Non-Newtonian Effects in Flow over circular cylinder at low Reynolds number

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Abstract— This paper tries to discuss numerically, the aerodynamic impact of non-Newtonian flow over circular cylinders at low Reynolds number. A 2-D laminar incompressible flow using Ostwald-de Waele power law model is used for accounting the non-Newtonian behavior. The analysis is performed for the range 75<Re<150 and 0.25<n<2, for eliminating the 3-D effects and to ensure numerical stability, where Re is the Reynolds number based on the diameter of the cylinder and n is the power law index. The numerical simulations are performed by using the open source CFD-Toolkit OpenFOAM (V4.1) and the post processing is done using Paraview. Analysis shows that increasing power law index reduce vortex shedding frequency and the amplitude of oscillation of force coefficients with a tradeoff of raised drag force on the cylinder.

I. INTRODUCTION

The flow past circular cylinder is a significant topic in aerodynamics. It is a key geometry used by researchers for studying flow physics in external flows. For more than a century, the topic had been a subject for many investigations such as boundary layer separation, wake instability, vortex shedding, turbulence, lift and drag forces during external flows, vortex-induced oscillations etc. Though most of the flow physics behind flow past circular cylinder is thoroughly followed by decades of investigation for large spectrum of Reynolds number, there’s not very much attention has been given to the non-Newtonian effects in flow over circular cylinders at low Reynolds. There are numerous works available on non-Newtonian viscosity models, yet the existing works and turbulence models are designed focusing on Newtonian transport model. This could be due to the preference towards high Reynolds number flow in external flows, since some of the desirable applications such as vortex suppression, drag reduction strategies in air- crafts, automobiles etc. occur at this regime. So, until now more time and energy of past numerical studies were utilized for external flows in key figures like circular cylinder and their behavior at various stages of Reynolds number considering only Newtonian transport model.

There are cases in numerical studies where viscosity models take temperature, pressure etc. as parameters. But variation of viscosity in flows with respect to velocity gradients or shear rate is often not considered in high Reynolds number region. This is because laminar sub-layer grows thin and turbulent boundary layer thickens at this regime, thus attaining free-stream velocity distribution at a very short distance from the walls. Hence, the non-linear strain rate or non-Newtonian behavior of a fluid is more appreciable when flow is at low Reynolds number, where the viscous forces are considerable compared to the inertial forces during this stage. Study of power law fluids over flat plate conducted by Asterios Pantokratoras et al. [1] verifies this point showing flow normal to a flat plate has less non-Newtonian effects at high Reynolds number.

In practice Newtonian fluids are in fact very rare in the industry and most of the fluids handled in production/manufacturing like oils, polymers, lubricants, foams, emulsions, food products etc. are non-Newtonian in nature, and most of the technologies in those industries like Food processing technologies, stirrers in medicinal plants, extrusion processes, shaping and manufacturing processes involving molten metal or plastic compounds, fluid couplings, journal bearings, dampers, shock absorbers, lubricated machineries, heat exchangers etc. are viable candidates of non-Newtonian flows at low Reynolds number.

The viscous nature of a fluid is greatly dependent on density of the fluid particles and their interaction at microscale. Despite the case, in a macroscale flow, Newton’s law of viscosity states that the resisting shear stress of fluid layers in an incompressible simple fluid follows a linear relationship with the shear strain rate of the same, indicating that fluid exhibit a constant coefficient of viscosity with the shear flow. Newton's viscosity law shows excellent agreement with most common fluid flows seen in nature such as winds, ocean currents etc. However, there are undeniably quite a lot of fluids (especially in liquids with the advent of polymers) that partly shows solid like features such as yield behavior and plastic like flow. Concerned to these difficulties in using viscosity law (practically made similar to the Hooke's law for solids) and the non-linearity in rheology of fluids has come up in the past few decades of research field of non-Newtonian fluid mechanics.
There are numerous literatures available discussing non-Newtonian flow over cylinders. The visco-elastic flow over circular cylinders is studied experimentally by Shiang et al. [2] using PIV method, and claimed that the non-Newtonian behavior surges by increasing the Deborah no.(De*), more variation occurs to the initial transient nature until it develops into a steady state streamline flow. That is increased time for relaxation and evident change in the visco-elastic boundary layer was found. Similar results were numerically investigated by Dodji et al. [3] using Oldroyd-B visco-elastic model and developed empirical model equations for such flows. Experimental investigation on flow of visco-elastic fluids over circular cylinder by Verhelst et al. [4] proved the drag characteristics after a critical velocity is proportional to square of the flow rate which is quite different from Newtonian models. Another effect of non-linear viscosity coefficient is observed by Pooja et al. [5] is that, In case of rotating cylinders in bingham plastic fluids, at higher Bingham plasticity, rotation suppresses the flow separation. Regarding the vortex shedding behavior of visco-elastic flow behind circular cylinder, Norouzi et al. [6] pointed out that the elasticity causes reduced vortex shedding frequency and amplitudes in oscillation in numerical simulations. The blockage effect in cylinder within confined walls was also surveyed and Huang et al. [7] showed wall blockage reduces wake length and raises drag in the flow and contrary to weak wall blockages. Chhabra et al. [8] ‘s work on flow power law fluids over cylinder shows same effect with blockage ratio and decreasing to increasing behavior in drag with increase in power law index from 0.2 to 1.4 at Re =20-40 range. This study focuses on flow of power law fluids over circular cylinder at low Reynolds number and their influence in vortex shedding by changes in power law index.

II. METHODOLOGY

A. Problem setup and Mathematical modeling

Numerical simulation of the problem was conducted in a two-Dimensional computational flow domain of size 40d x 10d, with origin fixed at the center of circular cylinder and d being the diameter of the cylinder. A schematic sketch of the domain is given in Fig 1. The inlet to the bounding box was kept closer to the front side of the cylinder and the outlet is kept farther, towards the rear side of the cylinder for capturing the development of wake in the downstream flow.

The governing partial differential equations, the continuity and the momentum equation are given by equations 1&2 and the non-Newtonian model used in this problem is Ostwald–de Waele power law for viscosity (equation 3).

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\[
\rho = \text{constant} \\
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla P + \nabla \cdot \mathbf{T} + \mathbf{F} \\
\mathbf{T} = \eta \left( \frac{\partial \mathbf{v}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y} \right) \\
\mathbf{F} = \mathbf{f} \\
\mathbf{v} = \mathbf{0} \\
\n\]
Fig. 2. Boundary conditions applied to the domain.

III. VALIDATION STUDY

A validation study is conducted for checking the reliability and accuracy of OpenFOAM CFD code. A case of laminar flow over circular cylinder with power law index, \( n=1.0 \) and Reynolds number, \( Re=100 \) was used to recreate Newtonian flow using the power law model, with \( n=1 \). The non-dimensional vortex shedding frequency called Strouhal number is compared against available experimental results from well-established literatures and is found good agreement in their values as shown in Table 1.

![Table 1](image)

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Reference</th>
<th>Strohal number</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Williamson et al. [9]</td>
<td>0.164</td>
</tr>
<tr>
<td>2</td>
<td>Norberg et al. [10]</td>
<td>0.164</td>
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<tr>
<td>3</td>
<td>Park et al. [11]</td>
<td>0.165</td>
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<td>4</td>
<td>Roshko et al. [12]</td>
<td>0.164</td>
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<tr>
<td>5</td>
<td>Present</td>
<td>0.168</td>
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IV. RESULTS AND DISCUSSION

Numerical simulations have been carried out for various power law indices at different Reynolds number. The purpose of these 2D simulations is to study the changes in vortex shedding frequency and the aerodynamic force coefficients on the cylinder. We restrict the Reynolds number between 75 and 150 to eliminate 3-Dimensional effects on the flow and to include transient nature of the flow. In addition, the power law index was kept in range \( n=0.25 \) to 2.0 for numerical stability. It is well known that the fluid becomes thicker, as the value of \( n \) increases.

Figs 3 and 4 represent the amplitude of oscillations of the aerodynamic force coefficients rms of the lift coefficient \( (C_{L_{rms}}) \) and the amplitude of the drag coefficient \( (|C_d|) \). It is fundamental that fluid damping increases with increase in power law index, \( n \) and is attributed to the decrease in the amplitudes of oscillating forces with power law index, \( n \). It is also observed that the trend is the same for all the four different Reynolds number used in the present study and finally, as expected, both the amplitudes of the oscillating forces increases with \( Re \).

![Fig. 3](image)

**Fig. 3.** Variation in RMS value of lift coefficient on the cylinder with respect to power law index \( (n) \)

![Fig. 4](image)

**Fig. 4.** Variation in the amplitudes of oscillating drag force coefficient on the cylinder with respect to power law index \( (n) \)

The dimensionless vortex shedding frequency i.e., strouhal number also follows the same trend as \( C_{L_{rms}} \) and \( |C_d| \) (Fig 5), and it is observed that the raise in power law index of a viscous fluid causes suppression towards vortex induced oscillations or damps the oscillations. From figures 3, 4 and
5 it can be concluded that the viscosity effects are more visible, at the lower Reynolds number regime in the study than at the higher Reynolds number flows considered.

Fig. 5. Variation in in vortex shedding frequency of flow with respect to power law index (n)

Fig. 6 shows the variation of time averaged drag coefficient (Cd) plotted against the power law index, n for different Re. It is found from this figure that with increase of power law index, n, the Cd decreases, attains a minimum value and then increases, for all the Re considered here. The shear thickening causes more drag to the cylinder and is attributed to such behavior of Cd. However, upon reaching a certain index, drag force was found to vary not much but appears to have reached a steady value for all Re.

Fig. 6. Drag characteristics at various Reynolds number with varying power law index (n)

Another observation was that shear thinning fluids hit a local minimum of drag force with respect to the index, and decreasing the index further only increases the drag force. This critical index was found to increase with Reynolds number.

V. CONCLUSION

The conclusive remarks on the topic is that

1) The higher power law index fluids can behave as a good damper to the vortex induced oscillations, and reduce the amplitudes of oscillations during external flows

2) The effect of power law index becomes weaker at high Reynolds number laminar flows

3) There exists a critical power law index for each Reynolds number flows, beyond which shear thinning the fluid causes only increase in the drag coefficient

4) The power law index that corresponds to the minimum drag coefficient tends to increase with raise in Reynolds number of flow and approaches a constant value at high Reynolds number.

5) The topic is especially beneficial to industrial processes involving low speed non-Newtonian flows and also may be aimed for passive vortex suppression of flows by introducing polymer additives to flows in the transition range

REFERENCES


