



Another reduction formula for Kampé de Fériet function

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Abstract

A reduction formula for the Kampé de Fériet function has been derived as a generalization with the help of the hypergeometric identity. Further, it is related with the reduction formula obtained by Cvijović and Miller as the special case with some particular values.

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1 Introduction

The logarithmic derivative $\psi(z)$ (digamma function or psi function) of Euler gamma function for real and positive values of z is defined

by

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)} \quad (1)$$

With the help of this function, several new summation formulae have been derived recently. Moreover, the Pochhammer symbol is defined by

$$(\alpha)_n = \frac{\Gamma(\alpha + n)}{\Gamma(\alpha)} = \alpha(\alpha + 1)(\alpha + 2) \cdots (\alpha + n - 1), n > 0 \quad (2)$$

where n is an integer and $(\alpha)_0 = 1$. Also, the sequence $(\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_p)$ is denoted by (α_p) and the product of p Pochhammer symbols $((\alpha_p))$ is defined by

$$((\alpha_p))_n \equiv (\alpha_1)_n \cdots (\alpha_p)_n \quad (3)$$

where an empty product $p = 0$ reduces to unity.

The hypergeometric function is one of the most significant special function which plays an imperative role in the field of Mathematical Physics, Engineering and Applied Mathematics [Kilbas et al. [4], Kiryakova [5], Mathai et al. [7], Samko et al. [9] and Virchenko et al. [11]]. This fact has stimulated the investigation of several generalizations of hypergeometric functions.

The Kampé de Fériet function (KdF function) is defined by

$$F_{q:s:v}^{p:r:u} \left[\begin{matrix} (a_p) : (c_r) : (f_u) \\ (b_q) : (d_s) : (g_v) \end{matrix} ; x, y \right] = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{((a_p))_{m+n}}{((b_q))_{m+n}} \frac{((c_r))_m ((f_u))_n}{((d_s))_m ((g_v))_n} \frac{x^m}{m!} \frac{y^n}{n!} \quad (4)$$

which is a generalized hypergeometric function in two variables. Exton [3], Srivastava and Karlsson [10] have studied KdF function and established some new properties of them as well as its convergence principles. Cvijović [1] and Miller [8] represented a generalized hypergeometric type series containing the psi function with the help of KdF function in two equal variables. Also, they have derived several summation formulas involving hypergeometric type series encompassing the psi function. In this paper, another reduction formula for the Kampé de Fériet function has been derived as a generalization with the help of the hypergeometric identity.

2 Reduction Formula

Considering

$$\begin{aligned}
 F(z, z) &= F_{1:1:-}^{1:2:1} \left[\begin{matrix} \alpha : \beta - \varepsilon, \gamma : \varepsilon \\ \beta : \delta \quad \quad : - \end{matrix} ; z, z \right] \\
 &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\alpha)_{m+n}}{(\beta)_{m+n}} \frac{(\beta - \varepsilon)_m (\gamma)_m}{(\delta)_m} (\varepsilon)_n \frac{z^m}{m!} \frac{z^n}{n!} \{By(4)\}
 \end{aligned} \tag{6}$$

Using the identity $(\alpha)_{m+n} = (\alpha)_m (\alpha + m)_n$, the above is reduced to

$$\begin{aligned}
 F(z, z) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\alpha)_m (\alpha + m)_n}{(\beta)_m (\beta + m)_n} \frac{(\beta - \varepsilon)_m (\gamma)_m}{(\delta)_m} (\varepsilon)_n \frac{z^m}{m!} \frac{z^n}{n!} \\
 &= \sum_{m=0}^{\infty} \frac{(\alpha)_m (\beta - \varepsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} \sum_{n=0}^{\infty} \frac{(\alpha + m)_n}{(\beta + m)_n} (\varepsilon)_n \frac{z^n}{n!} \\
 &= \sum_{m=0}^{\infty} \frac{(\alpha)_m (\beta - \varepsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} {}_2F_1 \left[\begin{matrix} \alpha + m, \varepsilon \\ \beta + m \end{matrix} ; z \right] \tag{5}
 \end{aligned}$$

According to (3.31) of [6],

$$\begin{aligned}
 {}_2F_1 \left[\begin{matrix} a, b \\ c \end{matrix} ; z \right] &= \\
 &= (1 - z)^{c-a-b} \frac{\Gamma(c) \Gamma(a+b-c)}{\Gamma(a) \Gamma(b)} {}_2F_1 \left[\begin{matrix} -b + c, -a + c \\ 1 - a - b + c \end{matrix} ; 1 - z \right] + \\
 &+ \frac{\Gamma(c) \Gamma(c-a-b)}{\Gamma(c-a) \Gamma(c-b)} {}_2F_1 \left[\begin{matrix} a, b \\ 1 + a + b - c \end{matrix} ; 1 - z \right] \tag{6}
 \end{aligned}$$

Substituting $a = \alpha + m$, $\beta = \epsilon$, $c = \beta + m$ in (6),

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} \alpha+m \\ \beta+m \end{matrix} ; z \right] &= (1-z)^{\beta+m-\alpha-m-\epsilon} \frac{[\alpha+m+\epsilon-\beta-m]}{[\alpha+m]_{\overline{\epsilon}}} \\
&\quad {}_2F_1 \left[\begin{matrix} -\epsilon+\beta+m & -\alpha-m+\beta+m \\ 1-\alpha-m-\epsilon+\beta+m \end{matrix} ; 1-z \right] + \\
&\quad \frac{[\beta+m]_{\overline{\beta+m-\alpha-m-\epsilon}}}{[\beta+m-\alpha-m]_{\overline{\beta+m-\epsilon}}} \\
&\quad {}_2F_1 \left[\begin{matrix} \alpha+m \\ 1+\alpha+m+\epsilon-\beta-m \end{matrix} ; 1-z \right] \\
&= (1-z)^{\beta-\alpha-\epsilon} \frac{[\beta+m]_{\overline{\beta+m-\alpha-\epsilon}}}{[\alpha+m]_{\overline{\epsilon}}} \\
&\quad {}_2F_1 \left[\begin{matrix} \beta-\epsilon+m & \beta-\alpha \\ 1-\alpha-\epsilon+\beta \end{matrix} ; 1-z \right] + \\
&\quad \frac{[\beta+m]_{\overline{\beta-\alpha-\epsilon}}}{[\beta-\alpha]_{\overline{\beta-\epsilon+m}}} {}_2F_1 \left[\begin{matrix} \alpha+m \\ 1+\alpha+\epsilon-\beta \end{matrix} ; 1-z \right]
\end{aligned}$$

Therefore, (4) reduced to

$$\begin{aligned}
F(z, z) &= \sum_{m=0}^{\infty} \left(\frac{(\alpha)_m (\beta-\epsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} \right) \\
&\quad \left(\left[(1-z)^{\beta-\alpha-\epsilon} \frac{[\beta+m]_{\overline{\beta+m-\alpha-\epsilon}}}{[\alpha+m]_{\overline{\epsilon}}} {}_2F_1 \left[\begin{matrix} \beta-\epsilon+m, \beta-\alpha \\ 1-\alpha-\epsilon+\beta \end{matrix} ; 1-z \right] \right. \right. \\
&\quad \left. \left. + \frac{[\beta+m]_{\overline{\beta-\alpha-\epsilon}}}{[\beta-\alpha]_{\overline{\beta-\epsilon+m}}} {}_2F_1 \left[\begin{matrix} \alpha+m, \epsilon \\ 1+\alpha+\epsilon-\beta \end{matrix} ; 1-z \right] \right] \right)
\end{aligned}$$

Splitting the summation into two parts,

$$\begin{aligned}
F(z, z) &= (1-z)^{\beta-\alpha-\epsilon} \frac{[\alpha+\epsilon-\beta]}{[\epsilon]} \sum_{m=0}^{\infty} \left[\frac{(\alpha)_m (\beta-\epsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} \right] \\
&\quad \left[\frac{[\beta+m]_{\overline{\beta+m-\alpha-\epsilon}}}{[\alpha+m]_{\overline{\epsilon}}} {}_2F_1 \left[\begin{matrix} \beta-\epsilon+m, \beta-\alpha \\ 1-\alpha-\epsilon+\beta \end{matrix} ; 1-z \right] \right] + \frac{[\beta-\alpha-\epsilon]}{[\beta-\alpha]} \\
&\quad \sum_{m=0}^{\infty} \frac{(\alpha)_m (\beta-\epsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} \left[\frac{[\beta+m]_{\overline{\beta-\alpha-\epsilon}}}{[\beta-\epsilon+m]} {}_2F_1 \left[\begin{matrix} \alpha+m, \epsilon \\ 1+\alpha+\epsilon-\beta \end{matrix} ; 1-z \right] \right]
\end{aligned}$$

Applying equation [2] in the RHS of the above equation and simplifying, one can obtain

$$\begin{aligned}
F(z, z) &= (1-z)^{\beta-\alpha-\varepsilon} \frac{\overline{\alpha+\varepsilon-\beta}}{\overline{\beta}} \sum_{m=0}^{\infty} \left[\frac{(\alpha)_m (\beta-\varepsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} \right] \\
&\quad \left[\frac{(\beta)_m \overline{\beta}}{(\alpha)_m \overline{\alpha}} {}_2F_1 \left[\begin{matrix} \beta-\varepsilon+m & \beta-\alpha \\ 1-\alpha-\varepsilon+\beta \end{matrix}; 1-z \right] \right] + \frac{\overline{\beta-\alpha-\varepsilon}}{\overline{\beta-\alpha}} \\
&\quad \sum_{m=0}^{\infty} \left[\frac{(\alpha)_m (\beta-\varepsilon)_m (\gamma)_m}{(\beta)_m (\delta)_m} \frac{z^m}{m!} \right] \\
&\quad \left[\frac{(\beta)_m \overline{\beta}}{(\beta-\varepsilon)_m \overline{\beta-\varepsilon}} {}_2F_1 \left[\begin{matrix} \alpha+m & \varepsilon \\ 1+\alpha+\varepsilon-\beta \end{matrix}; 1-z \right] \right] \\
&= (1-z)^{\beta-\alpha-\varepsilon} \frac{\overline{\alpha+\varepsilon-\beta}}{\overline{\beta}} \frac{\overline{\beta}}{\overline{\alpha}} \sum_{m=0}^{\infty} \left[\frac{(\beta-\varepsilon)_m (\gamma)_m}{(\delta)_m} \frac{z^m}{m!} \right] \\
&\quad \left[{}_2F_1 \left[\begin{matrix} \beta-\varepsilon+m & \beta-\alpha \\ 1-\alpha-\varepsilon+\beta \end{matrix}; 1-z \right] \right] + \frac{\overline{\beta-\alpha-\varepsilon}}{\overline{\beta-\alpha}} \frac{\overline{\beta}}{\overline{\beta-\varepsilon}} \\
&\quad \sum_{m=0}^{\infty} (\alpha)_m \frac{(\gamma)_m}{(\delta)_m} \frac{z^m}{m!} \left[{}_2F_1 \left[\begin{matrix} \alpha+m & \varepsilon \\ 1+\alpha+\varepsilon-\beta \end{matrix}; 1-z \right] \right] \\
&= (1-z)^{\beta-\alpha-\varepsilon} \frac{\overline{\alpha+\varepsilon-\beta}}{\overline{\beta}} \frac{\overline{\beta}}{\overline{\alpha}} \sum_{m=0}^{\infty} \frac{(\beta-\varepsilon)_m (\gamma)_m}{(\delta)_m} \frac{z^m}{m!} \\
&\quad \sum_{n=0}^{\infty} \frac{(\beta-\varepsilon+m)_n (\beta-\alpha)_n}{(1-\alpha-\varepsilon+\beta)_n} \frac{(1-z)^n}{n!} + \frac{\overline{\beta-\alpha-\varepsilon}}{\overline{\beta-\alpha}} \frac{\overline{\beta}}{\overline{\beta-\varepsilon}} \\
&\quad \sum_{m=0}^{\infty} (\alpha)_m \frac{(\gamma)_m}{(\delta)_m} \frac{z^m}{m!} \sum_{n=0}^{\infty} \frac{(\alpha+m)_n (\varepsilon)_n}{(1+\alpha+\varepsilon-\beta)_n} \frac{(1-z)^n}{n!}
\end{aligned}$$

$$\begin{aligned}
&= (1-z)^{\beta-\alpha-\varepsilon} \frac{[\alpha+\varepsilon-\beta]}{\mathbb{F}} \frac{[\beta]}{\bar{\alpha}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{(\beta-\varepsilon)_m (\gamma)_m}{(\delta)_m} \right] \\
&\quad \left[\frac{(\beta-\varepsilon+m)_n (\beta-\alpha)_n}{(1-\alpha-\varepsilon+\beta)_n} \right] \left[\frac{z^m (1-z)^n}{m! n!} \right] + \frac{[\beta-\alpha-\varepsilon]}{[\beta-\alpha]} \\
&\quad \frac{[\beta]}{[\beta-\varepsilon]} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\alpha)_m (\gamma)_m}{(\delta)_m} \frac{(\beta-\alpha)_n}{(1-\alpha-\varepsilon+\beta)_n} \frac{z^m (1-z)^n}{m! n!} \\
&= (1-z)^{\beta-\alpha-\varepsilon} \frac{[\alpha+\varepsilon-\beta]}{\mathbb{F}} \frac{[\beta]}{\bar{\alpha}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{(\beta-\varepsilon)_{m+n} (\gamma)_m}{(\delta)_m} \right] \\
&\quad \left[\frac{(\beta-\alpha)_n}{(1-\alpha-\varepsilon+\beta)_n} \frac{z^m (1-z)^n}{m! n!} \right] + \frac{[\beta-\alpha-\varepsilon]}{[\beta-\alpha]} \frac{[\beta]}{[\beta-\varepsilon]} \\
&\quad \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\alpha)_{m+n} (\gamma)_m}{(\delta)_m} \frac{(\varepsilon)_n}{(1+\alpha+\varepsilon-\beta)_n} \frac{z^m (1-z)^n}{m! n!} \\
&= (1-z)^{\beta-\alpha-\varepsilon} \frac{[\alpha+\varepsilon-\beta]}{\mathbb{F}} \frac{[\beta]}{\bar{\alpha}} F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta-\varepsilon : \gamma : \beta-\alpha \\ - : \delta : 1-\alpha-\varepsilon+\beta \end{matrix} ; z, 1-z \right] \\
&\quad + \frac{[\beta-\alpha-\varepsilon]}{[\beta-\alpha]} \frac{[\beta]}{[\beta-\varepsilon]} F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : \gamma : \varepsilon \\ - : \delta : 1+\alpha+\varepsilon-\beta \end{matrix} ; z, 1-z \right]
\end{aligned}$$

Hence

$$\begin{aligned}
&F_{1:1:-}^{1:2:1} \left[\begin{matrix} \alpha : \beta-\varepsilon, \gamma : \varepsilon \\ \beta : \delta : - \end{matrix} ; z, z \right] = (1-z)^{\beta-\alpha-\varepsilon} \frac{[\alpha+\varepsilon-\beta]}{\mathbb{F}} \frac{[\beta]}{\bar{\alpha}} \\
&F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta-\varepsilon : \gamma : \beta-\alpha \\ - : \delta : 1-\alpha-\varepsilon+\beta \end{matrix} ; z, 1-z \right] + \frac{[\beta-\alpha-\varepsilon]}{[\beta-\alpha]} \frac{[\beta]}{[\beta-\varepsilon]} \\
&F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : \gamma : \varepsilon \\ - : \delta : 1+\alpha+\varepsilon-\beta \end{matrix} ; z, 1-z \right] \quad (7)
\end{aligned}$$

This is valid when $|z| < 1$ since the convergence for the KdF function occurs when $|z| < 1$ (see [10, p. 27]).

3 Special Cases

Further, the convergence for ${}_3F_2$ function obviously occurs when $\left| \frac{z}{z-1} \right| < 1$. Thus, when z lies in the domain $|z| < 1$ and $\Re(z) < \frac{1}{2}$, by Eq (2.2) of [2], (5) becomes

$$\frac{1}{(1-z)^\alpha} {}_3F_2 \left(\begin{matrix} \alpha ; \beta-\varepsilon ; \delta-\gamma \\ \beta ; \delta \end{matrix} ; \frac{z}{z-1} \right) = (1-z)^{\beta-\alpha-\varepsilon} \frac{[\alpha+\varepsilon-\beta]}{\mathbb{F}} \frac{[\beta]}{\bar{\alpha}}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta - \varepsilon : \gamma : \beta - \alpha \\ - : \delta : 1 - \alpha - \varepsilon + \beta \end{matrix} ; z, 1 - z \right] + \frac{|\beta - \alpha - \varepsilon|}{|\beta - \alpha|} \frac{|\beta|}{|\beta - \varepsilon|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : \gamma : \varepsilon \\ - : \delta : 1 + \alpha + \varepsilon - \beta \end{matrix} ; z, 1 - z \right]$$

Therefore,

$${}_3F_2 \left(\begin{matrix} \alpha ; \beta - \varepsilon ; \delta - \gamma \\ \beta ; \delta \end{matrix} ; \frac{z}{z-1} \right) = (1-z)^{\beta-\varepsilon} \frac{|\alpha+\varepsilon-\beta|}{|\beta|} \frac{|\beta|}{|\alpha|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta - \varepsilon : \gamma : \beta - \alpha \\ - : \delta : 1 - \alpha - \varepsilon + \beta \end{matrix} ; z, 1 - z \right] + (1-z)^{\alpha+\beta-\varepsilon} \frac{|\beta-\alpha-\varepsilon|}{|\beta-\alpha|} \frac{|\beta|}{|\beta-\varepsilon|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : \gamma : \varepsilon \\ - : \delta : 1 + \alpha + \varepsilon - \beta \end{matrix} ; z, 1 - z \right]$$

When $\varepsilon = 1$, $\delta = \beta$, by Eq (2.21) of [2], (7) becomes

$$\frac{1}{(1-z)^\alpha} {}_3F_2 \left(\begin{matrix} \alpha ; \beta - 1 ; \beta - 1 \\ \beta ; \beta \end{matrix} ; \frac{z}{z-1} \right) = (1-z)^{\beta-\alpha-1} \frac{|\alpha+1-\beta|}{|\beta|} \frac{|\beta|}{|\alpha|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta - 1 : 1 : \beta - \alpha \\ - : \beta : \beta - \alpha \end{matrix} ; z, 1 - z \right] + \frac{|\beta-\alpha-1|}{|\beta-\alpha|} \frac{|\beta|}{|\beta-1|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : 1 : 1 \\ - : \beta : \alpha + 2 - \beta \end{matrix} ; z, 1 - z \right]$$

$${}_3F_2 \left(\begin{matrix} \alpha ; \beta - 1 ; \beta - 1 \\ \beta ; \beta \end{matrix} ; \frac{z}{z-1} \right) = (1-z)^{\beta-1} \frac{|\alpha+1-\beta|}{|\beta|} \frac{|\beta|}{|\alpha|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta - 1 : 1 : \beta - \alpha \\ - : \beta : \beta - \alpha \end{matrix} ; z, 1 - z \right] + (1-z)^\alpha \frac{|\beta-\alpha-1|}{|\beta-\alpha|} \frac{|\beta|}{|\beta-1|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : 1 : 1 \\ - : \beta : \alpha + 2 - \beta \end{matrix} ; z, 1 - z \right]$$

Further (7) becomes

$$F_{1:1:-}^{1:2:1} \left[\begin{matrix} \alpha : \beta - 1, 1 : 1 \\ \beta : \beta : - \end{matrix} ; z, z \right] = (1-z)^{\beta-\alpha-1} \frac{|\alpha+1-\beta|}{|\beta|} \frac{|\beta|}{|\alpha|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \beta - 1 : 1 : \beta - \alpha \\ - : \beta : \beta - \alpha \end{matrix} ; z, 1 - z \right] + \frac{|\beta-\alpha-1|}{|\beta-\alpha|} \frac{|\beta|}{|\beta-1|}$$

$$F_{-:1:1}^{1:1:1} \left[\begin{matrix} \alpha : 1 : 1 \\ - : \beta : \alpha + 2 - \beta \end{matrix} ; z, 1 - z \right]$$

4 Conclusion

Thus, the reduction formula has been obtained as a generalization for the KdF function with the support of the hypergeometric identity. Further, it is related with the reduction formula obtained by Cvijović and Miller [2] as the special case with some particular values.

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References

- [1] D. Cvijović, Closed-form summations of certain hypergeometric-type series containing the digamma function, *J. Phys. A: Math. Theor.*, **41** (2008), 455205-455211.
- [2] D. Cvijović and A.R. Miller, A reduction formula for the Kampé de Fériet function, *Applied Mathematics Letters*, **23** (2010), 769-771.
- [3] H. Exton, *Multiple hypergeometric functions and applications*, Chichester UK: Ellis Horwood (1976).
- [4] A.A. Kilbas and M. Saigo, *H-Transforms*, Chapman and Hall/CRC Press, Boca Raton, FL(2004).
- [5] V. Kiryakova, *Generalized Fractional Calculus and Applications*, Wiley & Sons. Inc., New York(1994).
- [6] C. Krattenthaler and K. Srinivasa Rao, *Hypergeometric identities: New from old, by the beta integral method* (preprint).

- [7] A.M. Mathai and R.K. Saxena, *Generalized Hypergeometric Functions With Applications in Statistics and Physical Sciences*, Springer-Verlag, Berlin (1973).
- [8] A.R. Miller, Summations for certain series containing the digamma function, *J. Phys. A: Math. Gen.*, **39** 2006, 3011-3020.
- [9] S.G. Samko, A.A. Kilbas and O.I. Marichev, *Fractional Integrals and Derivatives-Theory and Applications*, Gordon and Breach Science Publishers, New York (1993).
- [10] H.M. Srivastava and P.W. Karlsson, *Multiple Gaussian hypergeometric series*, Chichester UK: Ellis Horwood (1985).
- [11] N. Virchenko, S.L.Kalla and A. Al Zamel, Some results on a generalized hypergeometric function, *Integral Transforms and Special Functions*, **12** (2001), 89-100.

