

COMPLEXITY OF CHROMATIC STRONG DOMINATION IN BIPARTITE GRAPH

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Abstract

Let $G = (V, E)$ be a simple graph. The decision problem corresponding to the computation of chromatic weak domination number is found to be NP-complete even when restricted to bipartite graphs.

Keywords: Dominating set, Strong dominating set, Chromatic Strong dominating set.

1. Introduction

Let $G = (V, E)$ be a simple graph. NP denote the class of all decision problems which can be solved in polynomial time by a non-deterministic Turing Machine. NP stands for Non-Deterministic polynomial time. The fundamental open question in computational complexity is whether the class P equals the class NP. Clearly, P is a subclass of NP. Cook defined the class of NP-Complete problems. If any problem in NP-Complete class can be solved in polynomial time, then $P = NP$. M. R. Garey and D. S. Johnson, in their book "Computer and Interactability: A Guide to the theory of NP-Completeness" (Freeman, New York, 1979; MR80g: 68056), have given many problems in graph theory which have shown to be NP-Complete.

Definition 1.1 [4,5] A subset D of V is said to be strong dominating set if every vertex $u \in V - D$ is strongly dominated by v in D . The minimum cardinality of a strong dominating set is called the strong domination number of G and is denoted by $\gamma_s(G)$.

2. Chromatic Strong Dominating Set (CSDS)

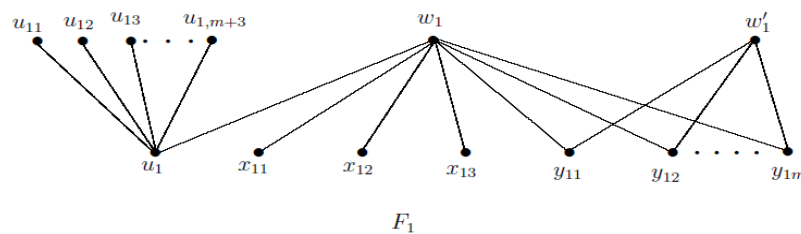
Definition 2.1 A subset D of V is said to be chromatic strong dominating set if D is strong dominating set and $\chi(<D>) = \chi(G)$. The minimum cardinality of a chromatic strong dominating set is called the chromatic strong domination number of G and is denoted by $\gamma_s^c(G)$.

INSTANCE : A graph $G = (V, E)$ and a positive integer $k \leq |V|$?

QUESTION : Is there a chromatic strong dominating set of cardinality at least k ?

Theorem 2.2 CSDS is NP-complete even for bipartite graphs.

Proof: We adopt the proof given in [4] for proving the NP-completeness of the problem of strong dominating sets in the class of bipartite graphs. We repeat the construction given in [4]. It is clear that CSDS is in NP. Let $X = \{x_1, x_2, \dots, x_{3q}\}$ and $C = \{c_1, c_2, \dots, c_m\}$ be an arbitrary instance of X3C. Corresponding to each variable x_i , associate a vertex u_i ($1 \leq i \leq 3q$). Corresponding to each set C_j ($1 \leq j \leq m$), associate a graph F_j , which is obtained from $K_{2,m} \cup K_{1,m+3}$ by joining a vertex of degree m in $K_{2,m}$ to the center of $K_{1,m+3}$. Let the center of $K_{1,m+3}$ be denoted by u_j and the vertex in $K_{2,m}$ to which it is joined be denoted by w_j . Let the other vertex of degree m in $K_{2,m}$ be denoted by w'_j . Join w_j with the vertices associated with the three elements of C_j . The graph thus obtained is bipartite.



Suppose C' is an exact 3-cover for X . Then $|C'| = q$. Then $S = \bigcup_{j=1}^m \{u_j, w'_j\} \cup \{w_j : C_j \in C'\}$ is a chromatic strong dominating set of G and $|S| = 2m + q$. Note that $\chi(< S >) = 2 = \chi(G)$.

Conversely, suppose S is a chromatic strong dominating set of G of cardinality at most k . Now $|(V(F_j) - \{w_j\}) \cap S| \geq 2$ for $j = 1, 2, \dots, m$. Let $S' = S \cap (X \cup C)$ where $C = \{w_1, w_2, \dots, w_m\}$. Then $|S'| + \sum_{j=1}^m |(V(F_j) - \{w_j\}) \cap S| = |S| \leq k$.

$|S'| \leq k - \sum_{j=1}^m |(V(F_j) - \{w_j\}) \cap S| = k - 2m = q$ ($k = 2m + q$). Let $|S \cap X| = \alpha$. Then $|S \cap C| \leq |S'| - |S \cap X| \leq q - \alpha$. Therefore $|N[S \cap C] \cap X| \leq 3(q - \alpha)$, (since any element of $S \cap C$ is adjacent to three elements of X). Therefore $|X - (S \cap C) - (N[S \cap C] \cap X)| \geq 3q - \alpha - 3(q - \alpha) = 2\alpha$. If $\alpha > 0$, then there exists $x_i \in X$ such that $x_i \notin N[S]$, a contradiction, since S strong dominates G . Therefore $\alpha = 0$. That is $S \cap X = \emptyset$. Therefore $S' = (S \cap X) \cup (S \cap C) = S \cap C \subset C$. Also S' strong dominates X . Let $C' = \{C_j : w_j \in S'\}$. Then $|C'| = |S'| \leq q$. And since S'

strong dominates X , C' must be a cover for X . Since every cover of X has cardinality atleast q . $|C'| \geq q$. Therefore $|C'| = q$. That is C' is an exact 3-cover for X .

References

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