

NON SPLIT HOP DOMINATION NUMBER FOR CARTESIAN PRODUCT OF TWO DISTINCT CYCLES

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Abstract:

In a graph $G = (V, E)$ let S be the subset of V . A set $S \subseteq V$ is a hop dominating set of G , if for every vertex $v \in V - S$ there exists $u \in S$ such that $d(u, v) = 2$. A set $S \subseteq V$ is a non split hop dominating set of G if S is a hop dominating set and $\langle V - S \rangle$ is connected. The minimum cardinality of non split hop dominating set is called non split hop domination number of G and it is denoted by $NSHD(G)$. In this paper we found $NSHD$ number for cartesian product of two distinct cycles.

Keywords: Nonsplit hop domination, Cartesian product.

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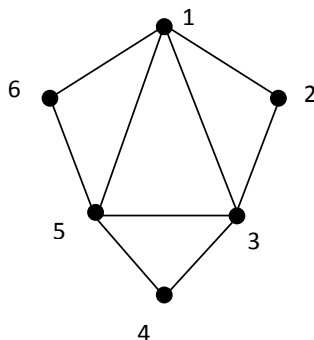
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I. Introduction:

Let G be a simple graph. For any two graph G and H let $G \square H$ denotes the cartesian product of G and H . $\lfloor y \rfloor$ denotes the largest integer less than or equal to y . $\lceil y \rceil$ denotes the smallest integer greater than or equal to y . A subset S of V is called a Dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S . $\gamma(G)$ is the minimum cardinality taken over all dominating sets in G which is called the domination number of G . If $\langle V - S \rangle$ is connected then S is said to be Non split dominating set of G . The minimum cardinality taken over all non split dominating set is the non split domination number and is denoted by $\gamma_{ns}(G)$. C. Natarajan, S.K. Ayyaswamy introduced the concept of Hop dominating set[6]. A set $S \subseteq V$ of a graph G is a Hop dominating set of G if for every $v \in V - S$ there exists $u \in S$ such that $d(u, v) = 2$. The minimum cardinality of a hop dominating set is called a hop domination number and it is denoted by $\gamma_h(G)$. A vertex u in a hop dominating set is said to be hop dominate an another vertex v if $d(u, v) = 2$. G.Mahadevan, V. Vijayalakshmi, C. Sivagnanam introduced the concept of Non split hop dominating set[4]. $S \subseteq V$ is a Non split hop dominating set of G if S is a hop dominating set and $\langle V - S \rangle$ is connected. The

minimum cardinality of non split hop dominating set is called non split hop domination number of G and it is denoted by $\text{NSHD}(G)$.

Example 1.1:



From the above figure Nonsplit hop dominating set = $\{1,2,3\}$; NSHD- number = 3.

II. NSHD NUMBER FOR CARTESIAN PRODUCT OF TWO DISTINCT CYCLES

In this section we found the exact value of the NSHD number for the product graph $C_m \square C_n$. Let C_m be a cycle with m vertices $\{x_1, x_2, \dots, x_m\}$. By the definition of hop domination, each x_i can hop dominate exactly two vertices $x_{i\pm 2}$ where $1 \leq i \leq n$. Let the vertices of $C_m \square C_n$ be $\{x_1^1, x_1^2, \dots, x_1^n, x_2^1, x_2^2, \dots, x_2^n, \dots, x_m^1, x_m^2, \dots, x_m^n\}$.

$$\text{Proposition 2.1: } \text{NSHD}(C_m \square C_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & \text{if } n \equiv 0, 3 \pmod{4} \\ \left\lceil \frac{n}{2} \right\rceil + 1 & \text{if } n \equiv 1, 2 \pmod{4} \end{cases}$$

where $m = 3, n \geq m$.

Proof: Let $S_1 = \{x_1^k, x_3^l : k \equiv 6, 7 \pmod{8}, l \equiv 2, 3 \pmod{8}\}$

$$\text{Now let } S = \begin{cases} S_1 & \text{if } n \equiv 0, 3 \pmod{4} \\ S_1 \cup \{x_1^n, x_2^n\} & \text{if } n \equiv 1, 2 \pmod{4} \end{cases}$$

Clearly S is the NSHD set for $C_m \square C_n$ where $m = 3, n \geq m$.

Proposition 2.2: $\text{NSHD}(C_m \sqcap C_n) = \begin{cases} \frac{4n}{6} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{4(n-1)}{6} + 1 & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{4(n-2)}{6} + 2 & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{4(n-3)}{6} + 3 & \text{if } n \equiv 3(\text{mod } 6); m = 4; \\ \frac{4(n-3)}{6} + 4 & \text{if } n \equiv 3(\text{mod } 6); m = 5; \\ \frac{4(n+2)}{6} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{4(n+1)}{6} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$

where $m = 4$ or 5 and $n \geq m$.

Proof: Let $S_1 = \{x_1^k, x_4^l : k \equiv 1, 2(\text{mod } 6), l \equiv 4, 5(\text{mod } 6)\}$

Now let $S = \begin{cases} S_1 & \text{if } n \equiv 0, 2, 5(\text{mod } 6); \\ S_1 \cup \{x_3^n\} & \text{if } n \equiv 1(\text{mod } 6); \\ S_1 \cup \{x_3^n\} & \text{if } n \equiv 3(\text{mod } 6); m = 4; \\ S_1 \cup \{x_3^n, x_4^n\} & \text{if } n \equiv 3(\text{mod } 6); m = 5; \\ S_1 \cup \{x_4^{n-1}\} & \text{if } n \equiv 4(\text{mod } 6); \end{cases}$

Clearly S is the NSHD set for $C_m \sqcap C_n$ where $m = 4$ or 5 , $n \geq m$.

Theorem 2.1: For $6 \leq m \leq n$, $\text{NSHD}(C_m \sqcap C_n) = \begin{cases} \frac{mn}{9} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{m(n-1)}{9} + \frac{m}{3} & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{m(n+4)}{9} & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{m(n+3)}{9} & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{m(n+2)}{9} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{m(n+1)}{9} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$

where $m \equiv 0(\text{mod } 6)$.

Proof:

$$\begin{aligned}
\text{Let } S_1 &= \{x_i^k, x_j^l : i \equiv 3(\text{mod } 6), j \equiv 0(\text{mod } 6), k \equiv 2, 3(\text{mod } 6), l \equiv 0, 5(\text{mod } 6), 1 \leq i, j \leq m, 1 \leq k, l \leq n\}, \\
S_2 &= \{x_i^1 : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}, \quad S_3 = \{x_i^2 : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}, \\
S_4 &= \{x_i^{n-1} : i \equiv 3(\text{mod } 6), 1 \leq i \leq m\}, \quad S_5 = \{x_i^{n-1} : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}, \\
S_6 &= \{x_i^n : i \equiv 4(\text{mod } 6), 1 \leq i \leq m\}.
\end{aligned}$$

Case 1: $n \equiv 0(\text{mod } 6)$

In this case S_1 is the NSHD set of $C_m \square C_n$.

Case 2: $n \equiv 1(\text{mod } 6)$

In this case $S_1 \cup S_3 \cup S_6$ is the NSHD set of $C_m \square C_n$.

Case 3: $n \equiv 2(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3 \cup S_4$ is the NSHD set of $C_m \square C_n$.

Case 4: $n \equiv 3(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3$ is the NSHD set of $C_m \square C_n$.

Case 5: $n \equiv 4(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_5$ is the NSHD set of $C_m \square C_n$.

Case 6: $n \equiv 5(\text{mod } 6)$

In this case $S_1 \cup S_2$ is the NSHD set of $C_m \square C_n$.

$$\text{Now let } S = \begin{cases} S_1 & \text{if } n \equiv 0(\text{mod } 6); \\ S_1 \cup S_3 \cup S_6 & \text{if } n \equiv 1(\text{mod } 6); \\ S_1 \cup S_2 \cup S_3 \cup S_4 & \text{if } n \equiv 2(\text{mod } 6); \\ S_1 \cup S_2 \cup S_3 & \text{if } n \equiv 3(\text{mod } 6); \\ S_1 \cup S_2 \cup S_5 & \text{if } n \equiv 4(\text{mod } 6); \\ S_1 \cup S_2 & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

It is clear that S is the NSHD set of $C_m \square C_n$ and hence

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{mn}{9} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{m(n-1)}{9} + \frac{m}{3} & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{m(n+4)}{9} & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{m(n+3)}{9} & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{m(n+2)}{9} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{m(n+1)}{9} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 0(\text{mod } 6)$ for all $n \geq 6$.

Theorem 2.2: For $7 \leq m \leq n$,

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m-1)n}{9} + \frac{n}{3} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{(m-1)(n-1)}{9} + \left\lfloor \frac{m}{3} \right\rfloor + \frac{n-1}{3} & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{(m-1)(n+1)}{9} + \frac{n-2}{3} + \frac{m-1}{3} & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{(m-1)n}{9} + \frac{n-3}{3} + \frac{m-1}{3} & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{(m-1)(n-1)}{9} + \frac{n-4}{3} + \frac{m-1}{3} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{(m-1)(n+1)}{9} + \frac{n+1}{3} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 1(\text{mod } 6)$.

Proof:

Let $S_1 = \{x_i^k, x_j^l : i \equiv 1(\text{mod } 6), j \equiv 4(\text{mod } 6), k \equiv 0, 5(\text{mod } 6), l \equiv 2, 3(\text{mod } 6), 1 \leq i, j \leq m, 1 \leq k, l \leq n\}$,

$S_2 = \{x_i^2 : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$, $S_3 = \{x_i^{n-1} : i \equiv 4(\text{mod } 6), 1 \leq i \leq m\}$,

$S_4 = \{x_i^1 : i \equiv 1(\text{mod } 6), 7 \leq i \leq m\}$, $S_5 = \{x_i^{n-1} : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$.

Case 1: $n \equiv 0(\text{mod } 6)$

In this case S_1 is the NSHD set of $C_m \square C_n$.

Case 2: $n \equiv 1(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_6$ is the NSHD set of $C_m \square C_n$.

Case 3: $n \equiv 2 \pmod{6}$

In this case $S_1 \cup S_3 \cup S_4 \cup S_2 - \{x_1^2\}$ is the NSHD set of $C_m \square C_n$.

Case 4: $n \equiv 3 \pmod{6}$

In this case $S_1 \cup S_4 \cup S_2 - \{x_1^2\}$ is the NSHD set of $C_m \square C_n$.

Case 5: $n \equiv 4 \pmod{6}$

In this case $S_1 \cup S_2 - \{x_1^2, x_m^2\} \cup S_3 - \{x_1^{n-1}, x_m^{n-1}\} \cup \{x_m^1, x_m^n\}$ is the NSHD set of $C_m \square C_n$.

Case 6: $n \equiv 5 \pmod{6}$

In this case $S_1 \cup S_5$ is the NSHD set of $C_m \square C_n$.

$$\text{Now let } S = \begin{cases} S_1 & \text{if } n \equiv 0 \pmod{6}; \\ S_1 \cup S_2 \cup S_6 & \text{if } n \equiv 1 \pmod{6}; \\ S_1 \cup S_3 \cup S_4 \cup S_2 - \{x_1^2\} & \text{if } n \equiv 2 \pmod{6}; \\ S_1 \cup S_4 \cup S_2 - \{x_1^2\} & \text{if } n \equiv 3 \pmod{6}; \\ S_1 \cup S_2 - \{x_1^2, x_m^2\} \cup S_3 - \{x_1^{n-1}, x_m^{n-1}\} \cup \{x_m^1, x_m^n\} & \text{if } n \equiv 4 \pmod{6}; \\ S_1 \cup S_5 & \text{if } n \equiv 5 \pmod{6}; \end{cases}$$

It is clear that S is the NSHD set of $C_m \square C_n$ and hence

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m-1)n}{9} + \frac{n}{3} & \text{if } n \equiv 0 \pmod{6}; \\ \frac{(m-1)(n-1)}{9} + \left\lceil \frac{m}{3} \right\rceil + \frac{n-1}{3} & \text{if } n \equiv 1 \pmod{6}; \\ \frac{(m-1)(n+1)}{9} + \frac{n-2}{3} + \frac{m-1}{3} & \text{if } n \equiv 2 \pmod{6}; \\ \frac{(m-1)n}{9} + \frac{n-3}{3} + \frac{m-1}{3} & \text{if } n \equiv 3 \pmod{6}; \\ \frac{(m-1)(n-1)}{9} + \frac{n-4}{3} + \frac{m-1}{3} & \text{if } n \equiv 4 \pmod{6}; \\ \frac{(m-1)(n+1)}{9} + \frac{n+1}{3} & \text{if } n \equiv 5 \pmod{6}; \end{cases}$$

where $m \equiv 1 \pmod{6}$ for all $n \geq 7$.

Theorem 2.3: For $8 \leq m \leq n$,

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m+4)n}{9} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{(m+4)(n-1)}{9} + \frac{m-8}{6} + \left\lfloor \frac{m}{6} \right\rfloor & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{(m+4)(n-2)}{9} + \frac{m-8}{3} + \left\lfloor \frac{m}{3} \right\rfloor & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{(m+4)(n-3)}{9} + \frac{m-8}{3} + \left\lfloor \frac{m}{3} \right\rfloor & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{(m+4)(n-1)}{9} + 2\left(\frac{m-8}{6}\right) - 1 & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{(m+4)(n+1)}{9} - 1 & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 2(\text{mod } 6)$.

Proof:

Let $S_1 = \{x_i^k, x_j^l : i \equiv 1(\text{mod } 6), j \equiv 4(\text{mod } 6), k \equiv 0, 5(\text{mod } 6), l \equiv 2, 3(\text{mod } 6), 1 \leq i, j \leq m, 1 \leq k, l \leq n\}$,
 $S_2 = \{x_i^1 : i \equiv 1(\text{mod } 6), 7 \leq i \leq m-2\}$, $S_3 = \{x_i^2 : i \equiv 2(\text{mod } 6), 7 \leq i \leq m-2\}$,
 $S_4 = \{x_i^{n-1} : i \equiv 4(\text{mod } 6), 1 \leq i \leq m\}$, $S_5 = \{x_i^n : i \equiv 2(\text{mod } 6), 8 \leq i \leq m\}$,
 $S_6 = \{x_i^{n-1} : i \equiv 1(\text{mod } 6), 8 \leq i \leq m\}$, $S_7 = \{x_m^1, x_m^j, x_m^{j+1} : i \equiv 2(\text{mod } 6), 2 \leq j \leq n-3\}$,
 $S_8 = \{x_i^1 : i \equiv 2(\text{mod } 6), 8 \leq i \leq m\}$, $S_9 = \{x_i^n : i \equiv 5(\text{mod } 6), 1 \leq i \leq m\}$.

Case 1: $n \equiv 0(\text{mod } 6)$

In this case $S_1 \cup S_7$ is the NSHD set of $C_m \square C_n$.

Case 2: $n \equiv 1(\text{mod } 6)$

In this case $S_1 \cup S_3 \cup S_7 \cup S_9$ is the NSHD set of $C_m \square C_n$.

Case 3: $n \equiv 2(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_7$ is the NSHD set of $C_m \square C_n$.

Case 4: $n \equiv 3(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3 \cup S_7$ is the NSHD set of $C_m \square C_n$.

Case 5: $n \equiv 4(\text{mod } 6)$

In this case $S_1 \cup S_5 \cup S_7 \cup S_8$ is the NSHD set of $C_m \square C_n$.

Case 6: $n \equiv 5 \pmod{6}$

In this case $S_1 \cup S_6 \cup S_7$ is the NSHD set of $C_m \square C_n$.

$$\text{Now let } S = \begin{cases} S_1 \cup S_7 & \text{if } n \equiv 0 \pmod{6}; \\ S_1 \cup S_3 \cup S_7 \cup S_9 & \text{if } n \equiv 1 \pmod{6}; \\ S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_7 & \text{if } n \equiv 2 \pmod{6}; \\ S_1 \cup S_2 \cup S_3 \cup S_7 & \text{if } n \equiv 3 \pmod{6}; \\ S_1 \cup S_5 \cup S_7 \cup S_8 & \text{if } n \equiv 4 \pmod{6}; \\ S_1 \cup S_6 \cup S_7 & \text{if } n \equiv 5 \pmod{6}; \end{cases}$$

It is clear that S is the NSHD set of $C_m \square C_n$ and hence

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m+4)n}{9} & \text{if } n \equiv 0 \pmod{6}; \\ \frac{(m+4)(n-1)}{9} + \frac{m-8}{6} + \left\lfloor \frac{m}{6} \right\rfloor & \text{if } n \equiv 1 \pmod{6}; \\ \frac{(m+4)(n-2)}{9} + \frac{m-8}{3} + \left\lfloor \frac{m}{3} \right\rfloor & \text{if } n \equiv 2 \pmod{6}; \\ \frac{(m+4)(n-3)}{9} + \frac{m-8}{3} + \left\lfloor \frac{m}{3} \right\rfloor & \text{if } n \equiv 3 \pmod{6}; \\ \frac{(m+4)(n-1)}{9} + 2\left(\frac{m-8}{6}\right) - 1 & \text{if } n \equiv 4 \pmod{6}; \\ \frac{(m+4)(n+1)}{9} - 1 & \text{if } n \equiv 5 \pmod{6}; \end{cases}$$

where $m \equiv 2 \pmod{6}$ for all $n \geq 8$.

$$\textbf{Theorem 2.4:} \text{ For } 9 \leq m \leq n, \quad \text{NSHD}(C_m \square C_n) = \begin{cases} \frac{mn}{9} & \text{if } n \equiv 0 \pmod{6}; \\ \frac{m(n-1)}{9} + \frac{m}{3} & \text{if } n \equiv 1 \pmod{6}; \\ \frac{m(n+4)}{9} & \text{if } n \equiv 2 \pmod{6}; \\ \frac{m(n+3)}{9} & \text{if } n \equiv 3 \pmod{6}; \\ \frac{m(n+2)}{9} & \text{if } n \equiv 4 \pmod{6}; \\ \frac{m(n+1)}{9} & \text{if } n \equiv 5 \pmod{6}; \end{cases}$$

where $m \equiv 3 \pmod{6}$.

Proof:

Let $S_1 = \{x_i^k, x_j^l : i \equiv 3(\text{mod } 6), j \equiv 0(\text{mod } 6), k \equiv 2, 3(\text{mod } 6), l \equiv 0, 5(\text{mod } 6), 1 \leq i, j \leq m, 1 \leq k, l \leq n\}$,
 $S_2 = \{x_i^j : j \equiv 0, 5(\text{mod } 6), 1 \leq j \leq n\}$, $S_3 = \{x_i^1 : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}$,
 $S_4 = \{x_i^2 : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}$, $S_5 = \{x_i^n, x_{m-1}^{n1} : i \equiv 4(\text{mod } 6), 1 \leq i \leq m\}$,
 $S_6 = \{x_i^{n-1} : i \equiv 3(\text{mod } 6), 1 \leq i \leq m\}$, $S_7 = \{x_i^n : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}$, $S_8 = \{x_i^{n-1} : i \equiv 0(\text{mod } 6), 1 \leq i \leq m\}$.

Case 1: $n \equiv 0(\text{mod } 6)$

In this case $S_1 \cup S_2$ is the NSHD set of $C_m \square C_n$.

Case 2: $n \equiv 1(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_4 \cup S_5$ is the NSHD set of $C_m \square C_n$.

Case 3: $n \equiv 2(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_6$ is the NSHD set of $C_m \square C_n$.

Case 4: $n \equiv 3, 4(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3 \cup S_7$ is the NSHD set of $C_m \square C_n$.

Case 5: $n \equiv 5(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_8 \cup \{x_1^{n-1}\}$ is the NSHD set of $C_m \square C_n$.

$$\text{Clearly } S = \begin{cases} S_1 \cup S_2 & \text{if } n \equiv 0(\text{mod } 6); \\ S_1 \cup S_2 \cup S_4 \cup S_5 & \text{if } n \equiv 1(\text{mod } 6); \\ S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_6 & \text{if } n \equiv 2(\text{mod } 6); \\ S_1 \cup S_2 \cup S_3 \cup S_7 & \text{if } n \equiv 3, 4(\text{mod } 6); \\ S_1 \cup S_2 \cup S_8 \cup \{x_1^{n-1}\} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

It is clear that S is the NSHD set of $C_m \square C_n$ and hence

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{mn}{9} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{m(n-1)}{9} + \frac{m}{3} & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{m(n+4)}{9} & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{m(n+3)}{9} & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{m(n+2)}{9} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{m(n+1)}{9} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 3(\text{mod } 6)$ for all $n \geq 9$.

Theorem 2.5: For $10 \leq m \leq n$,

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m-1)n}{9} + \frac{n}{3} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{(m-1)(n-1)}{9} + \frac{m-1}{3} + \frac{n-1}{3} + 1 & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{(m-1)(n-2)}{9} + \frac{2(m-1)}{3} + \frac{n-2}{3} + 1 & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{(m-1)(n-3)}{9} + \frac{2(m-1)}{3} + \frac{n-3}{3} + 1 & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{(m-1)(n+2)}{9} + \frac{n+2}{3} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{(m-1)(n+1)}{9} + \frac{n+1}{3} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 4(\text{mod } 6)$.

Proof:

Let $S_1 = \{x_i^k, x_j^l : i \equiv 1(\text{mod } 6), j \equiv 4(\text{mod } 6), k \equiv 0, 5(\text{mod } 6), l \equiv 2, 3(\text{mod } 6), 1 \leq i, j \leq m, 1 \leq k, l \leq n\}$,

$S_2 = \{x_i^2 : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$, $S_3 = \{x_i^{n-1} : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$,

$S_4 = \{x_i^{n-1} : i \equiv 4(\text{mod } 6), 1 \leq i \leq m\}$, $S_5 = \{x_i^1 : i \equiv 1(\text{mod } 6), 7 \leq i \leq m\}$.

Case 1: $n \equiv 0(\text{mod } 6)$

In this case S_1 is the NSHD set of $C_m \square C_n$.

Case 2: $n \equiv 1(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_4$ is the NSHD set of $C_m \square C_n$.

Case 3: $n \equiv 2 \pmod{6}$

In this case $S_1 \cup S_3 \cup S_4 \cup S_5$ is the NSHD set of $C_m \square C_n$.

Case 4: $n \equiv 3 \pmod{6}$

In this case $S_1 \cup S_2 \cup S_5$ is the NSHD set of $C_m \square C_n$.

Case 5: $n \equiv 4 \pmod{6}$

In this case $S_1 \cup S_2 \cup S_3$ is the NSHD set of $C_m \square C_n$.

Case 6: $n \equiv 5 \pmod{6}$

In this case $S_1 \cup S_3$ is the NSHD set of $C_m \square C_n$.

$$\text{Now let } S = \begin{cases} S_1 & \text{if } n \equiv 0 \pmod{6}; \\ S_1 \cup S_2 \cup S_4 & \text{if } n \equiv 1 \pmod{6}; \\ S_1 \cup S_3 \cup S_4 \cup S_5 & \text{if } n \equiv 2 \pmod{6}; \\ S_1 \cup S_2 \cup S_5 & \text{if } n \equiv 3 \pmod{6}; \\ S_1 \cup S_2 \cup S_3 & \text{if } n \equiv 4 \pmod{6}; \\ S_1 \cup S_3 & \text{if } n \equiv 5 \pmod{6}; \end{cases} \quad \text{It is clear that } S \text{ is the NSHD set of}$$

$C_m \square C_n$ and hence

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m-1)n}{9} + \frac{n}{3} & \text{if } n \equiv 0 \pmod{6}; \\ \frac{(m-1)(n-1)}{9} + \frac{m-1}{3} + \frac{n-1}{3} + 1 & \text{if } n \equiv 1 \pmod{6}; \\ \frac{(m-1)(n-2)}{9} + \frac{2(m-1)}{3} + \frac{n-2}{3} + 1 & \text{if } n \equiv 2 \pmod{6}; \\ \frac{(m-1)(n-3)}{9} + \frac{2(m-1)}{3} + \frac{n-3}{3} + 1 & \text{if } n \equiv 3 \pmod{6}; \\ \frac{(m-1)(n+2)}{9} + \frac{n+2}{3} & \text{if } n \equiv 4 \pmod{6}; \\ \frac{(m-1)(n+1)}{9} + \frac{n+1}{3} & \text{if } n \equiv 5 \pmod{6}; \end{cases}$$

where $m \equiv 4 \pmod{6}$ for all $n \geq 10$.

Theorem 2.6: For $11 \leq m \leq n$,

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m+1)n}{9} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{(m+1)(n-1)}{9} + \left\lceil \frac{m}{3} \right\rceil & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{(m+1)(n+4)}{9} & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{(m+1)(n+3)}{9} & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{(m+1)(n+2)}{9} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{(m+1)(n+1)}{9} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 5(\text{mod } 6)$.

Proof:

Let $S_1 = \{x_i^k, x_j^l : i \equiv 1(\text{mod } 6), j \equiv 4(\text{mod } 6), k \equiv 0, 5(\text{mod } 6), l \equiv 2, 3(\text{mod } 6), 1 \leq i, j \leq m, 1 \leq k, l \leq n\}$,
 $S_2 = \{x_i^2 : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$, $S_3 = \{x_i^n : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$,
 $S_4 = \{x_i^1 : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$, $S_5 = \{x_i^{n-1} : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$,
 $S_6 = \{x_i^n : i \equiv 1(\text{mod } 6), 1 \leq i \leq m\}$, $S_7 = \{x_i^{n-1} : i \equiv 4(\text{mod } 6), 1 \leq i \leq m\}$.

Case 1: $n \equiv 0(\text{mod } 6)$

In this case S_1 is the NSHD set of $C_m \square C_n$.

Case 2: $n \equiv 1(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_3$ is the NSHD set of $C_m \square C_n$.

Case 3: $n \equiv 2(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_4 \cup S_7$ is the NSHD set of $C_m \square C_n$.

Case 4: $n \equiv 3(\text{mod } 6)$

In this case $S_1 \cup S_2 \cup S_4$ is the NSHD set of $C_m \square C_n$.

Case 5: $n \equiv 4(\text{mod } 6)$

In this case $S_1 \cup S_5 \cup S_6$ is the NSHD set of $C_m \square C_n$.

Case 6: $n \equiv 5(\text{mod } 6)$

In this case $S_1 \cup S_5$ is the NSHD set of $C_m \square C_n$.

$$\text{Now let } S = \begin{cases} S_1 & \text{if } n \equiv 0(\text{mod } 6); \\ S_1 \cup S_2 \cup S_3 & \text{if } n \equiv 1(\text{mod } 6); \\ S_1 \cup S_2 \cup S_4 \cup S_7 & \text{if } n \equiv 2(\text{mod } 6); \\ S_1 \cup S_2 \cup S_4 & \text{if } n \equiv 3(\text{mod } 6); \\ S_1 \cup S_5 \cup S_6 & \text{if } n \equiv 4(\text{mod } 6); \\ S_1 \cup S_5 & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

It is clear that S is the NSHD set of $C_m \square C_n$ and hence

$$\text{NSHD}(C_m \square C_n) = \begin{cases} \frac{(m+1)n}{9} & \text{if } n \equiv 0(\text{mod } 6); \\ \frac{(m+1)(n-1)}{9} + \left\lceil \frac{m}{3} \right\rceil & \text{if } n \equiv 1(\text{mod } 6); \\ \frac{(m+1)(n+4)}{9} & \text{if } n \equiv 2(\text{mod } 6); \\ \frac{(m+1)(n+3)}{9} & \text{if } n \equiv 3(\text{mod } 6); \\ \frac{(m+1)(n+2)}{9} & \text{if } n \equiv 4(\text{mod } 6); \\ \frac{(m+1)(n+1)}{9} & \text{if } n \equiv 5(\text{mod } 6); \end{cases}$$

where $m \equiv 5(\text{mod } 6)$ for all $n \geq 11$.

The value of $\text{NSHD}(C_m \square C_n)$ for $6 \leq m, n \leq 19$ is shown in the following table:

m/n	6	7	8	9	10	11	12	13	14	15	16	17	18	19
6	4	6	8	8	8	8	8	10	12	12	12	12	12	14
7		9	10	10	10	12	12	15	16	16	16	18	18	21
8			10	10	11	15	16	17	18	18	19	23	24	25
9				14	14	16	16	19	22	22	22	24	24	28
10					16	16	16	20	23	23	24	24	24	28
11						16	16	20	24	24	24	24	24	28
12							16	20	24	24	24	24	24	28
13								25	28	28	28	30	30	35
14									30	30	31	35	36	39
15										34	34	36	36	41
16											36	36	36	42
17												36	36	42
18													36	42
19														49

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