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Interleaver graph of Brick product graph C(2n, 1, 3) and Hamiltonian Laceability

R.A.Daisy Singh
B N M Institute of Technology
Bengaluru, India
thavanya2010@ymail.com

R.Murali
Dr. Ambedkar Institute of Technology
Bengaluru, India
muralir2968@gmail.com

Abstract—A good interleaver for turbo codes can be constructed from 3-regular hamiltonian graphs having large girth. The girth of a graph is the shortest cycle contained in a graph. In topological point of view it is important for any interconnection network to have various graph theoretic properties which includes girth. In this paper we present a construction of interleaver graphs IG_N for $N \geq 8$ from the brick product graph C(2n, 1, 3) and explore its hamiltonian-t*-laceability properties.

Index Terms—brick product, interleaver graph, hamiltonian laceability, edge fault tolerance.

INTRODUCTION

Graphs are important structures by using which many applications of the real world such as communication networks and social networks are described. In [4] authors have proposed a new construction of interleavers for turbo codes from 3-regular hamiltonian graphs. Using the concept of brick products Alspach et.al. showed in [1] that all cubic cayley graphs over dihedral groups are hamiltonian. It is also conjectured that all brick products C(2n, 1, r) are hamiltonian laceable such that any two vertices at odd distance apart can be joined by a hamiltonian path. Also in [3] Alspach et.al. showed that most C(2n, 1, r) are hamiltonian laceable for $3 \leq r \leq 9$. Inspired by their work we continue with the development of interleaver graph IG_N, $N \geq 8$ constructed from the brick product graph C(2n, 1, 3) and discuss the Hamiltonian-t*-laceability properties for all even N and odd t such that $1 \leq t < 10$. Further fault tolerance is an important property in the network performance. Also an edge fault tolerance is an important issue for a network as the edges in the network may fail sometimes. Hsieh et.al. in [2] proposed the edge fault tolerant hamiltonicity to measure the performance of the hamiltonian property in the faulty networks. Extenting this we also explore the edge fault tolerant hamiltonian laceability of interleaver graphs IG_N, $N \geq 8$ for all even N and t such that $2 \leq t \leq 10$.

Definition:1 A connected graph G is hamiltonian laceable if between every pair of distinct vertices in G at an odd distance there exists a hamiltonian path. G is hamiltonian-t-laceable (hamiltonian-t*-laceable) if there exists in it a Hamiltonian path between every pair (atleast one pair) of distinct vertices u and v with the property $d(u, v) = t$ such that $1 \leq t \leq diam(G)$ where diam(G) is the diameter of G.

Definition:2 Let P be a path between the vertices a_i and a_j in a graph G and P^{0} be the path between the vertices a and a_k. Then the path $P \cup P$ is the path obtained by extending P from a_i to a_j to a_k through the common vertex a_j. That is if $P : a_i \rightarrow a_j$ and $P : a_j \rightarrow a_k$ then $P \cup P : a_i \rightarrow a_j \rightarrow a_k$.

Definition:3 Let m, n and r be the positive integers. Let $C_n = a_0, a_1, a_2, a_3 ..... a_{(2n-1)}, a_0$ be the cycle of order 2n. The (m, r)- brick product of C_{2n} denoted by C(2n, m, r) is defined for $m = 1$, we require r to be odd and greater than 1. Then C(2n, m, r) is obtained from C_{2n} by adding chords $C_{2n} = a_{2k}a_{(2k+r)}$, $K = 1, 2, 3, ...n$ where the computation is performed under modulo 2n.

Definition:4 The graph G^* is k-edge fault tolerant with respect to a graph G if every graph obtained by removing any K edges from G^* contains G. Further a graph G^* is K-edge fault tolerant hamiltonian laceable if $G^* - F$ remains hamiltonian laceable for every $F \subseteq E(G^*)$ with $|F| \leq k$.

PRELIMINARIES

Construction of interleavers from brick product graph C(2n, 1, 3):

Let G be a graph of C(2n, 1, 3). Let $2n = N$ be the number of vertices labelled from 1 to N placed on a cycle. Initially the fixed cycle becomes the hamiltonian cycle of the graph. To construct IG_N from G we first draw the upper and lower chain with N vertices. Note that the upper and lower chains are the two cycles of length N. If i is the vertex from the upper chain and j is the vertex from the lower chain then i and j should be connected in the IG_N if there is an edge between i and j in G which is not a part of the hamiltonian cycle of G.

We need to first introduce the following terminologies to establish results.

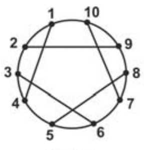
For each vertex a_{1i} and a_{2i} of IG_N, we shall write:

$a_{1i}P [N] = a_{1i}(a_{1(i+1)})(a_{1(i+2)})(a_{1(i+3)})(a_{1(i+4)}) ....(a_{1(i+N-1)})$.

$a_{1i}P^{-1}[N]=a_{1i}(a_{1(i-1)})(a_{1(i-2)})(a_{1(i-3)})(a_{1(i-4)}) ....(a_{1(i-N+1)})$.

$a_{2i}Q[N]=a_{2i}(a_{2(i+1)})(a_{2(i+2)})(a_{2(i+3)})(a_{2(i+4)})$ ....$(a_{2(i+N-1)})$.
$a_{1i}Q^{-1}[N]=a_{2i}(a_{2(i+1)})(a_{2(i+2)})(a_{2(i+3)})(a_{2(i+4)})$ ....$(a_{2(i-N+1)})$ for $i \le 1 \le N$.
Note that the symbols P [N], $P^{-1}[N]$, Q[N], $Q^{-1}[N]$ are the paths of order N where as the symbols R[m], $R^{-1[m]}$, S[m], $S^{-1[m]}$, T [m], $T^{-1[m]}$ I[m], $I^{-1[m]}$, J[m], $J^{-1[m]}$, K[m], $K^{-1[m]}$, L[m] are self explanatory.

For i = m = 1
$R[m] = a_{11} \to a_{1N}$; $R^{-1}[m] = a_{1N} \to a_{11}$
$S[m] = a_{21} \to a_{2N}$; $S^{-1}[m] = a_{2N} \to a_{21}$
$L[m] = a_{1(i+1)} \to a_{2(i+1)}$
For $1 \le i \le N - 3$ and for all odd i
$T[m] = a_{1i} \to a_{2(i+3)}$;
$T^{-1}[m] = a_{2(i+3)} \to a_{1i}$
$I[m] = a_{2i} \to a_{1(i+3)}$; $I^{-1}[m] = a_{1(i+3)} \to a_{2i}$ For i = m = 1, j = i + 1
$J[m]=a_{1j} \to a_{2(N-1)}$;
$J^{-1}[m] = a_{2(N-1)} \to a_{1j}$
$K[m]=a_{2j} \to a_{1(N-1)}$;
$K^{-1}[m] = a_{1(N-1)} \to a_{2j}$



C(10, 1, 3)
Figure 1.1(a)

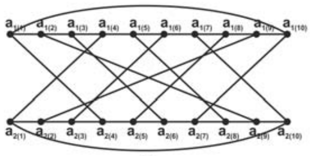

IG10
Figure 1.1(b)

figure 1.1(b) shows the IG_{10} constructed from the graph C(10, 1, 3) of figure 1.1(a).

RESULTS AND DISCUSSION

Theorem : 1 The graph of IG_N , N $\ge$ 8 constructed from C(2n, 1, 3) is Hamiltonian-t*-laceable for all even N and odd t such that $1 \le t < 10$.
Proof: Let IG_N be the interleaver graph of order N. Let $V = V_1 \cup V_2$ be the set of vertices such that $V_1 = \{a_{i(j)} : i = 1, 1 \le j \le N\}$,$V_2 = \{a_{i(j)} : i = 2, 1 \le j \le N\}$. Let E be the set of edges of IG_N such that $E = \{b_k : 1 \le k \le 3N\}$.
To establish the result we discuss the following cases.

case:1 In IG_N , $d(a_{1(1)}, a_{2(4)}) = 1$.

subcase (i): For N = 6r + 8, r = 0, 1, 2, 3...... P :
$a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r+1}S[m]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r+1}K[m]P[2]I^{-1}[m]Q^{-1}[2]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_r\ a_{2(4)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(4)}$.

subcase (ii): For N = 6r + 10, r = 0, 1, 2, 3...... P :
$a_{1(1)}P[2]J[m]\{Q^{-1}[2]T^{-1}[m]P[2]I^{-1}[m]\}_{r+1}Q^{-1}[2]K[m]\{P[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]\}_{r+1}P[2]I^{-1}[m]S[m]\{T^{-1}[m]P[2]I^{-1}[m]Q^{-1}[2]\}_{r+1}a_{2(4)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(4)}$.

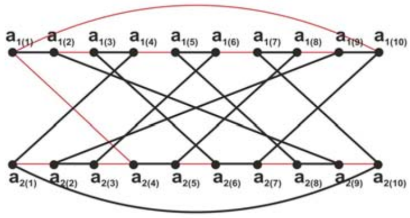


Figure 1.2

Hamiltonian path from $a_{1(1)}$ to $a_{2(4)}$

subcase (iii): For N = 24r + 12, r = 0, 1, 2, 3...... P :
$a_{1(1)}P[3]\{T[m]Q[2]I[m]P[2]\}_{3r}T[m]Q[2]I[m]P[3]\{I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]\}_{3r+1}I^{-1}[m]Q[3]\{I[m]p[2]T[m]Q[2]\}_{3r}I[m]P[2]T[m]Q[3]\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q^{-1}[2]\}_{3r+1}a_{2(4)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(4)}$.

subcase (iv): For N = 12r + 18, r = 0, 1, 2, 3...... P :
$a_{1(1)}P[2]J[m]\{Q[2]T^{-1}[m]P[2]I^{-1}[m]\}_{3r+4}Q[2]K[m]\{P[2]I^{-1}[m]Q[2]T^{-1}[m]\}_{3r+3}P[2]I^{-1}[m]Q[2]a_{2(4)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(4)}$.

subcase (v): For N = 24r, r = 1, 2, 3......
P : $a_{1(1)}P[3]\{T[m]Q[2]I[m]P[2]\}_{3r-1}T[m]Q[3]\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q^{-1}[2]\}_{3r-1}T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[3]\{I[m]P[2]T[m]Q[2]\}_{3r-1}I[m]P[3]\{I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]\}_{3r-1}I^{-1}[m]Q^{-1}[2]a_{2(4)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(4)}$.

case:2 In IG_N , $d(a_{1(1)}, a_{2(6)}) = 3$.

subcase (i): For N=6r + 8, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]\{I[m]P[2]T[m]Q^{-1}[2]\}_r I[m]P^{-1}[2]K^{-1}[m]Q[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_r P^{-1}[3]I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_r S[m]Q^{-1}[2]J^{-1}[m]P[2]T[m]a_{2(6)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(6)}$.

subcase (ii): For N=6r + 10, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_r\ P^{-1}[3]I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_r Q^{-1}[2]K[m]P[2]I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r+1}S[m]\{Q^{-1}[2]J^{-1}[m]P[2]\ T[m]\}_1 a_{2(6)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(6)}$.

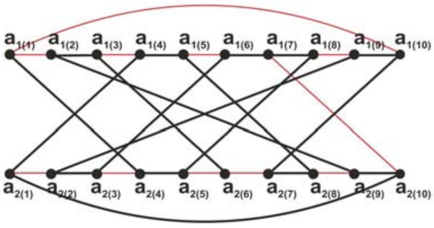


Figure 1.3

Hamiltonian path from $a_{1(1)}$ to $a_{2(6)}$

subcase (iii): For N = 12r, r = 1, 2, 3...... $a_{1(1)}P[2]J[m]\{Q[2]T^{-1}[m]P[2]I^{-1}[m]\}_{3r-1}$ $Q[3]I[m]P^{-1}[2]\{T[m]Q^{-1}[2]I[m]P^{-1}[2]\}_{3r-2}$ $K^{-1}[m]Q^{-1}[2]I[m]P^{-1}[2]T[m]a_{2(6)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(6)}$.

subcase (iv): For N=12r + 6, r = 1, 2, 3...... $a_{1(1)}P[2]J[m]\{Q[2]T^{-1}[m]P[2]I^{-1}[m]\}_{3r}Q^{-1}[3]I[m]P^{-1}[2]\{T[m]Q^{-1}[2]I[m]P^{-1}[2]\}_{3r}$ $K^{-1}[m]Q^{-1}[2]I[m]P^{-1}[2]T[m]a_{2(6)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(6)}$.

case:3 In IG_N, $d(a_{1(1)}, a_{2(8)}) = 5$.

subcase (i): For N=6r + 14, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_{r+1}$ $P^{-1}[2]K^{-1}[m]Q[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_{r+1}$ $P^{-1}[3]I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r}$ $Q^{-1}[2]T^{-1}[m]P^{-1}[2]J[m]Q[2]S^{-1}[m]I[m]P[2]$ $T[m]a_{2(8)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(8)}$.

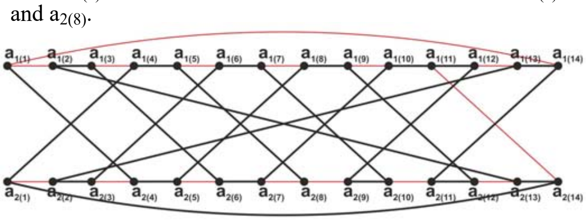


Figure 1.4

Hamiltonian path from $a_{1(1)}$ to $a_{2(8)}$

subcase (ii): For N=6r + 16, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_{r+1}P^{-1}[3]I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r+1}$ $Q^{-1}[2]K[m]P[2]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r+1}$ $I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]J[m]Q[2]S^{-1}[m]I[m]$ $P[2]T[m]a_{2(8)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(8)}$.

subcase (iii): For N=6r + 18, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]\{I[m]P[2]T[m]Q^{-1}[2]\}_{r+2}J^{-1}[m]$ $P[2]T[m]Q[2]\{I[m]P[2]T[m]Q^{-1}[2]\}_{r+1}I[m]P[3]$ $I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r+2}$ $Q^{-1}[3]I[m]P[2]T[m]a_{2(8)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(8)}$.

case:4 In IG_N, $d(a_{1(1)}, a_{2(14)}) = 7$.

subcase (i): For N=6r + 22, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]I[m]P[2]T[m]Q[2]I[m]$ $\{P[2]T[m]Q^{-1}[2]I[m]\}_{r+1}P^{-1}[2]$ $K^{-1}[m]\{Q[2]I[m]P[2]T[m]\}_2$ $\{Q^{-1}[2]I[m]P[2]T[m]\}_rQ^{-1}[2]I[m]$ $P^{-1}[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r+1}$ $I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]S[m]$ $Q^{-1}[2]J^{-1}[m]P[2]T[m]Q[2]I[m]P[2]$ $T[m]a_{2(14)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(14)}$.

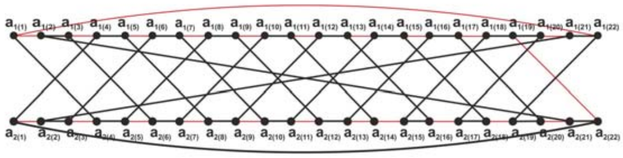


Figure 1.5

Hamiltonian path from $a_{1(1)}$ to $a_{2(14)}$

subcase (ii): For N=6r + 24, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]I[m]P[2]T[m]Q[2]I[m]$ $\{P[2]T[m]Q^{-1}[2]I[m]\}_{r+1}P^{-1}[3]I^{-1}[m]Q[2]$ $\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[2]\}_r\{T^{-1}[m]P^{-1}[2]$ $I^{-1}[m]Q^{-1}[2]\}_2K[m]P[2]\{I^{-1}[m]Q[2]T^{-1}[m]$ $P^{-1}[2]\}_{r+2}I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]$ $S[m]Q^{-1}[2]J^{-1}[m]P[2]T[m]Q[2]I[m]P[2]$ $T[m]a_{2(14)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(14)}$.

subcase (iii): For N=6r + 26, r = 0, 1, 2, 3......
$a_{1(1)}P[2]J[m]Q[2]S^{-1}[m]Q[2]K[m]P[2]$ $I^{-1}[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r+2}Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[3]I[m]P[2]T[m]Q[2]$ $I[m]P[2]T[m]Q^{-1}[2]I[m]P[3]\{I^{-1}[m]Q[2]$ $T^{-1}[m]P^{-1}[2]\}_2I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[3]$ $T[m]Q[2]I[m]P[2]T[m]a_{2(14)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(14)}$.

case:5 In IG_N, $d(a_{1(1)}, a_{2(16)}) = 9$.

subcase (i): For N=6r + 30, r = 0, 1, 2, 3......
$a_{1(1)}T[m]Q[2]I[m]P[2]T[m]Q[2]I[m]$ $\{P[2]T[m]Q^{-1}[2]I[m]\}_{r+2}P[3]I^{-1}[m]$ $\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r+2}$ $\{Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_1Q^{-1}[2]T^{-1}[m]$ $P^{-1}[2]J[m]\{Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r+3}$ $Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]Q^{-1}[3]I[m]P[2]$

T [m]Q[2]I[m]P [2]T [m]$a_{2(16)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(16)}$.

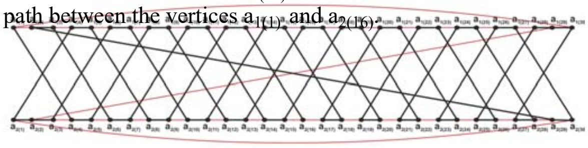


Figure 1.6

Hamiltonian path from $a_{1(1)}$ to $a_{2(16)}$

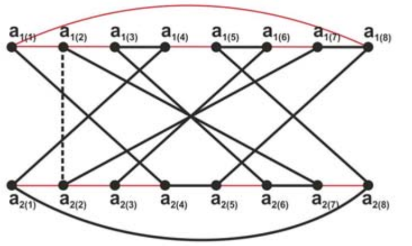


Figure 1.7

Hamiltonian path from $a_{1(1)}$ to $a_{2(3)}$

subcase (ii): For N=6r + 32, r = 0, 1, 2, 3...... $a_{1(1)}$T [m]Q[2]I[m]P [2]T [m]Q[2]{I[m]P [2]T [m] Q^{-1}[2]}$_{r+3}J^{-1}$[m]P^{-1}[2]T [m]Q[2]I[m]{P [2]T [m] Q[2]I[m]}$_{r+3}$P [3]I^{-1}[m]{Q[2]T^{-1}[m]P^{-1}[2] I^{-1}[m]}$_{r+3}Q^{-1}$[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]Q^{-1}[3] I[m]P [2]T [m]Q[2]I[m]P [2]T [m]$a_{2(16)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(16)}$.

subcase (iii): For N=6r + 34, r = 0, 1, 2, 3...... $a_{1(1)}$T [m]{Q[2]I[m]P [2]T [m]}$_2${Q^{-1}[2] I[m]P [2]T [m]}$_2Q^{-1}$[2]I[m]{P [2]T [m] Q^{-1}[2]I[m]}$_r$ P^{-1}[2]K^{-1}[m] {Q[2]I[m]P [2]T [m]}$_2Q^{-1}$[2]I[m]{P [2]T [m] Q^{-1}[2]I[m]}$_{r+2}P^{-1}$[3]I^{-1}[m]{Q[2]T^{-1}[m] P^{-1}[2]I^{-1}[m]}$_{r+2}Q^{-1}$[2]T^{-1}[m]P^{-1}[2] I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]J[m]Q[2] S^{-1}[m]I[m]P [2]T [m]Q[2]I[m]P [2] T [m]$a_{2(16)}$ is the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(16)}$.

Theorem : 2 The graph of IG_N, $N \geq 8$ is one edge fault tolerant hamiltonian-t*-laceable for all even N and t such that $2 \leq t \leq 10$.

Proof: Let IG_N be the interleaver graph of order N. Let $V = V_1 \cup V_2$ be the set of vertices such that $V_1 = \{a_{i(j)} : i = 1, 1 \leq j \leq N\}$, $V_2 = \{a_{i(j)} : i = 2, 1 \leq j \leq N\}$. Let E be the set of edges of IG_N such that $E = \{b_k : 1 \leq k \leq 3N\} \cup E^0$ where $E^0 = \{a_{1(2)} - a_{2(2)}\}$.

case:1 In IG_N, $d(a_{1(1)}, a_{2(3)}) = 2$.

subcase (i): For N = 6r + 2, r = 1, 2, 3...... the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is P : $P_1 \cup P_2 \cup P_3$ where P_1 : $a_{1(1)}${T [m]Q[2]I[m]P^{-1}[2]}$_r K^{-1}$[m]$a_{2(2)}$.
P_2 : $a_{2(3)}${I[m]P^{-1}[2]T [m]Q[2]}$_r$I[m]P^{-1}[2]T [m] S^{-1}[m]{I[m]P^{-1}[2]T [m]Q[2]}$_r$ J^{-1}[m]$a_{1(2)}$ and P_3 : $a_{1(2)}$L[m]$a_{2(2)}$.

subcase (ii): For N = 6r + 4, r = 1, 2, 3...... the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is P : $P_1 \cup P_2 \cup P_3$ where
P_1 : $a_{1(1)}${T [m]Q[2]I[m]P^{-1}[2]}$_r$T [m]S^{-1}[m] {I[m]P^{-1}[2]T [m]Q[2]}$_r$I[m]P^{-1}[2]K^{-1}[m]$a_{2(2)}$.
P_2:{I[m]P^{-1}[2]T [m]Q[2]}$_r J^{-1}$[m]$a_{1(2)}$
and P_3:$a_{1(2)}$L[m]$a_{2(2)}$.

subcase (iii): For N = 12r, r = 1, 3, 5, 7......, s = 1, 2, 3, 4.... the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is P : $P_1 \cup P_2 \cup P_3$ where P_1:$a_{1(1)}$R[m]I^{-1}[m]{Q^{-1} [2]T^{-1}[m]P^{-1}[2]I^{-1}[m]}$_{3s-3}Q^{-1}$[2] T^{-1} [m]p[3]{T [m]Q[2]I[m]P [2]}$_{3s-3}$T [m]Q[2]J^{-1} [m]$a_{1(2)}$.
P_2 : $a_{2(3)}$Q[3]{I[m]P [2]T [m]Q[2]}$_{3s-3}$I[m]P [2]T [m]S^{-1} [m] I[m]P^{-1}[2]{T [m]Q[2]I[m]P [2]}$_{3s-2}K^{-1}$[m]$a_{2(2)}$ and P_3:$a_{1(2)}$L[m]$a_{2(2)}$.

subcase (iv): For N = 6r + 12, r = 1, 5, 9, 13......, s = 1, 2, 3, 4.... the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is P : $P_1 \cup P_2 \cup P_3$ where P_1:$a_{1(1)}$R[m]I^{-1}[m]{Q^{-1} [2]T^{-1}[m]P^{-1}[2]I^{-1}[m]}$_{3s-2}$ Q^{-1} [2]T^{-1}[m]P [2]I^{-1}[m]S[m]T^{-1}[m]{P^{-1}[2]I^{-1}[m] Q^{-1} [2]T^{-1}[m]}$_{3s-2}P^{-1}$[3]T [m]Q[2]{I[m]P [2]T [m]Q[2]}$_{3s-2}$ J^{-1} [m]$a_{1(2)}$.
P_2:$a_{2(3)}$Q[3]I[m]P [2]{T [m]Q[2]I[m]P [2]}$_{3s-2}K^{-1}$[m]$a_{2(2)}$ and P_3:$a_{1(2)}$L[m]$a_{2(2)}$.

subcase (v): For N = 24r, r = 1, 2, 3, 4...... the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is P : $P_1 \cup P_2 \cup P_3$ where P_1 : $a_{1(1)}$T [m]Q[2]I[m]{P [2]T [m]Q[2]I[m]}$_{3r-1}P^{-1}$[2]K^{-1}[m]$a_{2(2)}$.
P_2 : $a_{2(3)}$I[m]{P [2]T [m]Q[2]I[m]}$_{3r-1}P^{-1}$[3]I^{-1}[m] {Q^{-1} [2]T^{-1}[m]P^{-1}[2]I^{-1}[m]}$_{3r-1}$S[m]Q^{-1}[3] {T^{-1} [m]P^{-1}[2]I^{-1}[m]Q^{-1} [2]}$_{3r-1}T^{-1}$[m]P^{-1}[2]$a_{1(2)}$ and P_3 : $a_{1(2)}$L[m]$a_{2(2)}$.

subcase (vi): For N = 6r + 24, r = 1, 5, 9, 13......, s = 1, 2, 3, 4.... the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(3)}$ is P : $P_1 \cup P_2 \cup P_3$ where P_1:$a_{1(1)}$T [m]{Q[2]I[m]P [2]T [m]}$_{3s}$Q[3]S^{-1}[m]I[m]

$\{P [2]T [m]Q[2]I[m]\}_{3s}P^{-1}[3]\{I^{-1}[m]Q^{-1}[2] T^{-1}[m]P^{-1}[2]\}_{3s}a_{1(2)}$. $P_2:a_{2(3)}I[m]\{P [2]T [m]Q[2]I[m]\}_{3s} P^{-1}[2]K^{-1}[m]a_{2(2)}$ and $P_3:a_{1(2)}L[m]a_{2(2)}$.

case:2 In IG_N , $d(a_{1(1)}, a_{2(1)}) = 4$.

subcase (i): For $N = 4r + 4$, $r = 1, 2, 3......$
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(1)}$ is $P : P_1 \cup P_2 \cup P_3$ where $P_1:a_{1(1)}T [m]Q^{-1}[2]\{I[m]P^{-1}[2]T [m]Q^{-1}[2]\}_rJ^{-1}[m]a_{1(2)}$. $P_2 : a_{2(1)}I[m]P^{-1}[2]\{T [m]Q^{-1}[2]I[m]P^{-1}[2]\}_rK^{-1}[m]a_{2(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

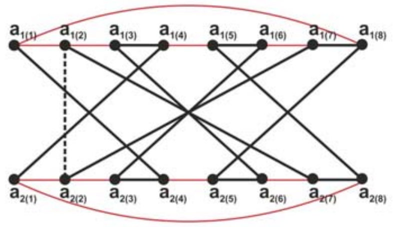


Figure 1.8

Hamiltonian path from $a_{1(1)}$ to $a_{2(1)}$

subcase (ii): For $N = 4r + 6$, $r = 1, 2, 3......$ the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(1)}$ is $P : P_1 \cup P_2 \cup P_3$ where
$P_1 : a_{1(1)}\{T [m]Q^{-1}[2]I[m]P^{-1}[2]\}_{r+1}K^{-1}[m]a_{2(2)}$
$P_2 : a_{2(1)}\{I[m]P^{-1}[2]T [m]Q^{-1}[2]\}_{r+1}J^{-1}[m]a_{1(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

case:3 In IG_N , $d(a_{1(1)}, a_{2(9)}) = 6$.

subcase (i): For $N = 6r + 10$, $r = 1, 2, 3......$
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(9)}$ is $P : P_1 \cup P_2 \cup P_3$ where $P_1:a_{1(1)}T [m]Q^{-1}[2]I[m]\{P^{-1}[2]T [m]Q^{-1}[2]I[m]\}_2$ $\{P [2]T [m]Q^{-1}[2]I[m]\}_{r-1}P [3]I^{-1}[m]Q[2]T^{-1}[m]$ $\{P^{-1}[2]I^{-1}[m]Q[2]T^{-1}[m]\}_{r-1}P [3]T [m]Q^{-1}[2]$ $\{I[m]P [2]T [m]Q^{-1}[2]\}_{r-1}J^{-1}[m]a_{1(2)}$.
$P_2:a_{2(9)}\{Q[2]T^{-1}[m]P [2]I^{-1}[m]\}_2Q[2]a_{2(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

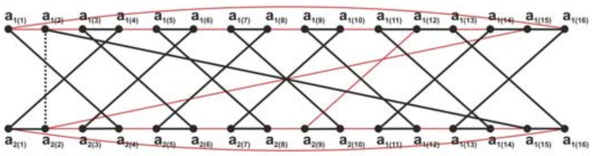


Figure 1.9

Hamiltonian path from $a_{1(1)}$ to $a_{2(9)}$

subcase (ii): For $N = 6r + 12$, $r = 1, 2, 3......$
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(9)}$ is $P : P_1 \cup P_2 \cup P_3$ where $P_1:a_{1(1)}T [m]\{Q^{-1}[2]I[m]P^{-1}[2]T [m]\}_2\{Q[2]I[m]P^{-1}[2]T [m]\}_r S^{-1}[m]\{I[m]P^{-1}[2]T [m]Q^{-1}[2]\}_{r+2}J^{-1}[m]a_{1(2)}$. $P_2:a_{2(9)}\{I[m]P^{-1}[2]T [m]Q[2]\}_rI[m]P^{-1}[2]K^{-1}[m]a_{2(2)}$
and $P_3:a_{1(2)}L[m]a_{2(2)}$.

subcase (iii): For $N = 6r + 14$, $r = 1, 2, 3......$
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(1)}$ is $P : P_1 \cup P_2 \cup P_3$ where $P_1:a_{1(1)}T [m]Q^{-1}[2]I[m]\{P^{-1}[2]T [m]Q^{-1}[2]I[m]\}_2\{P [2]T [m] Q^{-1}[2]I[m]\}_rP^{-1}[3]\ \{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_{r-1}I^{-1}[m]Q[2] T^{-1}[m]P^{-1}[3]\{T [m]Q^{-1}[2]I[m]P [2]\}_rT [m]Q^{-1}[2]J^{-1}[m]a_{1(2)}$.
$P_2:a_{2(9)}\{Q[2]T^{-1}[m]P [2]I^{-1}[m]\}_2Q[2]a_{2(2)}$
and $P_3:a_{1(2)}L[m]a_{2(2)}$.

case:4 In IG_N , $d(a_{1(1)}, a_{1(13)})=8$ where $(a_{1(1)}, a_{1(13)}) \in V_1$.

subcase (i): For $N = 24$, the hamiltonian path between the vertices $a_{1(1)}$ and $a_{1(13)}$ is $P : P_1 \cup P_2 \cup P_3$ where $P_1 : a_{1(1)}P [2]a_{1(2)}$.
$P_2 : a_{1(13)}T [m]Q[2]I[m]P [3]I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m] Q^{-1}[2]T^{-1}[m]P [2]I^{-1}[m]Q^{-1}[3]\{I[m]P^{-1}[2]T [m]Q[2]\}_2 I[m]P [2]T [m]Q[3]S^{-1}[m]\{I[m]P^{-1}[2]T [m]Q[2]\}_1 \{I[m]P [2]T [m]Q[2]\}_2I[m]P^{-1}[2]K^{-1}[m]a_{2(2)}$ and $P_3:a_{1(2)}L[m]a_{2(2)}$.

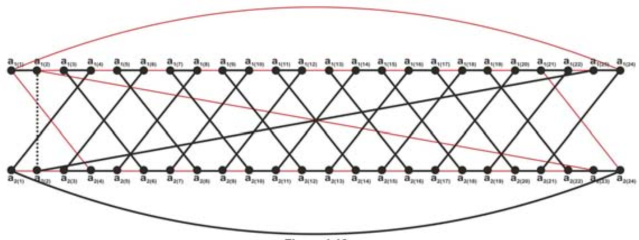


Figure 1.10

Hamiltonian path from $a_{1(1)}$ to $a_{1(13)}$

subcase (ii): For $N = 26$, the hamiltonian path between the vertices $a_{1(1)}$ and $a_{1(13)}$ is $P : P_1 \cup P_2 \cup P_3$ where
$P_1 : a_{1(1)}P [2]a_{1(2)}$.
$P_2 : a_{1(13)}T [m]Q[2]I[m]P [2]T [m]Q[3]S^{-}[m]\{I[m]P^{-1} [2] T [m]Q[2]\}_2I[m]P [2]T [m]Q[2]I[m]P^{-1}[3]I^{-1}[m]Q^{-1}[2] T^{-1}[m]Q^{-1}[2]I^{-1}[m]Q^{-1}[2]T^{-1}[m]P [2]I^{-1}[m]Q^{-1}[3] \{I[m]P^{-1}[2]T [m]Q[2]\}_2I[m]P [2]T [m]Q[2]I[m]P^{-1}[2] K^{-1}[m]a_{2(2)}$ and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

case:5 In IG_N , $d(a_{1(1)}, a_{2(15)}) = 8$.

subcase (i): For $N = 8r + 20$, $r = 1, 2, 3......$
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(15)}$ is $P : P_1 \cup P_2 \cup P_3$
where $P_1 : a_{1(1)}\{T [m]Q^{-1}[2]I[m]P [2]\}_2T [m]Q[2]I[m] \{P [2]T [m]Q[2]I[m]\}_rP^{-1}[2]K^{-1}[m]a_{2(2)}$.

$P_2 : a_{1(15)}I[m]\{P[2]T[m]Q[2]I[m]\}_r P^{-1}[3]$
$I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]$
$\{Q^{-1}[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]\}_{r-1}\{Q[2]T^{-1}[m]$
$P^{-1}[2]I^{-1}[m]\}_2 S[m]Q^{-1}[3]\{T^{-1}[m]P^{-1}[2]I^{-1}[m]$
$Q^{-1}[2]\}_r\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[2]\}_2 T^{-1}[m]$
$P^{-1}[2]a_{1(2)}$ and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

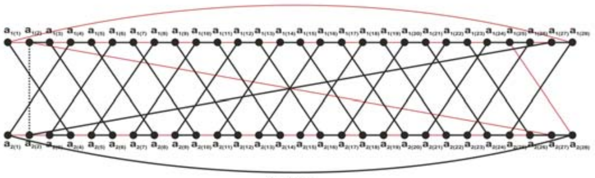


Figure 1.11

Hamiltonian path from $a_{1(1)}$ to $a_{2(15)}$

subcase (ii): For $N = 8r + 22$, r = 1, 2, 3......
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(15)}$ is $P : P_1 \cup P_2 \cup P_3$ where
$P_1{:}a_{1(1)}T[m]Q^{-1}[2]I[m]P[2]T[m]Q^{-1}[2]I[m]$
$\{P[2]T[m]Q[2]I[m]\}_{r+1}P[3]I^{-1}[m]\{Q^{-1}[2]T^{-1}[m]P^{-1}[2]$
$I^{-1}[m]\}_{r+1}Q[2]T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[2]T^{-1}[m]$
$P^{-1}[2]a_{1(2)}$. $P_2 : a_{2(15)}I[m]P[2]T[m]\{Q[2]I[m]P[2]T[m]\}_r$
$Q^{-1}[3]\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q^{-1}[2]\}_r$
$\{T^{-1}[m]P^{-1}[2]I^{-1}[m]Q[2]\}_3 a_{2(2)}$ and $P_3 : a_{1(2)}L[m]a_{2(2)}$.
subcase (iii): For $N = 8r + 24$, r = 1, 2, 3......
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(15)}$ $P : P_1 \cup P_2 \cup P_3$ where
$P_1{:}a_{1(1)}T[m]Q^{-1}[2]I[m]P[2]T[m]Q^{-1}[2]I[m]$
$\{P[2]T[m]Q[2]I[m]\}_{r+1}P[3]\{I^{-1}[m]Q^{-1}[2]T^{-1}[m]$
$P^{-1}[2]\}_{r+1}\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_2 a_{1(2)}$.
$P_2 : a_{2(15)}I[m]P[2]T[m]\{Q[2]I[m]P[2]T[m]\}_r$
$Q[3]S^{-1}[m]I[m]\{P[2]T[m]Q^{-1}[2]I[m]\}_2$
$\{P[2]T[m]Q[2]I[m]\}_{r+1}P^{-1}[2]K^{-1}[m]a_{2(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

subcase (iv): For $N = 8r + 26$, r = 1, 2, 3......
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(15)}$ $P : P_1 \cup P_2 \cup P_3$ where
$P_1{:}a_{1(1)}T[m]\{Q^{-1}[2]I[m]P[2]T[m]\}_2\{Q[2]I[m]P[2]$
$T[m]\}_{r+1}Q[3]S^{-1}[m]\{I[m]P[2]T[m]Q^{-1}[2]\}_2\{I[m]P[2]$
$T[m]Q[2]\}_{r+1}I[m]P^{-1}[3]\{I^{-1}[m]Q^{-1}[2]T^{-1}[m]P^{-1}[2]\}_{r+1}$
$\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_2 a_{1(2)}$.
$P_2 : a_{2(15)}\{I[m]P[2]T[m]Q[2]\}_{r+1}I[m]P^{-1}[2]K^{-1}[m]a_{2(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

case:6 In IG_N, $d(a_{1(1)}, a_{2(17)}) = 10$.

subcase (i): For $N = 6r + 26$, r = 1, 2, 3......
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(17)}$ $P : P_1 \cup P_2 \cup P_3$ where
$P_1{:}a_{1(1)}T[m]\{Q^{-1}[2]I[m]P^{-1}[2]T[m]\}_4\{Q[2]I[m]P^{-1}[2]$
$T[m]\}_{r+1}S^{-1}[m]\{I[m]P^{-1}[2]T[m]Q^{-1}[2]\}_3\{I[m]P^{-1}[2]$
$T[m]Q[2]\}_{r+2}J^{-1}[m]a_{1(2)}$.

$P_2 : a_{2(17)}\{I[m]P^{-1}[2]T[m]Q[2]\}_{r+1}I[m]P^{-1}[2]K^{-1}[m]a_{2(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

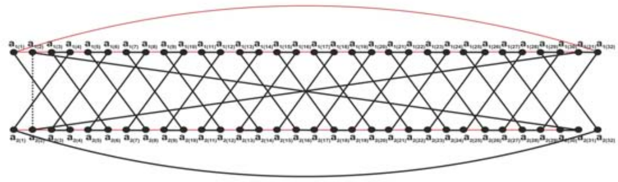


Figure 1.12

Hamiltonian path from $a_{1(1)}$ to $a_{2(17)}$

subcase (ii): For $N = 6r + 28$, r = 1, 2, 3......
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(17)}$ $P : P_1 \cup P_2 \cup P_3$ where
$P_1{:}a_{1(1)}\{T[m]Q^{-1}[2]I[m]P^{-1}[2]\}_4\{T[m]Q[2]I[m]P^{-1}[2]\}_{r+1}$
$T[m]Q[2]J^{-1}[m]a_{1(2)}$. $P_2{:}a_{2(17)}I[m]P^{-1}[2]T[m]\{Q[2]I[m]$
$P^{-1}[2]T[m]\}_{r+1}S^{-1}[m]\{I[m]P^{-1}[2]T[m]Q^{-1}[2]\}_3$
$\{I[m]P^{-1}[2]T[m]Q[2]\}_{r+2}I[m]P^{-1}[2]K^{-1}[m]a_{2(2)}$ and
$P_3 : a_{1(2)}L[m]a_{2(2)}$.

subcase (iii): For $N = 6r + 30$, r = 1, 2, 3......
the hamiltonian path between the vertices $a_{1(1)}$ and $a_{2(17)}$ is $P : P_1 \cup P_2 \cup P_3$ where
$P_1{:}a_{1(1)}\{T[m]Q^{-}[2]I[m]P^{-}[2]\}_4\{T[m]Q^{-}[2]I[m]P[2]\}_{r+1}$
$T[m]Q^{-1}[2]I[m]P^{-1}[3]\{I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[2]\}_r$
$I^{-1}[m]Q[2]T^{-1}[m]P^{-1}[3]\{T[m]Q^{-1}[2]I[m]P[2]\}_{r+1}$
$T[m]Q^{-1}[2]I[m]P^{-1}[2]K^{-1}[m]a_{2(2)}$.
$P_2 : a_{2(17)}\{Q[2]T^{-1}[m]P[2]I^{-1}[m]\}_4 S[m]Q^{-1}[2]J^{-1}[m]a_{1(2)}$
and $P_3 : a_{1(2)}L[m]a_{2(2)}$.

Conclusion

The study of hamiltonicity and hamiltonian laceability has lot of significance in computer networks. In this paper we proposed the hamiltonian-t*- laceability of an interleaver graph constructed from brick product graph C(2n, 1, 3). Precisely, we have shown that the interleaver graphs IG_N for even N, $N \geq 8$ is hamiltonian-t*- laceabile for all odd t such that $1 \leq t < 10$ and 1-edge fault tolerant hamiltonian-t*- laceable for all even t such that $2 \leq t \leq 10$. This concludes that the existence of hamiltonian path in such networks suffice to solve data communication problems.

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REFERENCES

[1] B.Alspach , C.Q.Zhang, Hamilton cycles in cubic cayley graphs on dihedral groups.Ars Combinatoria, 28, 1989, PP : 101-108.

[2] S.Y.Hsieh, C.H.Chen, C.W. Ho, Fault free Hamiltonian cycles in fauly arrangement graphs. IEEE Trans. Parallel Distributed systems 10(32),
1999, PP : 223-237.

[3] Brian Alspach,,C.C.Chen, Kevin McAvaney, On a class of Hamiltonian Laceable 3-regular graphs. Elsevier,151,1996 ,PP : 19-38.

[4] Arya Mazumdar,A.K.Chaturvedi,Adrish Banerjee, Construction of turbo code interleavers from 3-regular Hamiltonian graphs, IEEE, vol-10(4),
2006, PP : 284-286.

[5] Yilun Shang, " On the Hamiltonicity of Random bipartite graphs"., Indian J.Pure Appl.Math 46(2), 2015, PP : 163-173

[6] Annapoorna M, Murali R, Hamiltonian Laceability in the line graph of the (w l,n, k) graph. International Journal of Computer Application (2250-1797) Volume 5 No. 5, 2015 pp : 8-15.

[7] Frank Harary, John.P.Hayes,"Edge fault tolerance in graphs"., Networks: An international Journal, Issue-2, Vol.23, 1993, PP : 135-142.

[8] R.Murali, K.S.Harinath , Hamiltonian n* laceable graphs, Far east journal of applied mathematics.(391) 1999, PP : 59-79.

[9] Leena N Shenoy, R.Murali, Laceability on a class of regular graphs, International journal of computational science and Mathematics, 20(3)
2010, PP : 397-406.

[10] R.A.Daisy Singh, R.Murali, Hamiltonian laceability properties in the Hierarchical hypercube network,International journal of computer application, 6, 2016, PP : 1-10.

[11] R.A.Daisy Singh, R.Murali Hamiltonian t laceability property of Edge fault Hierarchical hypercube network, Annals of pure and applied mathematics, vol-12(1) 2016, PP : 101-109.

[12] M.S.Annapoorna, R.Murali, B. Shanmukha, Laceability in the modified distance graph of grid graphs, vol-7, 2017, PP : 17-35.

[13] P.Gomathi, R.Murali, Hamiltonian t* laceability in the Cartesian product of paths, International journal of Mathematics and computation, vol-27, Issue-2, 2016, PP : 95-102.