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Compressed Sensing Magnetic Resonance Imaging –A Review

Deepak M D[1], Karthik P[2], B K Raghavendra[3]

[1]*Research Scholar, Dept. of CSE. VTU, Belagavi, India.*
[2]*Professor, Dept. of ECE. KSSEM, Bengaluru, India*
[3]*Professor and Head, Dept. of CSE. KSSEM, Bengaluru, Inida*
Corresponding author E-mail:
*[*1]deepak.m.d@kssem.edu.in*
*[*2]karthik.p@kssem.edu.in*
*[*3]hod.cse@kssem.edu.in*

Abstract:

Even though the compressed sensing (Compressed sensing) has emerged as the promising field a decade ago for research in the field of Image/Signal Processing, gained a lot of attention due to its exploitation of signal sparsity. Sparsity an inherent characteristics of many natural signals, enables the signal to be stored in few samples. The essential ambition of compressed sensing (CS) is to reconstruct signals and images from few measurements (samples) than actually needed. CS provides a new approach for data acquisition in terms of reduced or less number of samples that violates the Niquist rate. Nyquist theorem states that signal can be reconstructed exactly and uniquely only if signal should be sampled at-least twice of its maximum frequency. This paper gives a brief survey on the compressed sensing on MRI. The wavelet transform is used along with the compressed sensing technique to provide the compression and reconstruction of MR Images. The wavelet transform is applied along with the compressed sensing technique. Wavelet is a mathematical function used to denoising and compression of two dimensional signals, such as images. When applied on images, the wavelet transforms will give four bands like LL(Low-Low), LH(Low-High), HL(High-Low), HH(High-High). The LL band is having the much information of the image i.e. the low frequency components of the images. The other bands are neglected. By using only LL band, we can reconstruct the original signal.

Keywords: *compressed sensing, Magnetic Resonance Imaging, Matching Pursuit, Orthogonal Matching Pursuit, sparsity.*

1. Introduction

MRI that uses the electromagnetic waves and the radio waves to capture the internal organs of human body. The MRI signals are generated by protons in the body, mostly consisting of water molecules. It has long examination time to capture the details of human body. During this long duration of scanning, patient will not maintain his body steadily. He under goes a lot of disturbances in many ways like changing his position in some manner, moving by to his sides gently, increasing his heart beats due to new medical environments(Scanning Machine) etc. Thus best expression of the image is selected, thus shortening the time of MRI scanning, reducing the pain of the patient and improving the quality of the image. Compressive sensing is a signal processing technique for acquiring signals fastly and reconstructing the signal. The acquired signals are vey less compared to the Niquist rate and based on the concept of optimisation that is, the signal can be reconstructed from far few samples than what the Shannon Nyquist rule says. There are two situations under which the reconstruction can be achieved.1) Sparsity which requires the signal to be sparse in some domain (wavelet transform) and 2) incoherence which is applied through the isometric property which is sufficient for sparse signals. During the scanning, the patient undergoes long duration of scanning. The compressed sensing method reduces the scanning time by increasing the acquisition speed of the signal. So that a very less no of samples are acquired that may be less than the Nyquist rate. Generally data points in MRI are complex in frequency domain with magnitude and phase components. These constitute a matrix called k-space. Some of the other parameters that influence data acquisition of MRI are – longitudinal relaxation time T1 and transverse relaxation time T2 which vary from tissue to tissue. The work was going on to develop a technique for a long time to reduce the scanning time and to improve the data acquisition technology. As a result of compressed sensing [1] technique came to improve the image or data acquisition methods. The CS employs the concept of random under-sampling which may reduce the number of k-space samples to be measured during data acquisition in an MRI machine and hence reduce scan time. Compressive sensing can be used in image compression, radar system, A to D converter, Medical Imaging, speech compression.

2. Compressed Sensing Model

Applying CS to Magnetic Resonance Imaging (MRI) provides significant scan time reductions, with benefits for patients and health care economics. The CS requires three things [2, 3] which are as follows.

a. Transform sparsity: The desired image should have a sparse representation in a known transform domain (i.e., it must be compressible by transform coding).

b. Incoherence of under sampled artifacts: The artifacts in linear reconstruction caused by *k*-space under sampling to be incoherent in the transform domain.

c. Nonlinear reconstruction: The image should be reconstructed by a nonlinear method that enforces both sparsity of the image representation and consistency of the reconstruction with the acquired samples should be incoherent (noise like) in the sparsifying transform domain.

The sparsity which is implicit in MR images is exploited to significantly under sample *k*-space. Images with a sparse representation can be recovered from randomly under sampled *k*-space data. Transform-based compression is a widely used strategy adopted in the JPEG, JPEG-2000, and MPEG standards. S Space is a signal which has only non zero coefficients. Vectors are often used to represent large amounts of data which can be difficult to store or transmit. By using a sparse approximation the amount of space needed to store the vector would be reduced to a fraction of what was originally needed. The transform sparsity of MR images and the coded nature of MR acquisition are two key properties enabling CS in MRI.

The sparsity of the signal in mathematical terms is defined as: Let us suppose, we have a discreet time signal x in R^N which can be expressed in terms of a set of orthogonal basis or support of vectors $[\Psi i]^N_{i=1}$as follows: $X=\sum^N_{i=1} S_i\Psi_i$ where si is the coefficient sequence of x. In matrix form, we can simply write the above equation as $X=\Psi_S$. Then, signal x is said to be k-sparse if only k entries of s are non-zeroes and remaining n-k entries are zero. In order to measure all N coefficients of x, we consider a vector y of dimension M X 1(M<N) such that y=Φx. Where $\{\Phi_j\}^M_{j=1}$ is a MxN matrix or collection of vectors called measurement matrix with Φ_J^T as rows. The measurement matrix plays a vital role in the process of recovering the original signal. How should one design a measurement matrix Φ that is a collection of N vectors in K dimensions? The random measurement matrix and the predefined measurement matrix are the 2 types of measurement matrices. if a signal **x** composed of *N* samples is sparse then the actual signal can be reconstructed using $M \geq O(\mathrm{Klog}\left(\frac{N}{K}\right))$ N linear projections of x onto another basis.

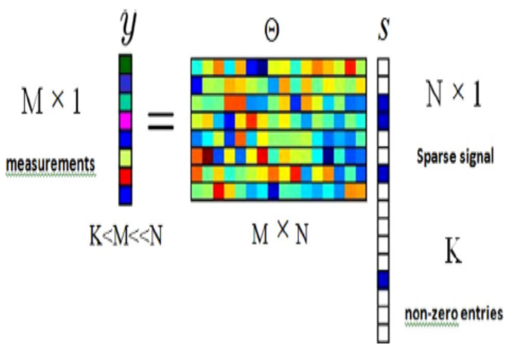

Fig. 1 CS model

3. Wavelet Transforms

The wavelet transform is similar to the Fourier transform. Fourier transform decomposes the signal into sines and cosines, i.e. the functions localized in Fourier space but wavelet transform uses functions that are localized in both the real and fourier space. The Fourier transform is expressed as

$$F(a,b) = \int_{-\infty}^{\infty} f(x)\, \psi^*_{(a,b)}(x)\, \mathrm{d}x$$

Where * --> complex conjugate symbol and function ψ --> some function.

The discrete wavelet transform (DWT) is an implementation of the wavelet transform using a discrete set of the wavelet scales and translations obeying some defined rules. Continuous wavelet transform (CWT) is an implementation of the wavelet transform using arbitrary scales and almost arbitrary wavelets.

4. Reconstruction Algorithms

The greedy approach [8] is a step-by-step iterative method. In each iteration, the solution is updated by selecting only those columns of reconstruction matrix, which are highly correlated with the measurements. The selected columns are called atoms. Generally, the atoms selected once, are not included in subsequent iterations of the algorithm. This idea lowers the computational complexity of the algorithm. Here, the solution is approached in a greedy passion and hence, the name. The advantages of this approach are simple operation, low computational complexity and faster execution. It includes the matching pursuit algorithms such as Matching Pursuit(MP)[5], Orthogonal Matching Pursuit(OMP), Stagewise Orthogonal Matching Pursuit[20], (StOMP), Compressive Sampling Matching Pursuit (CoSaMP), and the gradient pursuit algorithms such as GP (Gradient Pursuit) and CGP (Conjugate Gradient Pursuit). Here we analyze two types of greedy algorithms: matching pursuit and gradient pursuit, including MP,OMP, StOMP, CoSaMP, GP, CGP, etc. The second one is the convex relaxation AND third one is combinatorial methods. In restricted isometric property (RIP), the CS exploits such sparsity (number of non-zero elements) to dictate the far fewer sampling recourses, are needed than the traditional approach. CS predicts that the image can be reconstructed from far fewer samples.

3.1 Matching Pursuit:

Matching pursuit is applied to signal or image video encoding etc. It performs better than DCT coding for bit rate in both the efficiency of coding and quality of image. The matching pursuit has the problem with the computational complexity of the encoder. Matching pursuit is related to the field of compressed sensing and has been extend by researchers to Orthogonal Matching Pursuit (OMP), stagewise OMP (StOMP), Compressed Sampling Matching Pursuit (CosaMP), Generalised OMP, Multipath Matching Pursuit (MMP). Matching pursuit is a class of iterative algorithms that decomposes a signal into a linear expansion of functions that form a dictionary. Given a signal f in a Hilbert space H and a finite-size dictionary $\Phi = \{ \varphi_\gamma \}$ of M unit norm vectors ($\forall_\gamma \in [1..M]$, $\|\varphi\ \gamma\|^2 = 1$) in H, called atoms, find the smallest expansion of f in Φ up to a reconstruction error ϙ : min $\|\alpha\|0$ such that $\|f - \sum^M_{\gamma=1} \alpha_\gamma \varphi_\gamma\|^2 \leq \varepsilon ϙ$ (1) where $\|\alpha\|_0$ is the number of non-zero elements in the sequence of weights $\{\alpha_\gamma\}$.

The best approach for CS recovery of signal is OMP. The solution is the one with minimum l0 norm due to sparsity. But commonly used technique includes l1 minimization such as Basis Pursuit (BP) and greedy pursuit algorithm such as Orthogonal Matching Pursuit algorithm. In each iteration OMP calculates the new signal approximation X^n. The approximation error r^n is calculated as r^n = X-X^n. This is used in the next iteration to determine which new element is to be selected. The selection is based on the inner product between current residual r^n and column vector Φ_i. Let the inner product be in $\alpha = \Phi_i^T r_n$..

Stagewise Orthogonal Matching pursuit (StOMP) transforms the signal into negligible residuals starting from initial residual r0=y where y=Φx. Here enters each stage in StOMP. Here OMP takes many states but StOMP takes fixed number of stages (say 10).

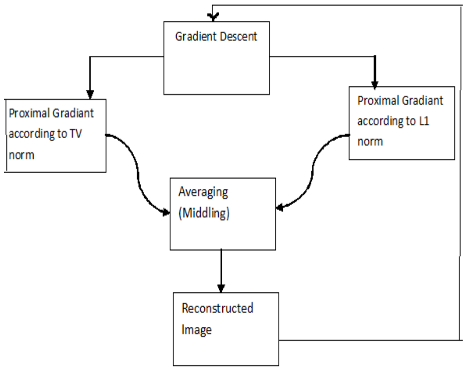

Fig. 2 The reconstructed image is obtained from missing solution of the sub problem in a repetition passion.

The compressed sensing MR images using wavelet transforms introduces the L1 Norm and total variance (TV) regularisation. The problem is divided into total variation and L1 norm subset problem respectively. The reconstructed image is obtained from missing solution of the sub problem in a repetition passion [4]. The process is shown in the following fig 1.

The compressed sensing MRI with total variation and frame based regularisation provides a new sparse regularisation mode with CS-MRI for frame based image reconstruction. In frame based l1 regularisation image restoration, there are 3 forms 1) synthesis based 2) analysis based 3) Balence based approaches.

The Compressed sensing based MRI can be expressed as y=SFU+V =BU+V where B=SF. Where $U \in X^n$ denotes the unknowns. The original MR image $y \in X^n$ (mxn) is the measured k space data. $V \in X^n$ is the complex attribute white Gaussian noise with standard variance (σ) $CS \in X^{mxn}$ is the under sampled operator. And $F \in X^{mxn}$ denotes the fourier transform B=SF.

5. Applications of compressive sensing:

CS has seen major applications in diverse fields, ranging from image processing to gathering geophysics data. Most of this has been possible because of the inherent sparsity of many real world signals like sound, image, video, etc. These applications of CS are the main focus of our survey paper, with added attention given to the application of this signal processing technique [12].

5.1 CS in Cameras:

CS has far reaching implications on compressive imaging systems and cameras. It reduces the number of measurements, hence, power consumption, computational complexity and storage space without sacrificing the spatial resolution. With the advent of single pixel camera (SPC) by Rice University, imaging system has transformed drastically. The camera is based on a single photon detector adaptable to image at wavelengths which were impossible with conventional charge coupled device (CCD) and complementary metal oxide semiconductor (CMOS) images.[13][14][15].

5.2 Medical imaging:

CS is being actively pursued for medical imaging, particularly in magnetic resonance imaging (MRI). MR images, like angiograms, have sparsity properties, in domains such as Fourier or wavelet basis. Generally, MRI is a costly and time consuming process because of its data collection process which is dependent upon physical and physiological constraints. However, the introduction of CS based techniques has improved the image quality through reduction in the number of collected measurements and by taking advantage of their implicit sparsity. MRI is an active area of research for CS community and in recent past, a number of CS algorithms have been specifically designed for it [16].

5.3 Seismic Imaging:

Seismic Imaging is a tool that bounces sound waves off underground rock structures to reveal possible crude oil– and natural gas–bearing formations. Seismologists use ultrasensitive devices called geophones to record the sound waves as they echo within the earth. Seismic data is usually high-dimensional, incomplete and very large. Seismology exploration techniques depend on collection of massive data volume which is represented in five dimensions; two for sources, two for receivers and one for time. However, because of high measurement and computational cost, it is desirable to reduce the number of sources and receivers which could reduce the number of samples. Therefore, sampling technique must require less number of samples while maintaining quality of image at the same time. CS solves this problem by combining sampling and encoding in one step, by its dimensionality reduction approach. This randomized sub sampling is advantageous because linear encoding does not require access to high resolution data. A CS based successful reconstruction theory is developed in this sense known as curvelet based recovery by sparsity-promoting inversion (CRSI) [17].

5.4 CS in RADAR:

CS theory contributes to RADAR system design by eliminating the need of pulse compression matched filter at receiver and reducing the analog to digital conversion bandwidth from Nyquist rate to information rate, simplifying hardware design. Resolution is improved by transmitting incoherent deterministic signals, eliminating the matched filter and reconstructing received signal using sparsity constraints. CS has successfully been demonstrated to enhance resolution of wide angle synthetic aperture RADAR[18][19]. CS has successfully been demonstrated to enhance resolution of wide angle synthetic aperture RADAR. CS imaging is also .applicable in sonar and ground penetrating RADARs (GPRs).

5.5 Analog-to-Information Converters (AIC)

Information content of the signal is much smaller than its bandwidth, it maybe a wastage of precious hardware and software resources to sample the whole signal. Compressive sampling solves the problem by replacing 'analog to digital conversion (ADC)' by 'AIC'. The approach of random non-uniform sampling used in ADC is bandwidth limited with present hardware devices whereas AIC utilizes random sampling for wideband signals for which random non-uniform sampling fails. AIC is based on three main components: demodulation, filtering and uniform sampling.

6. Conclusion

The MRI is a popular imaging tool but with slow data acquisition process. This leads to long time exposure of patient to the scanning machine. In recent years significant efforts are made towards using Compressive Sensing technique to reduce the amount of acquired data. Since CS method exploits sparsity for successful image reconstruction, various sparsifying transforms are also described. New directions with promises of better quality image procured in less time are emerging using Compressed Sensing MRI with numerous contributions of research works. In the direction of better representations of MRI data, recent developments of Greedy algorithms such as Matching pursuit, FOCUSS etc which are tractable replacements of NP-hard Minimization, l0 l1, l2 minimization have proved to be efficient for reconstruction of MRI data.

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