



An Inventory Model for Deteriorating Products with Two Parameter Weibull Distribution Demand and Salvage Value

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Abstract

In this paper we discussed an inventory model for deteriorating products with demand rate is two parameter Weibull distribution and deterioration rate follows two parameter exponential distributions. This model allows shortages which are completely backlogged. The salvage value is used for deteriorated items. This model is solved analytically to obtain the minimum total cost. The results are illustrated with numerical example and graphical representation.

Key Words: Deteriorating items, shortages, salvage value, weibull distribution.

1. Introduction

Inventory models for deteriorating products with shortages which were completely backlogged have been proposed and analysed in the past. In the classical inventory models demand rate and deterioration rate were assumed as constants. Researchers are developing inventory models for deteriorating products by assuming time dependent demand rate and deterioration rate. Now many inventory models have been developed by considering the salvage of the deteriorated units. Venkateswarlu R and Mohan R developed their model by considering Quadratic demand and salvage value for the deteriorated units. Also shown that how the retarded decline and accelerated decline of a product will be useful in a real situation. Nita H Shah ,Ajay S Gov and H.M Wee presented an inventory system based on the joint vendor-buyer scheme for deteriorating items with salvage value. Mishra U and Tripathy considered an inventory model with three parameter Weibull distribution deterioration rate and salvage value for direct application of business enterprises. Kumar Mishra .V applied salvage value in a production Inventory model for remanufacturing the returned item.

Mishra V considered an model for obtaining optimal procurement quantity for three parameter Weibull distribution along with salvage value. Vinoth Kumar Mishra (2012) made an inventory model more realistic for taking Weibull distribution deterioration rate in association with salvage value and made the model very useful for industries when both the holding cost and deterioration rate are function of time. Misess enterprises which can reduce the deterioration rate significantly .Srichandan Mishra , et al (2014) hra V k (2015) attempted an inventory model with the use of preservation technology for businderived an optimal replenishment policy by assuming time varying demand and salvage value which is associated to the deteriorated units during the cycle time. Narges Khanlarzde (2012) applied an active packaging concept to decrease the deterioration rate of items in a two-echelon inventory control system with genetic algorithm.

This paper is an extended work of Amutha R and Chandrasekaran.E entitled as deteriorating inventory model for two parameter Weibull demand with shortages by considering that the salvage value which is incorporated with the deteriorated items and obtained the total minimum cost .

2. Assumptions and Notations

The following assumptions and notations have been considered in this inventory system.

- The inventory system deals with single item.
- A finite planning horizon is assumed.
- Lead time is zero.
- Shortages are allowed and are completely backlogged.
- A is a set up cost per order.

- C_1 is the deterioration cost per unit time.
- C_2 is the shortages cost per unit time.
- $I(t)$ is the inventory level at time t .
- T_1 is the time at which the inventory level reaches zero.
- $K(T_1, T)$ is the total cost per unit time.
- C is the inventory holding cost.
- Q is the initial inventory level.
- The salvage value γ , ($0 \leq \gamma \leq 1$) is constant.
- The demand rate of any time is $\alpha\beta t^{(\beta-1)}$ two parameter Weibull distribution, where $0 < \alpha \leq 1$, $\beta > 0$ scale and shape parameter respectively.
- $\theta(t)$ is the deteriorated rate which follows the two parameter exponential distribution

$$f(t, \mu, \theta) = \frac{1}{\theta} e^{-\frac{t-\mu}{\theta}}, t \geq \mu \text{ and } t \geq 0, \text{ where } \mu \text{ is a location parameter and } \theta > 0 \text{ is scale parameter.}$$

3. Mathematical Model

The initial inventory level of the inventory system is assumed to be Q . Either by demand or by deterioration the inventory level becomes zero when $t=T_1$. Shortages are completely backlogged. The differential equation for the instantaneous state over $(0, T)$ are given by

$$\frac{dI(t)}{dt} + \frac{1}{\theta} I(t) = -\alpha\beta t^{\beta-1}, 0 \leq t \leq T \quad (1)$$

$$\frac{dI(t)}{dt} = -\alpha\beta t^{\beta-1}, T_1 \leq t \leq T \quad (2)$$

with boundary conditions $I(T_1) = 0$ and $I(0) = Q$

The solutions of equations (1) and (2) with boundary conditions $I(T_1) = 0$ and $I(0) = Q$ are

$$I(t) = \alpha\beta \left(\frac{T_1^\beta - t^\beta}{\beta} + \frac{T_1^{\beta+1} - t^{\beta+1}}{\theta(\beta+1)} + \frac{T_1^{\beta+2} - t^{\beta+2}}{2\theta^2(\beta+2)} \right) - \frac{\alpha\beta}{\theta} \left(\frac{T_1^\beta t - t^{\beta+1}}{\beta} + \frac{tT_1^{\beta+1} - t^{\beta+2}}{\theta(\beta+1)} + \frac{tT_1^{\beta+2} - t^{\beta+3}}{2\theta^2(\beta+2)} \right) + \frac{\alpha\beta}{2\theta^2} \left(\frac{t^2 T_1^\beta - t^{\beta+2}}{\beta} + \frac{t^2 T_1^{\beta+1} - t^{\beta+3}}{\theta(\beta+1)} + \frac{t^2 T_1^{\beta+2} - t^{\beta+4}}{2\theta^2(\beta+2)} \right), \quad 0 \leq t \leq T_1 \quad (3)$$

$$I(t) = \alpha(T_1^\beta - t^\beta), \quad T_1 \leq t \leq T \quad (4)$$

The initial order Quantity at $t=0$ is

$$I(t) = \alpha\beta \left(\frac{T_1^\beta}{\beta} + \frac{T_1^{\beta+1}}{\theta(\beta+1)} + \frac{T_1^{\beta+2}}{2\theta^2(\beta+2)} \right) \quad (5)$$

Therefore the total cost consists of the following four components, DC, SHC, HC, SC and SV

Setup Cost Per Unit Time

$$SC = \frac{A}{T} \quad (6)$$

Inventory Holding Cost Per Unit Time

$$HC = \frac{C}{T} \int_0^{T_1} I(t) dt$$

$$= \frac{\alpha \beta C T_1^{\beta+4}}{12\theta^3 T(\beta+4)} + \frac{\alpha \beta C T_1^{\beta+1}}{T(\beta+1)} + \frac{\alpha \beta C T_1^{\beta+2}}{2\theta T(\beta+2)} - \frac{\alpha \beta C T_1^{\beta+3}}{6\theta^2 T(\beta+3)} + \frac{\alpha \beta C T_1^{\beta+5}}{12\theta^4 T(\beta+5)} \quad (7)$$

Shortages Cost Per Unit Time

$$SHC = \frac{-C_2}{T} \left[\int_{T_1}^T \alpha (T_1^\beta - t^\beta) dt \right]$$

$$= \frac{C_2 \alpha \beta T_1^{(\beta+1)}}{T} + \frac{C_2 \alpha T^\beta}{\beta+1} - \alpha C_2 T_1^\beta \quad (8)$$

Deterioration Cost Per Unit Time

$$DC = \frac{C_1}{T} \left[Q - \int_0^{T_1} D(t) dt \right] \quad (9)$$

$$= \frac{\alpha \beta C_1 T_1^{(\beta+1)}}{T \theta (\beta+1)} + \frac{\alpha \beta C_1 T_1^{(\beta+2)}}{2T \theta^2 (\beta+2)}$$

Salvage Value of Deteriorated Unit Time

$$SV = \gamma * DC$$

$$= \frac{\alpha \beta \gamma C_1 T_1^{(\beta+1)}}{T \theta (\beta+1)} + \frac{\alpha \beta \gamma C_1 T_1^{(\beta+2)}}{2T \theta^2 (\beta+2)} \quad (10)$$

Using equations 6,7 8,9 and 10 we can get the average total cost per unit time $K(T_1, T)$ as

$K(T_1, T)$ = Set up cost + Deterioration cost + Holding cost + Shortages – Salvage value

$$= \frac{A}{T} + \frac{\alpha \beta C T_1^{\beta+4}}{12\theta^3 T(\beta+4)} + \frac{\alpha \beta C T_1^{\beta+1}}{T(\beta+1)} + \frac{\alpha \beta C T_1^{\beta+2}}{2\theta T(\beta+2)} - \frac{\alpha \beta C T_1^{\beta+3}}{6\theta^2 T(\beta+3)} + \frac{\alpha \beta C T_1^{\beta+5}}{12\theta^4 T(\beta+5)} +$$

$$\frac{\alpha \beta C_1 T_1^{(\beta+1)}}{T \theta (\beta+1)} + \frac{\alpha \beta C_1 T_1^{(\beta+2)}}{2T \theta^2 (\beta+2)} + \frac{C_2 \alpha \beta T_1^{(\beta+1)}}{T} + \frac{C_2 \alpha T^\beta}{\beta+1} - \alpha C_2 T_1^\beta -$$

$$\frac{\alpha \beta \gamma C_1 T_1^{(\beta+1)}}{T \theta (\beta+1)} + \frac{\alpha \beta \gamma C_1 T_1^{(\beta+2)}}{2T \theta^2 (\beta+2)}$$

Now our objective is to minimize the average cost per unit time $K(T_1)$. The necessary conditions for total cost $K(T_1, T)$ to be minimum are

$$\frac{\partial K(T_1, T)}{\partial T_1} = 0 \text{ and } \frac{\partial K(T_1, T)}{\partial T} = 0 \text{ provided } \left(\frac{\partial^2 TC}{\partial T_1^2} \right) \left(\frac{\partial^2 TC}{\partial T^2} \right) - \left(\frac{\partial^2 TC}{\partial T_1 \partial T} \right)^2 > 0,$$

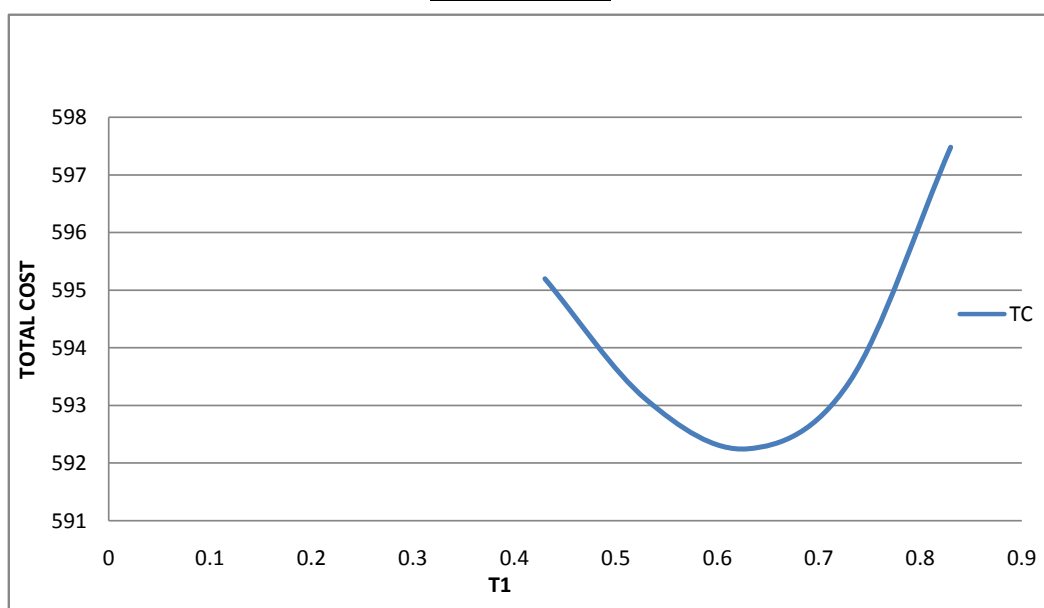
$$\left(\frac{\partial^2 TC}{\partial T^2} \right) > 0 \text{ and } \left(\frac{\partial^2 TC}{\partial T_1^2} \right) > 0$$

Numerical Examples and Sensitivity Analysis

In this section, a numerical example and sensitivity analysis are given to illustrate this model.

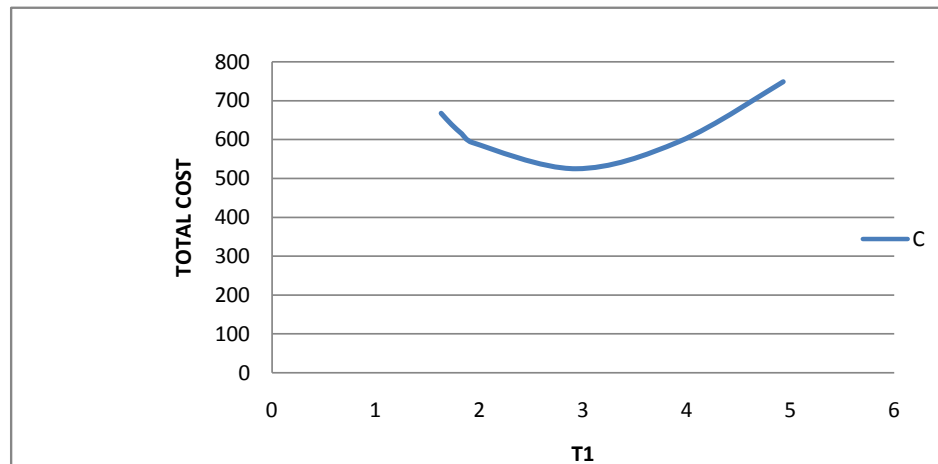
Total Cost Vs T_1 at $T = 1.9390$

T_1^*	$K(T_1^*)$
0.43	595.1976
0.53	593.1105
0.63	592.2490
0.73	593.4039
0.83	597.4786



Total Cost Vs T at $T_1 = 0.6315$

T^*	$K(T^*)$
1.73	638.6799
1.83	614.3767
1.93	592.2490
2.93	524.7527
3.93	595.1619
4.93	748.6100



4. Conclusion

We have developed an inventory model for deteriorating products when the demand rate is assumed to be two parameter Weibull distributions and deterioration rate follows two parameter exponential distribution. Salvage value is considered to keep the quality of the products as good as possible. The analytic solution of the model has given that minimizes the total inventory cost. This research may further be extended by combining the parameters partial backlogging, inflation; priced is count, trade credit.

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