . Then $G \cong G_{33}$. Let v_1 be adjacent to u_1 and v_5 be adjacent to u_2 . If $d(v_1) = 5$, then $G \cong G_3$.

Case (10) $\langle S \rangle = C_5$

Let v_i $i = 1$ to 5 be the vertices of C_5

Sub case (a) $|V(K)|$ is even. Let v_i be adjacent to u_i . Then $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD- set of G hence $\gamma_{cp} = 5$ so that $p = 7$ hence $K = K_2$. In this case the possible graph is $C_5(P_3)$. If $d(v_2), d(v_3), d(v_4), d(v_5) > 2$, then we get a contradiction.

Sub case (b) $|V(K)|$ is odd. Let v_i be adjacent to u_i . Then $\{v_3, v_4, v_5, u_j\}$ forms a CPD-set of G $\gamma_{cp} = 4$, which is a contradiction.

Case (11) $\langle S \rangle = C_4 \cup K_1$

Let v_i $i = 1$ to 4 be the vertices of C_4 and v_5 be the isolated vertex.

(i) Let any one of vertex say v_i and v_5 are adjacent to u_1 (or)

(ii) Let any one of vertex say v_i be adjacent to u_i and v_5 be adjacent to u_j .

Sub case (a) $|V(K)|$ is even. Then the above (i),(ii) cases $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD-set of G hence $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$. In this case the possible graph is $C_4(P_4)$. Let v_1 and v_5 are adjacent to u_1 . Then $G \cong G_{34}$.

Sub case (b) $|V(K)|$ is odd. The above (i),(ii) $\{v_3, v_4, v_5, u_i\}$ forms a CPD-set of G hence $\gamma_{cp} = 4$, which is a contradiction. Hence no new graph exists.

Case (12) $\langle S \rangle = C_4(P_2)$

Let v_i $i = 1$ to 4 and v_5 be the path vertex of v_1 .

(i) Let u_i be adjacent to v_1 or

(ii) Let u_i be adjacent to any one of $\{v_2, v_3\}$ or

(iii) Let u_i be adjacent to v_5

Sub case (a) $|V(K)|$ is even. In (i) and (ii), $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD-set of G hence $\gamma_{cp} = 5$ so that $p = 7$ and hence $K = K_2$. In this case the possible graphs is $C_4(u(P_3, P_2), 0, 0, 0)$. In (iii) $\gamma_{cp} = 4$, which is a contradiction. Hence no new graph exists.

Sub case (b) $|V(K)|$ is odd. The above (i),(ii),(iii) $\{v_3, v_4, v_5, u_i\}$ forms a CPD-set of G $\gamma_{cp} = 4$, which is a contradiction. Hence no new graph exists.

Case (13) $\langle S \rangle = P_3(P_2, P_2, 0)$

Let v_1 be adjacent to v_4, v_2, v_5 and v_2 be adjacent to v_3

(i) Let u_i be adjacent to v_1 or v_2

(ii) Let u_i be adjacent to any one of $\{v_3, v_4, v_5\}$

Sub case (a) $|V(K)|$ is even. In (i) and (ii) $\{v_3, v_4, v_5, u_i, u_j\}$ forms a CPD-set of G hence $\gamma_{cp} = 5$ so that $K = K_2$. In this case the possible graphs are $G \cong P_3(u(2P_2, P_3), 0), P_2(u(P_2, P_3), 0)$ and $G \cong G_{35}$

Sub case (b) $|V(K)|$ is odd. The above (i),(ii) $\{v_3, v_4, v_5, u_i\}$ forms a CPD-set of G $\gamma_{cp} = 4$, which is a contradiction. Hence no new graph exists.

If G does not contains a clique K on $p-5$ vertices, then it is not possible to obtain graphs satisfying the hypothesis. Hence no graph exists.

For all the remaining cases K_5 , $K_4 \cup K_1$, $K_4 \cup P_2$, $K_3 \cup K_2$, $K_5 - e$, W_5 , F_2 , $C_3(P_3)$, $C_3(2P_2)$, $C_3(P_2) \cup K_1$, $B(2, 1)$, $K_4 - e \cup K_1$ and the graph H_1 to H_7 it can be verified that no graph exists.

Proposition. 3.2 Let G be a connected graph with $\gamma_{cp}(G) = p-6$ and $\chi(G) = p-1$. Then $\gamma_{cp}(G) + \chi(G) = 2p-7$ if and only if G is isomorphic to the family of $G \in \mathcal{G}_3 = \{K_8(P_2), K_6(p_1), K_7(P_2), 2 \leq p_1 \leq 5, 1 \leq p_2 \leq 6\}$.

Proof. Let $\gamma_{cp}(G) = p-6$. Since $\chi(G) = p-1$, G contains a clique K on $p-1$ vertices. Let $V(G) - V(K) = v$. Since G is connected v is adjacent to at least a vertex in K .

Case 1 Suppose $|V(K)| = \text{even}$. Let $d(v) = 1$ and let $vu_1 \in E$. Then $\{v, u_1, u_2\}$ $u_1 u_2 \in V(K)$ is a γ -set of G and hence $p = 9$. This gives $K = K_8$. Hence G is isomorphic to $K_8(P_2)$. Let $d(v) \geq 2$ and let $vu_1 \in E$. Then u is a CPD - set of G so that $p = 7$. This gives $K = K_6$. Then G is isomorphic to $K_6(p_1)$, $2 \leq p_1 \leq 5$.

Case 2 Suppose $|V(K)| = \text{odd}$. Then $\{v, u_1\}$ $u_1 \in V(K)$ is a CPD - set of G so that $p = 8$. This gives $K = K_7$. Hence G is isomorphic to $K_7(p_2)$, $1 \leq p_2 \leq 6$. The converse is obvious.

Theorem. 3.3

For any connected graph G , $\gamma_{cp}(G) + \chi(G) = 2p-7$ if and only if $G \cong \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$

Proof. Let $\gamma_{cp}(G) + \chi(G) = 2p-7$. Then all the possible cases are (i) $\gamma_{cp}(G) = p$ and $\chi(G) = p-7$ (ii) $\gamma_{cp}(G) = p-1$ and $\chi(G) = p-6$, (iii) $\gamma_{cp}(G) = p-2$ and $\chi(G) = p-5$, (iv) $\gamma_{cp}(G) = p-3$ and $\chi(G) = p-4$, (v) $\gamma_{cp}(G) = p-4$ and $\chi(G) = p-3$, (vi) $\gamma_{cp}(G) = p-5$ and $\chi(G) = p-2$, (vii) $\gamma_{cp}(G) = p-6$ and $\chi(G) = p-1$, (viii) $\gamma_{cp}(G) = p-7$ and $\chi(G) = p$. The cases, (ii), (iv), (vi), (viii) $<V-S>$ has odd number of vertices it not possible to form a perfect matching. Hence in the all these cases, no graph exists. For the remaining cases, the proof follows from Theorem 3 and proposition 3.1, 3.2. The converse is obvious.

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