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RELIABILITY EVALUATION AND ANALYSIS OF MULTISTAGE INTERCONNECTION NETWORKS

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ABSTRACT

In this paper, a new graph theoretic method is proposed for calculating two important reliability measures viz. terminal and broadcast reliability of multistage interconnection networks (MINs) respectively. The proposed method views MIN as reliability logic graph (RLG) and reduces it to sub graph containing the minimal path sets/broadcast trees. Then it generates minimal cut sets of these sub graphs. The idea behind this reduction is to avoid searching and validating minimal cut sets of entire RLG. The proposed method is well supported by two new algorithms for calculating the terminal and broadcast reliability respectively. The proposed algorithms are compared with some existing analytical methods in terms of computed reliability values. The comparison ensures the proposed algorithms to be quite competent in estimating the reliability values of MINs. Two terminal as well as broadcast reliability of eight different types of MINs of greater importance have been evaluated and compared on the same working environment.

Keywords: Reliability, Multistage interconnection network, Reliability logic graph

INTRODUCTION

Almost all of the parallel interconnection schemes can be broadly classified into two groups: static networks and dynamic networks [1]. The dynamic networks are built out by crossbar switches and are used in shared memory-multiprocessors. Some of the important examples of fault tolerant candidates of dynamic networks (also called as tightly coupled systems) are Extra stage cube, Multipath omega network, Shuffle exchange network [1-4]. The main advantages associated with these networks are high bandwidth, low diameter, constant degree switches for which they have been used for various commercial machines including super computers [4].

A static network on the other hand, has no crossbar type of switching elements and represents a fixed scheme of interconnection connecting a collection of standalone processors [5]. This type of architecture is used as distributed memory multi computers and is also called as loosely coupled system. Improved performance and increased reliability are the two distinct advantages attributed to interconnection topology [6]. With the increase in size and complexity of the parallel interconnection systems, their reliability becomes extremely important. There are many reliability measures of interest, out of which the node-pair (two terminals) reliability is an important performance measure in parallel computer interconnection systems. Two terminal reliability addresses the probability that a given source-destination pair has at least one fault free path between them.

Reliability prediction of parallel computer interconnection networks has been widely investigated by many researchers in the past. Methods for reliability calculation of interconnection network broadly classified in to two categories viz. analytical method [8-15] and simulation methods [16-21]. The important analytical approaches are continuous time Markov chains (CTMC) [7] and reliability block diagram (RBD) [], [12]. Although continuous time Markov chains have been used effectively to compute the reliability, the exponential growth of the state space as the network size increases restricts its use. Reliability block diagram (RBD) is the graphical representation of the components of a system and the relationships between them, which can be used to determine the overall system reliability. Gunawn [12] used analytical method to evaluate the reliability of shuffle exchange network and its two variants viz. SEN+

and SEN+2. He discussed the effect of reliability on adding extra stages to shuffle exchange network. In his subsequent work [13], he proposed a bound for gamma networks. Bistouni et al. [14], [15] did the same work as reported in [12] using reliability block diagram. Although the analytical methods evaluate accurate value of reliability of multistage interconnection networks still they are network specific i.e. each network has a separate formula for reliability evaluation. It is quite difficult even sometimes impossible to make generalization of these reliability evaluation formula which would work for a similar group of MIN. So, in order to go for generalization, one may opt for simulation methods.

Some of important simulation methods are: Binary Decision Diagram (BDD) [17-18], Path-set/cut-set based methods [16], [18] [19] and, and Decomposition [20] etc. Out of these methods, the network properties of MINs allow path/cut based methods for easier implementation on them to evaluate their reliability. The path-set/cut-sets based methods require all minimal paths/cuts to be enumerated in advance. Then, the minimal paths/cuts are manipulated to get the counterparts in sum-of-disjoint product form. The main demerit associated with these methods is that much computational efforts are required for disjointing of the minimal path/cut sets. Further, the reliability calculated by using path-set/cut-set based methods is generally a function of link reliability. However, for MINs the switching elements are more prone to failure than links. This leads to restrict the use of these methods for evaluating reliability of MINs as the switching elements are generally denoted by nodes in their equivalent reliability logic diagram.

From the discussions carried out so far as well as the network properties of MINs, it is quite clear that path-set/cut-set based methods are much easier to implement on MINs to evaluate their reliability. Moreover, cut-sets based should be preferred when the switching elements may fail. However, the computational tasks required for generating valid cut-sets as well as the complex architecture of MINs do not attract more researcher to work on it. Hence, in this paper, in order to avoid searching and validating the minimal cut-sets, the reliability logic graph of MIN is reduced to sub graph containing the minimal path-sets between source to destination or the broadcast trees among a given source to all destinations. Rest of the paper is organized as follows: Section II proposes a new method to evaluate the reliability of interconnection network. Two new algorithms have also been proposed in this Section. Simulated results along with comparison of the proposed method against some existing methods are presented in Section III. Section IV concludes the paper with its future scope.

Proposed Method to Compute the Reliability of Interconnection Network

Definition 1. Terminal Reliability

The two terminal reliability is the probability that a communication path must exist between a specific pair of nodes of a network under a predefined working environment.

Definition 1. Broadcast Reliability

Broadcast reliability is defined as the probability that asingle-input switch is able to broadcast data or connectedto all the output switches

Definition 3. Minimal Path

A Minimal Path is a path whose proper subsets are no longer paths

Definition 3. Minimal Cut-set

A Minimal Cut-set is a cut-set whose proper subsets are no longer cut-sets

a. Brief Description on the Proposed Method

The multistage interconnection network is first converted into its equivalent reliability logic graph G(V,E.) where the node represents the switching elements (SEs) and edge represents the links. The nodes are numbered sequentially from the source node (s) to the sink node (t) using . The adjacency matrix of G(V,E) is generated. The proposed method reduces the RLG of MIN to sub graph containing the minimal path-sets for two terminal reliability evaluation or broadcast trees for evaluating broadcast reliability of MINs. The valid minimal cut-sets can be easily generated by traversing this sub graph. The numbers of cut-sets generated are generally a function of the number of stages of MINs. Reliability can then be evaluated by applying inclusion-exclusion principle on these cut-sets.

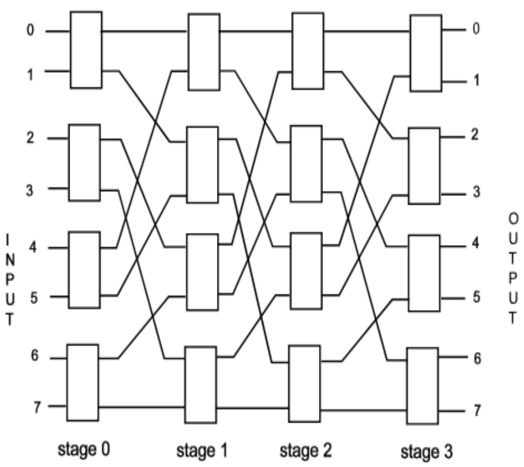

Fig. 1. Extra stage shuffle exchange network

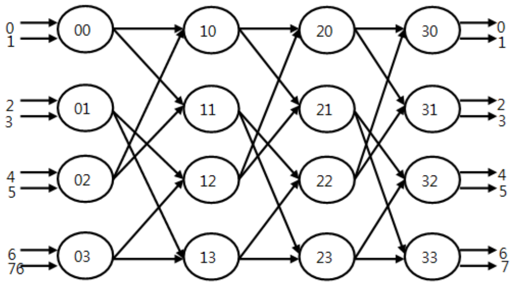

Fig. 2. Reliability logic graph of Extra stage shuffle exchange network

b. Proposed Algorithms to compute the Reliability of Interconnection Network

Algorithm for evaluation of two terminal reliability of MIN

Input:

- Adjacency matrix (A) of RLG, G(V,E)
- $s-t$ (source and destination nodes)
- n being the no. of stages of MIN

Output:

Two terminal reliability (TR)

Two_Terminal_Rel(A, n,s, t)

```
begin
Generate a MP sayPi preserving the node sequence between s and t.
S=S U Pi, where S is the path set
for x ∈ Pi where x being intermediate node
Find the next path Pi+1 from Pi s.t Pi+1 must contain {Pi~x}
if ( no such path)then
 break
 else
 S = S ∪ Pi+1
end if
minimal_path(Pi+1 )
next x
end for
j=0
for each stage i of S
Cj++= node set
Next i
end for
QT = P(∪i Ci)
TR=1-QT
return(TR)
end
```

Function definition of minimal path (P_k)

```
begin
for each intermediate node eεPk generate the next path Pk+1 from Pk s.t Pk+1 must contains {Pk~e}
if(no such path)
 return
else
 S = S ∪ (Pk+1~Pk)
minimal_path(Pk+1 )
end for
return(S)
end
```

Time Complexity of Two_Terminal_Rel(A, s, t)

c. The proposed algorithm has the following three main operations along with their time complexity

1. the construction of the sub-graph containing the minimal paths (O(V^2))
2. generation of cut-sets (O(n))

3. application of inclusion-exclusion principles on cut-sets (O(n))

Thus, the construction of the sub-graph superceeds other operations in the proposed algorithm viz. generation of cut-sets as well as the application of inclusion-exclusion principles on cut-sets. Hence, the time complexity of thepropsed algorithm is found to be $O(V^2)$.

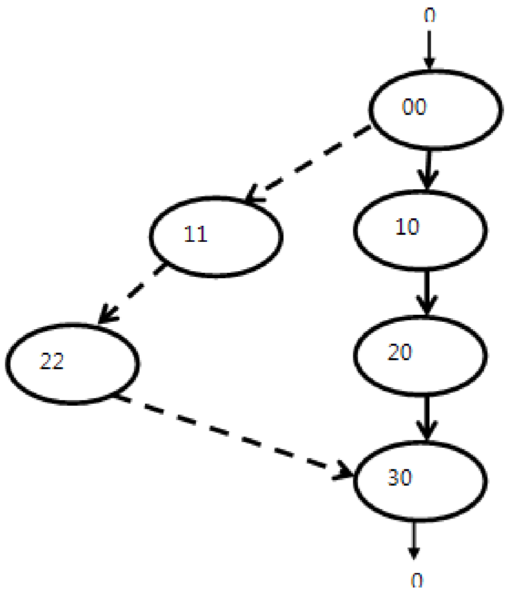

Fig. 3. Sub-graph containing minimal path sets

Minimal Cut-sets Node set

C_1 {00}

C_2 {10, 11}

C_3 {20, 22}

C_4 {30}

By applying Inclusion-exclusion principle on above minimal cut-sets, the unreliability expression is

$$QT = P(C_1 \cup C_2 \cup C_3 \cup C_4)$$

$$\begin{aligned} &= P(C_1) + P(C_2) + P(C_3) + P(C_4) - P(C_1 \cap C_2) \\ &\quad - P(C_1 \cap C_3) - P(C_1 \cap C_4) - P(C_2 \cap C_3) \\ &\quad - P(C_2 \cap C_4) - P(C_3 \cap C_4) \\ &\quad + P(C_1 \cap C_2 \cap C_3) + P(C_1 \cap C_2 \cap C_4) \\ &\quad + P(C_1 \cap C_3 \cap C_4) + P(C_2 \cap C_3 \cap C_4) \\ &\quad - P(C_1 \cap C_2 \cap C_3 \cap C_4) \end{aligned}$$

$$= 2Q + 2Q^2 - 4Q^3 - Q^2 - Q^4 + 2Q^5 + 2Q^4 - Q^6$$

$$= 2Q + Q^2 - 4Q^3 + Q^4 + 2Q^5 - Q^6$$

$$= 0.206119$$

Hence, the two-terminal reliability of ESEN between input-output pair (0, 0) is

$TR = 1 - QT = 0.793881$ (when reliability of each SE=0.9)

Algorithm for evaluation of Broadcast reliability of MIN

Input:

- Adjacency matrix (A) of RLG, G(V,E)
- Source node s and all output nodes

Output: Broadcast reliability (BR)

Broadcast_Rel(A,s)

```
begin
S=Null
T=Binary_Tree(A,s)
S=S∪ T
for each intermediate node x ∈ T
if (Tx=Binar_Tree(A~x, s) ≠ null)then
S=S∪ Tx
else
break
end if
end for
x=root of S
y= null
V= {x}, j=1
for l=2 to n-1
for each node pair x, y ∈ Vat level l
If( x → child ∩ y → child ≠ null)then
if ((x or y ) ∈ V)then
V=V~ {x or y}
V=V ∪ {xy}
Cj++={x,y}
else
Cj++={x}
Cj++={y}
end if
end if
end for
end for
```

29: $QB = P(\cup_i C_i)$
30: BR=1-QB
31: Return(BR)
32: end

Binary_Tree(A,s)

Begin

1: **If**(construct a binary tree (T) with root connecting the input and leaf nodes connecting all output)**then**
2: return T
3: **else**
4: return null

Time Complexity of Two_Terminal_Rel(A, s, t)

The proposed algorithm has the following three main operations along with their time complexity

1. the construction of broadcast tree ($O(V^2)$)
2. generation of cut-sets $O(n \times V)$
3. application of inclusion-exclusion principles on cut-sets (O(n))

Thus, the construction of thebroadcst tree superceeds other operations in the proposed algorithm viz. generation of cut-sets as well as the application of inclusion-exclusion principles on cut-sets. Hence, the time complexity of thepropsed algorithm is found to be $O(V^2)$.

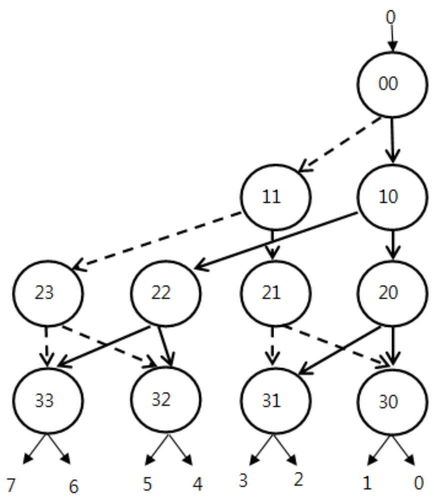

Fig. 4. Sub-graph containing broadcast tree

Minimal Cut-sets Level Node sets

C_1 1 {00}
C_2 2 {10,11}
C3 3 {20,21}
C_4 3 {22,23}
C_5 4 {30}
C_6 4 {31}
C_7 4 {32}
C_8 4 {33}

By applying Inclusion-exclusion principle on above minimal cut-sets, the unreliability expression is

$$QT = P(C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8)$$

Hence, the broadcast reliability of ESEN is

$BR = 1 - QT$=0.548 (when reliability of each SE=0.9)

1. RESULTS AND DISCUSSION

a. Comparison

In order to validate the accuracy of the proposed method in estimating the two terminal reliability of multistage interconnection network, it is compared against the existing methods [12] and [14]. SEN and ESEN are taken as benchmark networks for this comparison. The reliability of switching elements is varied from 0.9 to 0.99 for these networks. The comparison shows two existing methods [8] 14] have same values of terminal reliability for these networks which is obvious as both use analytical methods to compute the reliability of said networks. The proposed method though being a simulation approach estimates reliability of these networks more accurately (refer Table 1 and 2).

Table -1 Comparison of Two terminal reliability of Shuffle exchange networks

Switching Element	Terminal Reliability of SEN Computed by			Terminal Reliability of ESEN Computed by		
	Method [12]	Method [14]	Proposed method	Method [12]	Method [14]	Proposed method
0.9000	0.7290	0.7290	0.7290	0.7808	0.7808	0.7939
0.9100	0.7536	0.7536	0.7536	0.8036	0.8036	0.8147
0.9200	0.7787	0.7787	0.7787	0.8264	0.8264	0.8356
0.9300	0.8044	0.8044	0.8044	0.8491	0.8491	0.8564
0.9400	0.8306	0.8306	0.8306	0.8716	0.8716	0.8772
0.9500	0.8574	0.8574	0.8574	0.8939	0.8939	0.8980
0.9600	0.8847	0.8847	0.8847	0.9159	0.9159	0.9187
0.9700	0.9127	0.9127	0.9127	0.9376	0.9376	0.9392
0.9800	0.9412	0.9412	0.9412	0.9589	0.9589	0.9596
0.9900	0.9703	0.9703	0.9703	0.9797	0.9797	0.9799

Table -2. Comparison of Broadcast reliability of Shuffle exchange networks

Switching Element	Broadcast Reliability of SEN Computed by			Broadcast Reliability of ESEN Computed by		
	Method [12]	Method [14]	Proposed method	Method [12]	Method [14]	Proposed method
0.9000	0.47829	0.478	0.478	0.5548	0.54712	0.548
0.9100	-	0.516	0.516	-	0.58613	0.590
0.9200	0.55784	0.557	0.557	0.63284	0.62680	0.631
0.9300	-	0.601	0.601	-	0.66906	0.670
0.9400	0.64847	0.648	0.648	0.71696	0.71284	0.713
0.9500	0.69833	0.698	0.698	0.76119	0.75804	0.760
0.9600	0.75144	0.751	0.751	0.80675	0.80454	0.805
0.9700	-	0.807	0.807	-	0.85218	0.8561
0.9800	0.86812	0.868	0.868	0.01462	0.90079	0.9021
0.9900	0.93206	0.932	0.932	0.95033	0.95015	0.9541

b. **Reliability analysis of Multistage Interconnection Network**:

Here, eight numbers of different multistage interconnection networks have been considered for calculating as well as for analysing their different reliabilities measures viz. two terminal reliability and broadcast reliability. The network characteristics of these networks are presented in Table -3.

Table -3. Network characteristics of Multistage interconnection networks

Sl. No.	Multistage Interconnection Network	No. of input (N)	No. of Switching element (SE)	No. of stages
1	Shuffle Exchange Network (SEN)	8	12	3
2	SEN with an Extra Stage (ESEN)	8	16	4
3	Extra Stage Cube (ESC)	8	16	4
4	Phi Network (PHN)	8	10	3
5	Double tree network (DOT)	8	13	5
6	Modified Double Tree Network (MDOT)	8	13	5
7	Fault-tolerant Double Tree (FDOT) Network	8	15	3
8	Extra Group Network (EGN)	8	12	2
9	Quad tree (QT)	16	26	5

The reliability analysis of MINs are classified into two types viz. time independent analysis and time dependent analysis.

c. Type independent reliability analysis of MINs

For time independent reliability analysis, the reliability of each SEs is set to values 0.9 to 0.99. The corresponding two terminal and broadcast reliability of said MINs are evaluated and plotted against the reliability of SE (Figure 5 &6). From these figures, it can be observed that Quad tree has the highest reliability values followed by FDOT while the reliability of SEN is the least.

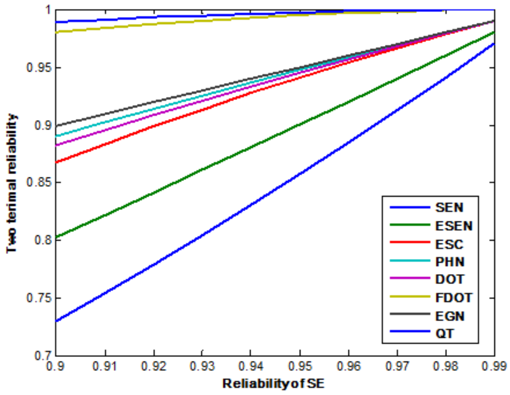

Fig. 5. Two terminal reliability of Multistage Interconnection Networks (time independent)

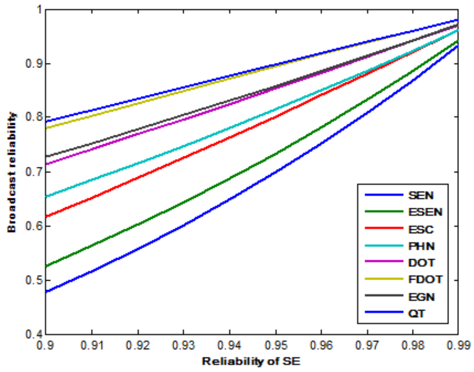

Fig. 6. Broadcast reliability of Multistage Interconnection Networks (time independent)

d. Type dependent two terminal reliability analysis of MINs

Under this case, the reliability of each SEs is obtained from the following equation

$$p(SE) = e^{-\lambda t}$$

Where,

P(SE)- probability of success of each SE
λ= switching failure rate
t= mission time in hours

The following parameters are set as per [14]:

The mission time is set values ranging from 0 to 90000 hours.

The switching failure rate of each switch is set to 0.00001.

Setting all these parameters the reliability of said MINs are evaluated and plotted for the purpose of comparison (Figure 7 & 8). From these comparisons Quad tree has the highest values of two-terminal reliability as well as broadcast reliability while these reliability values for SEN are the lowest.

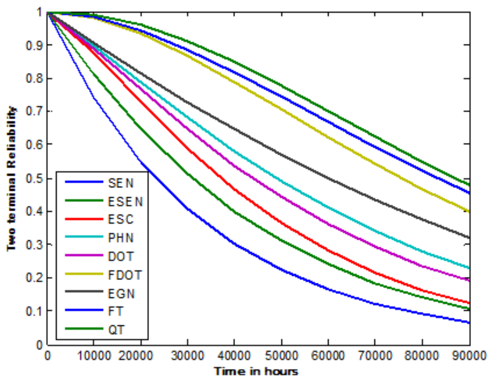

Fig. 7. Two terminal reliability of Multistage Interconnection Networks (time dependent)

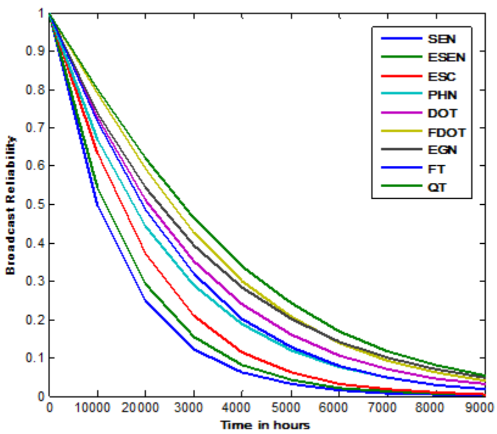

Fig. 8. Broadcast reliability of Multistage Interconnection Networks (time dependent)

2. CONCLUSION

In this paper, a new graph theoretic based method has been proposed. The proposed method is well supported by two algorithms for evaluating terminal reliability and broadcast reliability of MINs. The first algorithm reduces the reliability logic graph(RLG) of the MIN to a sub-graph containing the minimal path-sets while the second one reduces the RLG of MINs to a sub-graph containing broadcast tress. Then the cut-sets are generated from these sub-graphs. The reliability is calculated by applying inclusion-exclusion principle on these cut-sets. The comparisons of the proposed algorithm against existing methods ensure the proposed algorithm to be much efficient and competent with respect to its counterpart analytical methods. Terminal reliability and broadcast reliability of eight number of fault tolerant MINs have been evaluated under the same environment. The work carried out in this paper may be extended to compute the network reliability of MINs.

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