

New Approach to Solve Fully Fuzzy Transportation Problem using Ranking Technique.

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Abstract

In the literature, there are several methods for finding a fuzzy initial basic feasible solution to the fully fuzzy transportation problem. Proposed method was illustrated by the numerical examples with a triangular fuzzy number. Transportation problem is a linear programming which considers minimum cost for shipping a product from some origins to other destinations such as from factories to warehouse, or from a warehouse to supermarkets. In real projects costs and the value of supply and demands are fuzzy numbers and it is expected that optimal solutions for determining the value of commodities transported from a source to a destination be obtained as a fuzzy. So the first idea is to present the in the full fuzzy condition and then an algorithm which is of importance for solving such a problem.

Keywords: - Fuzzy Transportation Problem, Triangular fuzzy numbers, Roubast Ranking Method, North - west corner method, optimal solution

Introduction

The aim of Fuzzy transportation is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Many previous papers [1, 2, 5, 6] have presented arithmetic operations, alpha level and simple ranking by operations. P. Pandian [4, 5] has presented some methods for fuzzy

transportation problem. Finally all these research papers present the solutions of FTP by alpha level, simple ranking method and alpha level standard value. In this paper we are presenting a ranking technique with alpha optimal solution for solving fuzzy transportation problem, where fuzzy demand and supply all are in the form of triangular fuzzy numbers. A fuzzy transportation problem is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. In this paper we investigate more realistic problems, namely the transportation problem with fuzzy costs a_{ij} . Since the objective is to minimize the total cost or to maximize the total profit, subject to some fuzzy constraints, the objective function is also considered as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. On the basis of this idea the Roubast Ranking method [7] with the help of α solution has been adopted a transform the fuzzy transportation problem. The idea is to transform a problem with fuzzy parameters in the form of Linear programming problem and solve it by the Vogel Approximation Method. Preliminaries Zadeh [9] in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness

in everyday life. Fuzzy set: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval $[0, 1]$ i.e. $A = \{(x, \mu_A(x)) ; x \in X\}$, Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A . These membership grades are often represented by real numbers ranging from $[0,1]$. Triangular fuzzy number: For a triangular fuzzy number $A(x)$, it can be represented by $A(a, b, c; 1)$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1, & x = b \\ \frac{(c-x)}{(c-b)} & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

The data of the model include

1. The level of supply at each source and the amount of demand at each destination.
2. The unit transportation cost of the commodity from each source to each destination.

Since there is only one commodity, a destination can receive its demand from more than one source. The objective is to determine how much should be shipped from each source to each destination so as to minimise the total transportation cost. A transportation model in which the total supply and total demand are unequal is called unbalanced. It is always possible to balance an unbalanced transportation problem.

Methods of Solving Transportation Problem

The Methods of solving transportation problem are

Step 1: Formulate the problem.

Formulate the given problem and set up in a matrix form. Check whether the problem is a

balanced or unbalanced transportation problem. If unbalanced, add dummy source (row) or dummy destination (column) as required.

Step 2: Obtain the initial feasible solution.

The initial feasible solution can be obtained by any of the following three methods:

Northwest Corner Method (NWC)

Least Cost Method (LCM)

Vogel's Approximation Method (VAM)

The transportation cost of the initial basic feasible solution through Vogel's approximation method, VAM will be the least when compared to the other two methods which gives the value nearer to the optimal solution or optimal solution itself. Algorithms for all the three methods to find the initial basic feasible solution are given.

Formulating the Problem

As a first step, it is necessary to clearly understand the problem situations. It is important to know how it is characterized and what is required to be determined.

Initially it is important to identify the key decision and the objective from the problem. Then, it involves determination of the number of decision variables and the relationship between variables. The measurable guaranties that are represented through these variables are notified.

Finally the practical limitations or constraints are also inferred from the problem.

The advantages of mathematical modeling are many:

Models exactly represent the real problem situations.

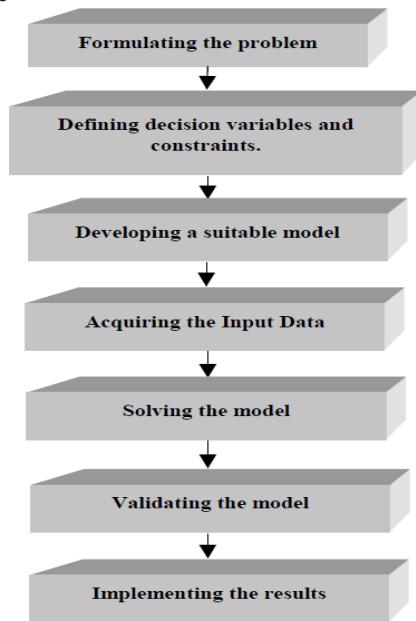
Models help managers to take decisions faster and more accurately.

They typically offer convenience and cost advantages over other means of obtaining the required information on reality.

Large and complex problems can be solved with ease.

Models act as communicators to others by providing information and impact in changing conditions.

The methodology adopted in solving problems is as follows



Robust Ranking Technique:

Robust ranking technique which satisfy compensatio, linearity, and additively properties and provides results which are consist human intuition. If $\tilde{a}$ is a fuzzy number then the Robust Ranking is defined by $R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L a_\alpha^U) d\alpha$, where $(a_\alpha^L a_\alpha^U)$ is the α level cut of the fuzzy number $\tilde{a}$. In this paper we use this method for ranking the objective values. The Robust ranking index $R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

Numerical Example:- A company has four sources S_1, S_2, S_3 , and S_4 , and four destinations D_1, D_2, D_3 , and D_4 ; the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is C_{ij} where $[c_{ij}]_{4 \times 4} =$

$$\begin{pmatrix} (3,6,9)(3,6,12)(3,9,12)(3,6,9) \\ (3,6,12)(3,9,12)(3,6,9)(6,9,12) \\ (3,6,12)(6,9,12)(6,9,12)(3,6,9) \\ (6,9,15)(3,6,9)(6,12,18)(6,9,15) \end{pmatrix}$$

And fuzzy availability of the product at source are $((6, 9, 12), (3, 6, 9), (12, 18, 24), (9, 12, 15))$ and the fuzzy demand of the product at destinations are $((15, 18, 21), (6, 9, 12), (3, 9, 12), (6, 9, 15))$ respectively.

The fuzzy transportation problems are

(3,6,9)	(3,6,12)	(3,9,12)	(3,6,9)	(6,9,12)
(3,6,12)	(3,9,12)	(3,6,9)	(6,9,12)	(3,6,9)
(3,6,12)	(6,9,12)	(6,9,12)	(3,6,9)	(12,18,24)
(6,9,15)	(3,6,9)	(6,12,18)	(6,9,15)	(9,12,15)
(15,18,21)	(6,9,12)	(3,9,12)	(6,9,15)	

$$\begin{aligned} \text{Min } Z = & R(3,6,9)x_{11} + R(3,6,12)x_{12} + R(3,9,12)x_{13} + R(3,6,9)x_{14} + R(3,6,12)x_{21} \\ & + R(3,9,12)x_{22} + R(3,6,9)x_{23} + R(6,9,12)x_{24} + R(3,6,12)x_{31} + R(6,9,12)x_{32} \\ & + R(6,9,12)x_{33} + R(3,6,9)x_{34} + R(6,9,15)x_{41} + R((3,6,9)x_{42} + R(6,12,18)x_{43} + R(6,9,15)x_{44} \end{aligned}$$

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L a_\alpha^U) d\alpha$$

$$(a_\alpha^L, a_\alpha^U) = ((b-a)\alpha + a, (c-a)\alpha + c)$$

$$R(3,6,9) = \int_0^1 (0.5)(3\alpha + 3,9 - 3\alpha) d\alpha$$

$$R(3,6,9) = \int_0^1 (0.5)(12) d\alpha = 6$$

Similarly

$$\begin{aligned} R(3,6,12) &= 6.75, R(3,9,12) = 8.25, R(3,6,9) = 6, R(6,9,12) = 9, \\ R(3,9,12) &= 8.25, R(3,6,9) = 6, R(6,9,12) = 9, R(3,6,9) = 6, R(3,6,12) = 6.75, R(6,9,12) = 9, \\ R(6,9,12) &= 9, R(3,6,9) = 6, R(6,9,15) = 9.75, R(3,6,9) = 6, R(6,12,18) = 12, R(6,9,15) = 9.75 \end{aligned}$$

$$\text{Rank of all supply } R(6,9,12) = 9, R(3,6,9) = 6, R(12,18,24) = 18, R(9,12,15) = 12$$

Rank of all demands $R(15,18,21) = 18$,
 $R(6,9,12) = 9$, $R(3,9,12) = 8.25$,
 $R(6,9,15) = 9.75$,
 The fuzzy transportation problems are

TABLE 1

	FD_1	FD_2	FD_3	FD_4	Fuzzy Capacity
FO_1	6	6.75	8.25	6	9
FO_2	6.75	8.25	6	9	6
FO_3	6.75	9	9	6	18
FO_4	9.75	6	12	9.75	12
Fuzzy Demand	18	9	8.25	9.75	

Starting the algorithm finding an initial basic feasible solution

We consider two ways of constructing initial basic feasible solutions for a transportation problem, i.e. allocations with $n + m - 1$ basic variables which satisfy all the constraint equations

The North-West Corner Method

The north-west corner method generates an initial allocation according to the following procedure:

Allocate the maximum amount allowable by the supply and demand constraints to the variable x_{11} (i.e. the cell in the top left corner of the transportation tableau).

If a column (or row) is satisfied, cross it out. The remaining decision variables in that column (or row) are non-basic and are set equal to zero. If a row and column are satisfied simultaneously, cross only one out (it does not matter which).

Adjust supply and demand for the non-crossed out rows and columns.

Allocate the maximum feasible amount to the first available non-crossed out element in the next column (or row).

When exactly one row or column is left, all the remaining variables are basic and are assigned the only feasible allocation.

TABLE 2

	FD_1	FD_2	FD_3	FD_4	Fuzzy Capacity
FO_1	6(9)	6.75	8.25	6	9
FO_2	6.75(6)	8.25	6	9	6
FO_3	6.75(3)	9(9)	9(6)	6	18
FO_4	9.75	6	12(2.25)	9.75(9.75)	12
Fuzzy Demand	18	9	8.25	9.75	

Number of basic variables = $m + n - 1 = 4 + 4 - 1 = 7$.

The total **transportation cost** is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

Solution: In the above examples it has been shown that the total optimal cost obtained by our methods remains as that obtained by defuzzifying the total fuzzy optimal cost by applying the Robust's Ranking method.

RESULT: For the fuzzy transportation problem with fuzzy objective function $\min z = (6 \times 9) + (6.75 \times 6) + (6.75 \times 3) + (9 \times 9) + (9 \times 6) + (12 \times 2.25) + (9.75 \times 9.75) = 371.8125$.

Conclusion

Studying fully fuzzy transportation problems due to its close connection with human life, is considered to be of great importance. The preliminary of solving

the transportation problem has to do with fully fuzzy linear programming. Therefore, in this article through using a proposed algorithm in fully fuzzy transportation problem and getting ideas from robust ranking method using north west corner method optimal solution was obtained. The proposed algorithm which is significant for being used in interactive methods for making any comment by related managers and achieving the logical solutions. For further studies, solving fully fuzzy transportation problem through taking into account the minimal cost and time is recommended. In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of triangular numbers has been transformed into crisp transportation problem using Robust's ranking indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using Robust's ranking method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by Robust's ranking method effectively. This technique can also be used in solving other types of problems like, project schedules, assignment problems and network flow problems.

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