QUEUING THEORY APPLICATION
IN THE TRAFFIC FLOW OF
INTERSECTION

R. Manimaran\textsuperscript{1} and R. Venkatraman\textsuperscript{2}
\textsuperscript{1,2}SRM Institute of Science and Technology,
Vadapalani Campus, Chennai-600026, T.N, India.

Abstract

Much effort has been spent in developing more efficient control systems for signalized intersections to adapt the capacity of the network to the variability of the demand. This variability is partly due to time-dependent factors but also to the stochastic nature of the demand itself. The famous British transportation researcher, F. V. Webster, developed a series of useful traffic theories, which have had a very big influence on the modern traffic analysis since the 1950's. However, based on this study, Webster's minimum delay cycle length equation overestimates the optimal cycle length compared to the results based on the HCM 2016 method. This is due to the restructuring of the HCM 2016 delay equation as compared to the original Webster's delay calculation. For an isolated intersection, based on Webster's delay equation, the delay will become infinity when the degree of saturation of a lane group approaches one, which is unrealistic, while the delay based on HCM 2016 method can accommodate some random failures and short-term over saturation situations. The HCS software was used to conduct experiments for a typical four-phase intersection over a wide range of volume and lost time scenarios, and the results were used to modify the original Webster minimum delay cycle length equation. The new minimum delay cycle length equations based on this study significantly improve the accuracy of
predicting the optimal cycle length for isolated intersections at high traffic volume conditions.

AMS Subject Classification:
Key Words and Phrases: Traffic Control, Optimal Cycle Length, Isolated Intersection, Delay, HCM

1 Introduction

In the 1950’s, Webster conducted a series of experiments on pre timed isolated intersection operations [1]. Two traffic signal timing strategies came from his study. One is signal phase splits. Webster demonstrated, both theoretically and experimentally, that pre timed signals should have their critical phases timed for the equal degrees of saturation for a given cycle length to minimize the delay. The other is the minimum delay cycle length equation, which is shown as Equation 1. In developing the equation for the optimal minimum delay cycle length, it was assumed that the effective green times of the phases were in the ratio of their respective $y$ values (flow ratios).

$$C_0 = \frac{1.5L + 5}{1 - Y}$$  \hspace{1cm} (1)

where $C_0$ = the optimal minimum delay cycle length, sec; $L$ = total lost time within the cycle, sec; and $Y$ = the sum of critical phase flow ratios [2].

The above two strategies are very useful for traffic design and planning. When the two rules are applied together, one can practically minimizes the resulting delay at an isolated pre timed signalized intersection. However, when the traffic demand of an intersection is high, which causes a high value of degrees of saturation, the optimal cycle length based on Webster’s equation will become extremely high, may be 30 to 40 seconds higher than the value based on the HCM 2016 delay calculation. The optimal cycle length overestimation of Webster’s equation has not been addressed yet based on our literature reviews. The purpose of this paper is to find out the reason for the higher cycle length prediction by Webster equation and provide more accurate models.
This paper is organized as follows. A series of experiments were conducted on a hypothetical isolated pre timed four-leg traffic signal by using the HCS software and an Excel spreadsheet model implementing Webster’s equation. The optimal cycle lengths for different traffic demand situations were calculated based on both HCM method and Webster’s equation. Comparisons were made and alternatives proposed. Finally, a summary and conclusions were provided.

With the introduction of the Markov Chain technique we provide a faster generation of data than with micro simulation. In this way we are able to analyze the validity of the available formulae and to detect and quantify the errors committed when applying these formulae.

Among others, these limits preclude the utilization of the available analytic models to assignment and optimization problems:

- Time dependent queuing models are valid only if the mean flow rate is constant for the whole evaluation period;
- Only an initial queue equal to zero is admitted;
- The models don’t cover the estimation of decreasing queues, occurring when the initial queue is larger than the equilibrium one;
- The models are usually suited for certain time steps, typically 15 minutes.

To assess the importance of the new model we compare first the results with the available models in a test scenario with time-varying flows. Later on we apply the novel model to a realistic scenario showing the differences that we obtain in the case of an assignment problem involving both route choice and departure time choice.

2 Theoretical Background

Webster Delay Equation

The delay calculation for the Webster method is expressed as Equation (2).

\[ d = \frac{c(1 - \lambda)^2}{2(1 - \lambda x)} + \frac{x^2}{2q(1 - x)} - 0.65 \left( \frac{c}{q^2} \right)^{\frac{1}{3}} x^{2+5\lambda} \]  

(2)
where \( d \) = average delay per vehicle on the particular lane group of
the intersection, sec/veh;
\( c \) = cycle length, sec;
\( q \) = flow, vehicles/sec;
\( \lambda \) = proportion of the effective green with respect to cycle length
(i.e. \( g/c \) and \( g \) is effective green, sec); and
\( x \) = the degree of saturation. This is the ratio of the actual flow
to the maximum flow which can be passed through the intersection
from this lane group, and is given by
\( x = g/\lambda s \), where \( s \) is the saturation flow in vehicles per second.

The first term of Equation 2 represents the delay when the
traffic is assumed to be arriving uniformly. The second term of
the equation makes some allowance for the random nature of the
arrivals. It is an expression for the delay experienced by vehicles
arriving randomly in time at a “bottleneck”, queueing up, and
leaving at constant headways. The third term of the equation is
an empirical correction term to give a closer fit for all values of
flow. Normally, the last term is relatively small comparing to the
total delay and frequently is omitted by reducing ten percent of
the first two terms[3].

**HCM 2016 Delay Equation**

The average control delay per vehicle for a given lane group in the
HCM 2016 is calculated by using the following equation

\[
d = d_1 \times PF + d_2 + d_3
\]  

(3)

where \( d \) = control delay per vehicle, s/veh
\( d_1 \) = uniform control delay assuming uniform arrivals, s/veh;
\( PF \) = uniform delay progression adjustment factor, which accounts
for effects of signal progression (in this paper, \( PF = l \) because an
isolated intersection is assumed);
\( d_2 \) = incremental delay to account for effect of random arrivals and
over saturation queues, adjusted for duration of analysis period
and type of signal control; this delay component assumes no initial
queue for a lane group at the start of analysis period, s/veh; and
\( d_3 \) = initial queue delay, which accounts for delay to all vehicles
in analysis period due to an initial queue at the start of analysis
period, s/veh. A zero initial queue is assumed in this paper.
The equation used to calculate the uniform control delay, described in Equation 4, is essentially the same as the first term of Webster’s delay formulation and is widely accepted as an accurate depiction of delay for the idealized case of uniform arrivals. Note that degrees of saturation beyond 1.0 are not used in the computation of $d_1$.

$$d_1 = \frac{0.50c(1 - \frac{g}{c})^2}{1 - \left[\min(1, x)^\frac{2}{x}\right]}$$  \hspace{1cm} (4)

where the terms in the equation are the same as defined before.

Equation 5 is used to estimate the incremental delay due to non uniform arrivals and temporary cycle failures (random delay) as well as delay caused by sustained periods of over saturation (over saturation delay). The equation assumes that there is no unmet demand that causes initial queues at the start of the analysis period. The incremental delay term, $d_2$, is valid for all values of $x$, including highly over saturated lane groups.

$$d_2 = 900T \left[ (x - 1) + \sqrt{(x - 1)^2 + \frac{8kIx}{cT}} \right]$$  \hspace{1cm} (5)

where $T =$duration of analysis period, hour; $k =$incremental delay factor that is dependent on actuated controller settings; $I =$upstream filtering/metering adjustment factor; $C =$lane group capacity, $vph$; and $x =$lane group $v/c$ ratio or degree of saturation.

There are significant differences between the second term of Webster’s delay equation and HCM 2016’s second term of delay calculation. When the degree of saturation is close to one, the delay based on the Webster’s equation will approach infinity, which is unrealistic. However, the HCM 2016 delay will be somewhat along the solid line of Figure 1 for saturated and over saturated conditions.

The Level of Service is closely related to the average control delay of the intersection. For easy reference, the HCM 2016 Level of Service criteria based on the average control delay are listed in Table 1 [4].
Optimal Minimum Delay Cycle Length Equation

For an isolated intersection, optimum cycle length is corresponding to the minimum total delay of the intersection. This minimum total delay situation can be obtained by selecting an appropriate cycle length and green splits. For a given cycle length, the effective green phases can be selected in proportion to the critical flow ratio of the phases. One way to obtain the optimal cycle length is to take the derivative of the expression for total delay of the intersection with respect to cycle length and set equal to zero. Because the delay calculations are different between the Webster and HCM 2016 method, as shown previously, one would expect that the optimal cycle length equation will be different. Figure 2 shows the relationship between cycle length and delay based on a sample intersection described in Webster’s paper [1]. From the graph, one can see that the optimal cycle lengths corresponding to the minimum delay of the intersection are similar at the low traffic volume, i.e., 1600 vph. However, when the traffic volume is high, the optimal cycle lengths are significantly different. For example, the optimal cycle length from Webster is 40 seconds higher than that from the HCM 2016 when the volume is equal to 3000 vph.

Webster found the minimum cycle, \( c_m \), is just long enough to allow all the traffic which arrives in one cycle (assume uniform flow) to pass through the intersection in the same cycle, which can be expressed using Equation 6.

\[
C_m = L + \sum_{i=1}^{n} \frac{q_i c_m}{s_i} = \frac{L}{1 - Y} \tag{6}
\]

where \( q_i \) is the arrival volume at lane group \( i \); and \( s_i \) is the saturation flow at lane group \( i \).

Theoretically, the minimum cycle length will cause infinite delays because of the random nature of the traffic flow. Webster further developed the following linear approximation for the optimal cycle length for the practical application purposes:

\[
C_0 = \frac{KL + 5}{1 - Y} = C_m + \Delta_c = \frac{L}{1 - Y} + \frac{0.5L + 5}{1 - Y} \tag{7}
\]

where \( K \) is a regression parameter. \( K \) is equal to 1.5 according to Webster which gives Equation 1. From the above theoretical
analysis and experimental results shown in Figure 2, one would expect that Equation 1 should be modified correspondingly to accommodate the development of HCM 2000 delay equation.

3 Conclusions

In this paper, the minimum delay cycle lengths for a wide range of different traffic and lost time situations were computed using HCS software based on HCM 2016. After comparing and modifying Webster's minimum delay cycle length equation, the paper reached the following conclusions:

1. The first delay term in the HCM 2016 delay model is based on the first term of the Webster's original delay equation. Until v/c ratio is equal to 1, the first term of HCM is the same as Webster's first term delay equation. However, the second term of the delay model in HCM 2016 is different from the Webster's original second term of delay equation. When the degree of saturation is approaching one, the delay based on Webster's delay will become infinity, which is unrealistic. The HCM 2016's delay model is time-dependent, thus can handle the random failure and short-term over saturated situations.

2. Because the delay calculations are different for the HCM 2016 method and Webster's method, the Webster's optimal cycle length equation should be modified accordingly. Based on our experimental results, at the low traffic volume conditions, for LOS C or better, Webster's optimal equation is still a good estimation. However, for high traffic volume conditions, the modified Webster's optimal equation developed in this study showed better results. In addition, an exponential type regression model was developed in this paper. The Exponential cycle length model fits both high volume and low volume situations.

3. The modified Webster's optimal cycle length model and the Exponential cycle length model are useful in signal timing design and analysis. The models can be adopted in HCS-type software as an optimization tool to provide initial estimate on optimal cycle length.
4. This study is limited to the four-phase intersection’s optimal cycle length analysis. Further studies should be conducted on two, three and other multiphase situations to develop a more generalized model. In addition, the analysis duration $T$ is limited as 15 minutes. The effect of $T$ should also be included in the generalized model. Nevertheless, similar research methodology as proposed in this study could be applied. Due to the lack of computing power in the 1950’s, the generality of Webster’s model has never been established either.

References


