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Reduced Order Modelling of Linear Time Invariant Systems by using Improved Modal Method

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Abstract: In this paper a new model order reduction technique is proposed for the order reduction of linear dynamic systems. The reduced order models obtained by the proposed method are always stable if the original system is stable. The denominator coefficients of the reduced model are obtained by the preserving of dominant poles of the original system and the numerator coefficients are computed by using a simple mathematical algorithm discussed in the proposed scenario. The proposed method guarantees the retention of fundamental features of the original system such as steady state and transient responses. The accuracy and superiority of the proposed method are shown by using various numerical examples taken from the literature. The proposed method is also validated with the help of a real timeflexible-missile control system.

Keywords: Dominant poles algorithm; order reduction; reduced order modeling;stability; transfer function.

1. Introduction

For the analysis and design of large scale systems, the computer simulation is a necessary step [1]. The exact mathematical models of a higher order system tend to be large and therefore their computer simulations are computationally expensive. The accurate and reduced models of the large scale systems can help fasten their simulation and analysis. The model reduction is a promising method which reduces the order and complexity of higher order mathematicalmodels while retaining their fundamental characteristics [2].

Mostly model reduction techniques have been proposed to ease the analysis and control ofthe higher order systems [3], [4]. In [5], model reduction has been applied to measure the thermal conductivity and the volumetric heat capacity of gases of thermal micro-sensor. Sikander and Prasad applied the model reduction technique for the design of controller in two wheeled mobile robot [6]. For reducing the computational cost concerned with evaluating frequencyresponses of antennas and microwave devices, model compression methods such as the reducedbasis method, or model simplification method are used in [7]. In electromagnetic field, model order reduction techniques are widely used for the various applications such as,reduction of time consumption forbroadband computation of parameter sweeps concerning rigid body motion [8], generation of equivalent circuits of electromagnetic devices [9].In [10], [11], discussed the application of model order reduction techniques in power system. Model order reduction techniques are widely used in the control systems for the design of controllers [12]-[14].

To date, various model reduction techniques have been proposed, in frequency domainsuch as Pade approximation [15], Routh approximation [16], Routh stability [17], stability equation [18], pole clustering [19] methodsand in time domain, balanced truncation [20], singular perturbation [21],aggregation [22], Krylov subspace [23], Hankel norm [24], [25] methods. On the basis of these techniques several mixed methods are proposed in frequency domain [26]-[30] and in time domain [31]-[33].Among these methods, mostly model order reduction methods are based on the retention of dominant poles of original systems in the reduced model such as [19]-[22], [34], [35]. Because the dominant poles of the dynamical systems are used to control their performance and non-dominant poles are used to ensure that the transfer function of controller can be realized by physical components [36]. On the basis of preservation of dominant poles of original system in the reduced model, several mixed model order reduction technique have been proposed [37]-[40]

The main objective of this contribution is to propose a new model simplification technique for the linear dynamic systems. In this method, the denominator polynomial of the reduced model is obtained by modal method [38] and the numerator coefficients are calculated by a simple mathematical algorithm discussed in [41]. Since the denominator polynomial is obtained by modal method, which preserves the dominant poles or desired poles. Therefore reduced models are always stable if the original system is stable. Due to the preservation of dominant poles of original system in the reduced model, the lower order model guarantees the preservation of transient response of original system in the reduced model. The numerator of reduced model is obtained by a simple mathematical algorithm which guarantees the preservation of steady state value of original system in the reduced model [41]. Therefore the proposed method guarantees the retention of stability, steady state and transient responses of the original systems in the lower order models.

The proposed technique is very simple compared to other existing model reduction methods with less restriction. The minor difficulty of this method is that it requires to determine the dominant poles of the large scale system for the computation of reduced models. The truncation method is the only easier method compared to the proposed method but this method has two major limitations, it guarantees the stability of reduced model up to the fourth order reduced model and applicable for certain types of higher order systems [42]. The remaining paper is arranged as: in Section 2, the proposed objective of the paper is discussed for single input and single output (SISO) as well as multi input and multi output (MIMO) systems. The proposed method is explained in Section 3. The proposed technique has been applied on various numerical examples taken from the literature for the illustration and validation of presented technique in Section 4. The conclusions and future scopes of the proposed method are drawn in Section 5.

2. Problem Statement

Consider an nth-order transfer function of higher order SISO linear time invariant (LTI)systems

$$G(s) = \frac{N(s)}{D(s)} = \frac{d_0+d_1s+\cdots+d_{n-1}s^{n-1}}{e_0+e_1s+e_2s^2+\cdots+e_ns^n} \tag{1}$$

The objective of the paper is to compute the unknown scalar constants of rth-order ($r < n$) reduced model and its nature and performance are approximately same as the original systems and it is defined as the following transfer function

$$R_r(s) = \frac{Q_r(s)}{P_r(s)} = \frac{q_0+q_1s+q_2s^2+\cdots+q_{r-1}s^{r-1}}{p_0+p_1s+p_2s^2+\cdots+p_{r-1}s^{r-1}+p_rs^r} \tag{2}$$

3. Basic Procedure of the Proposed Technique

For preserving the stability, steady state value and transient response oflarge scale systems in its reduced model anew model order reduction technique has been proposed. This techniquecombines a very simple mathematical algorithm [41] and dominant pole approach.The proposed technique is described in two steps. In first step the computation of denominator of ROM and in second stepthe calculation of numeratorcoefficients areexplained.

3.1.Procedure for obtaining the denominator polynomial of the reduced order model

To obtain therth order simplified model, $r$number dominant poles or r number of desired poles of the large scale dynamical system are preserved in lower order model and the denominator polynomial of the lower order model is obtained as

$$P_r(s) = \prod_{k=1}^{r}(s - \lambda_k) \tag{3}$$

$$= p_0 + p_1 s + p_2 s^2 + \cdots + p_{r-1}s^{r-1} + p_r s^r \tag{4}$$

whereλ_k for $(k = 1, 2, \ldots, r)$ are the dominant poles or desired poles of the higher order original system. The poles lying in left-half s-plane and closer to the origin of s-plane (dominant poles) give rise to transient responses which will falloffrelativelygradually, while the non-dominant poles comparative to the dominant poles which are far away from the origin of s-plane correspond to quickly decaying the time responses [36], [37].

3.2. Procedure for computingthe numerator polynomial of the reduced order model

The numerator polynomial of the reduced model is determined by using a mathematical algorithm discussed in [41]. In this mathematical procedure, the transfer function of large scale system is compared with the transfer function of lower order model.

$$\frac{d_0+d_1s+\cdots+d_{n-1}s^{n-1}}{e_0+e_1s+e_2s^2+\cdots+e_ns^n} = \frac{q_0+q_1s+q_2s^2+\cdots+q_{r-1}s^{r-1}}{p_0+p_1s+p_2s^2+\cdots+p_{r-1}s^{r-1}+p_rs^r} \tag{5}$$

After cross-multiplication of Equation (5), equating the same powers of 's' from s^0 to s^{r-1} on both sides and it gives "r" number equations.

$$\begin{cases} d_0p_0 = e_0q_0 \\ d_0p_1 + d_1p_0 = e_0q_1 + e_1q_0 \\ d_0p_2 + d_1p_1 + d_2p_0 = e_0q_2 + e_1q_1 + e_2q_0 \\ \vdots \end{cases} \tag{6}$$

The coefficients $(q_0, q_1, \ldots, q_{r-1})$ of the numerator polynomial are calculated by solving "r" number of above equations in which the coefficients of the denominator polynomial are already computed in step1.

4. Numerical Examples

In order to validate the accuracy, effectiveness and superiority of the proposed technique with some other popular model order reduction methods, the integral square error (ISE), relative integral square error (RISE), integral absolute error (IAE) and integral time weighted absolute error (ITAE) of their lower order models are calculated and which are defined as [6], [43]

$$\begin{cases} ISE = \int_0^\infty [y(t_i) - y_r(t_i)]^2 dt \\ RISE = \int_0^\infty [y(t_i) - y_r(t_i)]^2 dt \,/\, \int_0^\infty [\hat{y}(t_i)]^2 dt \end{cases} \tag{7}$$

$$\begin{cases} IAE = \int_0^\infty |y(t_i) - y_r(t_i)| dt \\ ITAE = \int_0^\infty t|y(t_i) - y_r(t_i)| dt \end{cases} \tag{8}$$

where$y(t_i)$ and $y_r(t_i)$ are the step responses of higher order system and lower order model respectively at t_i time. The final time for the simulation is taken as 100 seconds with a sampling interval of 0.1 second.

For illustrating the accuracy and verifying the preservation of important characteristics of large scale systems in the reduced models, the proposed technique has been applied on twostandard numerical examples.

Example 1: Consider the transfer function of a flexible-missile control system containing compensators and the rigid body loop, given in [44]-[46]

$$G(s) = \frac{-s^6 + 3.06 \times 10^2 s^5 - 4.96 \times 10^4 s^4 + 3.577 \times 10^6 s^3 - 6.303 \times 10^7 s^2 - 1.246 \times 10^{10} s + 5.906 \times 10^{11}}{s^8 + 52.99 s^7 + 3.05 \times 10^4 s^6 + 1.375 \times 10^6 s^5 + 1.839 \times 10^8 s^4 + 5.232 \times 10^9 s^3 + 3.422 \times 10^{11} s^2 + 2.823 \times 10^{12} s + 1.442 \times 10^{14}} \quad (9)$$

The poles of the original systems are

$$0.50 + j24.59, 0.50 - j24.59, 22.72 + j46.29, 22.72 - j46.29, 1.28 + j62.46, 1.28 - j62.46, 1.99 + j151.56, 1.99 - j151.56$$

The dominant poles of the original systems are

$$0.50 + j24.59, \qquad 0.50 - j24.59$$

By using these dominant poles, the denominator of the second order reduced model is obtained as

$$P_r(s) = (s + 0.50 + j24.59)(s + 0.50 - j24.59j)$$

$$= s^2 + s + 605 \quad (10)$$

The coefficients of the numerator polynomial are calculated by using Equation (6) and the second order reduced model obtained by proposed technique is

$$R_3(s) = \frac{Q_r(s)}{P_r(s)} = \frac{-0.0967s + 2.478}{s^2 + s + 605} \quad (11)$$

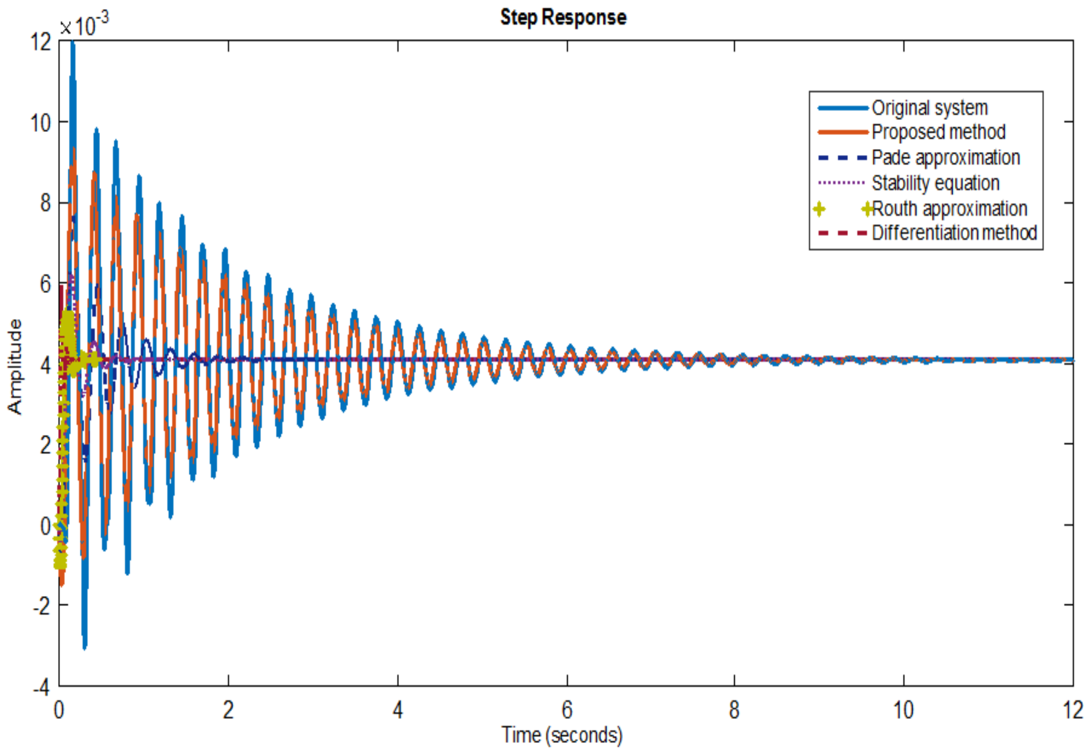

Figure 1: Comparison of step responses of original system and its various reduced order models.

Table 1: Comparison of various model order reduction techniques with respect to ISE, RISE, IAE and ITAE.

Reduction technique	Reduced model	ISE	RISE	IAE	ITAE
Routh stability and Pade approximation [47], [48],	$\frac{-2.3535\times10^{10}s+5.906\times10^{11}}{1.8794\times10^{11}s^2+1.1903\times10^{11}s+1.442\times10^{14}}$	5.4693×10^{-4}	0.0035	0.1441	0.4195
Routh stability and factor division [49]	$\frac{-2.3534\times10^{10}s+5.906\times10^{11}}{1.8794\times10^{11}s^2+1.1903\times10^{11}s+1.442\times10^{14}}$	5.4692×10^{-4}	0.0035	0.1441	0.4195
Routh stability method [17]	$\frac{4.3787\times10^{6}s+5.906\times10^{11}}{1.8794\times10^{11}s^2+1.1903\times10^{11}s+1.442\times10^{14}}$	4.6355×10^{-4}	0.0029	0.1232	0.3064
Modified pole clustering and modified Pade [50]	$\frac{2.8024}{s^2+1.0788s+6.8422\times10^2}$	4.0735×10^{-4}	0.0026	0.1064	0.1991
Modified pole clustering and FD [51], [52]	$\frac{-1.585\times10^{-4}s+2.8024}{s^2+1.0788s+6.8422\times10^2}$	4.0715×10^{-4}	0.0026	0.1064	0.1991
Modified pole clustering and Pade [53]	$\frac{-0.1096s+2.8024}{s^2+1.0788s+6.8422\times10^2}$	3.7474×10^{-4}	0.0024	0.0960	0.1822
Pole clustering and Pade [54]	$\frac{-0.3603s+9.1013}{s^2+2.4046s+2.2221\times10^3}$	3.4588×10^{-4}	0.0022	0.1041	0.2167
Improved pole clustering method [41]	$\frac{-0.1096s+2.8024}{s^2+1.0788s+6.8422\times10^2}$	3.4588×10^{-4}	0.0022	0.1041	0.2167
Routh approximation [16]	$\frac{-0.1663s+7.88}{s^2+37.67s+1924}$	2.4165×10^{-4}	0.0015	0.0884	0.1803
Routh and Pade approximations [55], [56]	$\frac{-0.1662s+7.8801}{s^2+37.67s+1924}$	2.4164×10^{-4}	0.0015	0.0884	0.1778
Differentiation and Pade approximation [57]	$\frac{-4.26\times10^{14}s+1.1906\times10^{16}}{2.464\times10^{14}s^2+1.423\times10^{16}s+2.907\times10^{18}}$	2.3979×10^{-4}	0.0015	0.0882	0.1804
Differentiation method [58]	$\frac{-4.187\times10^{13}s+1.191\times10^{16}}{2.464\times10^{14}s^2+1.423\times10^{16}s+2.907\times10^{18}}$	2.3424×10^{-4}	0.0015	0.0885	0.2403
Stability equation and continued-fraction [46]	$\frac{-2.67\times10^{-4}s+2.4209}{s^2+11.5726s+591.1932}$	2.2688×10^{-4}	0.0014	0.0875	0.2174
Truncation method [42]	$\frac{-1.246\times10^{10}s+5.906\times10^{11}}{3.422\times10^{11}s^2+2.823\times10^{12}s+1.442\times10^{14}}$	2.1574×10^{-4}	1.4×10^{-3}	0.0848	0.1768
Factor division and stability equation method [59]	$\frac{-1.0664\times10^{10}s+5.906\times10^{11}}{2.4399\times10^{11}s^2+2.823\times10^{12}s+1.442\times10^{14}}$	2.1459×10^{-4}	0.0014	0.0852	0.1763
Stability Equation [18], Pade and stability equation [44], [45]	$\frac{-1.246\times10^{10}s+5.906\times10^{11}}{2.4399\times10^{11}s^2+2.823\times10^{12}s+1.442\times10^{14}}$	2.1272×10^{-4}	0.0013	0.0849	0.1762
Pade approximation [15]	$\frac{-0.0626s+1.99}{s^2+4.479s+485.8}$	1.9682×10^{-4}	1.211×10^{-3}	0.0831	0.2077
Time moment matching [60]	$\frac{-1.2886\times10^{-4}s+4.09\times10^{-3}}{2.0585\times10^{-3}s^2+9.2198\times10^{-3}s+1}$	1.9682×10^{-4}	1.21×10^{-3}	0.0830	0.206
Proposed method, Pade and dominant pole [38]	$\frac{-0.0967s+2.478}{s^2+s+605}$	3.7709×10^{-5}	2.3918×10^{-4}	0.0297	0.0582

The comparison of unit step responses of original system with its reduced models obtained by various model reduction techniques has been depicted in Figure 1. From this figure it is clearly visible that the response of reduced model obtained by the proposed method is excellently matched with the response of the original system as compared to the responses of various reduced models obtained by other existing popular model reduction methods. In order to compare the accuracy and superiority of the proposed method, the performance error indices of various reduction methods have been calculated and tabulated in Table 1. This table demonstrates that the proposed method gives least error indices compared to other existing methods. It is also clear that the proposed method and method discussed in [38] give same transfer function and performance error indices but the proposed method is simpler compared to existing technique.

Example 2: Let us consider an eighth order system described by following transfer function is [17]

$$G(s) = \frac{35s^7+1086s^6+13285s^5+82402s^4+278376s^3+511812s^2+482964s+194480}{s^8+33s^7+437s^6+3017s^5+11870s^4+27470s^3+37492s^2+28880s+9600} \quad (12)$$

The poles and zeros of $G(s)$ are

Zeros: $-1.035 \pm 0.631j,\ -2.639,\ -3.835,\ -4.902,\ -7.801,\ -9.785$

Poles: $-1, -1 \pm 1j, -3, -4, -5, -8, -10.$

The fourth order reduced model obtained by the proposed method discussed in Section 3 is:

$$R_4(s) = \frac{33.6045s^3+151.6093s^2+219.8063s+121.55}{s^4+6s^3+13s^2+14s+6} \quad (13)$$

The Figure 2, represents the unit step responses of original model and its reduced models computed by different model reduction techniques. In this figure it is clear that the reduced order model obtained by proposed method gives approximately same time response as given by the original system. The performance indices of various model reduction techniques have been tabulated in Table 2. This table indicates that the proposed method gives least error indices compared to some other popular [16]-[20] and recently proposed techniques [50]-[54], [57], [59]. Hence it is concluded that the proposed method is better approximation technique compared to some existing techniques.

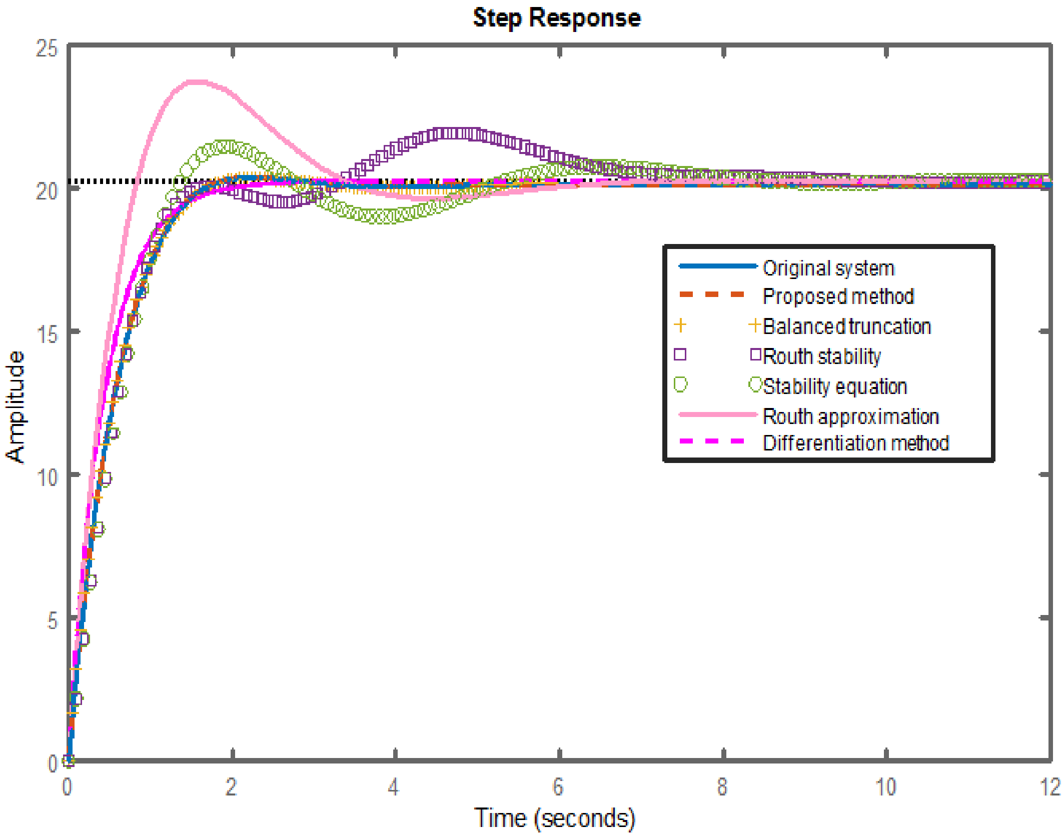

Figure 2: Comparison of step responses of original system and its various lower order models.

Table 2:Comparison of different model reduction techniques with respect to ISE, RISE, IAE and ITAE.

Model order reduction method	Reduced order model	ISE	RISE	IAE	ITAE
Pade and Differentiation [57]	$\frac{-7714484964757.8s^3 + 257589205006.9s^2 - 8526456536.6s + 326725862.4}{284880s^4 + 3296400s^3 + 13497120s^2 + 24259200s + 16128000}$	9.736×10^{12}	2.22×10^{9}	1.2552×10^{7}	1.249×10^{7}
Pade and pole clustering [54]	$\frac{-12729000s^3 + 425050s^2 - 14054s + 541.9825}{s^4 + 10.95s^3 + 33.28s^2 + 44.66s + 26.75}$	3.251×10^{12}	7.42×10^{8}	8.6171×10^{6}	1.205×10^{7}
Pade and modified pole clustering [53]	$\frac{-5740700s^3 + 191720s^2 - 6326.6s + 247.7772}{s^4 + 8.5272s^3 + 21.1699s^2 + 25.2853s + 12.2309}$	1.369×10^{12}	3.126×10^{8}	4.2359×10^{4}	7.233×10^{4}
Pade and dominant poles [38]	$\frac{-2790800s^3 + 93209s^2 - 3071.3s + 121.549}{s^4 + 6s^3 + 13s^2 + 14s + 6}$	8.01×10^{11}	1.83×10^{8}	4.9978×10^{6}	8.8906×10^{6}
Stability and Pade approximation [45], [46]	$\frac{-4365200000s^3 + 145840000s^2 - 4782700s + 194480}{10308s^4 + 23868s^3 + 37083s^2 + 28880s + 9600}$	2.39×10^{11}	5.46×10^{7}	3.5034×10^{6}	1.0177×10^{7}
Routh stability and factor division method [49]	$\frac{18826888.108s^3 - 10632856.88s^2 + 408393.07s + 194480}{6817.2s^4 + 14847.1s^3 + 31694s^2 + 25199s + 9600}$	4.5859×10^{9}	4.487×10^{5}	4.487×10^{5}	1.356×10^{6}

Factor division and modified pole clustering method [51], [52]	$\frac{-2606406.66s^3+886146.45s^2-9588.98s+16263.25}{s^4+47.99s^3+495.38s^2+894.78s+802.79}$	9.8979× 10^8	2.26× 10^5	1.0316× 10^5	9.687× 10^4
Routh stability and Pade approximation method [47], [48]	$\frac{-77186000s^3+147790000s^2-4857300s+194480}{6817.2s^4+14847.1s^3+31694s^2+25199s+9600}$	4.27× 10^8	9.7467× 10^4	1.4267× 10^5	4.3378× 10^5
Modified Pade and modified pole clustering [50]	$\frac{298.452s^3+152.5693s^2-6326.6s+247.7772}{s^4+8.5272s^3+21.1699s^2+25.2853s+12.2309}$	9.9379× 10^5	2.268× 10^2	4.2359× 10^4	7.233× 10^4
Pole clustering method [19]	$\frac{79.3s^3+533.2s^2+880.2s+542.1}{s^4+10.95s^3+33.28s^2+44.66s+26.75}$	463.9612	1.06× 10^{-1}	103.5677	438.5651
Routh approximation [16]	$\frac{41.2369s^3+97.7602s^2+87.309s+30.3835}{s^4+3.5811s^3+5.6212s^2+4.5118s+1.4998}$	293.0804	0.0669	95.3986	169.4458
Stability equation and factor division method [59]	$\frac{209755.227s^3+503526.341s^2-482964s+194480}{10308s^4+23868s^3+37083s^2+28880s+9600}$	108.367	2.47× 10^{-2}	77.1285	239.3885
Truncation method[42]	$\frac{278376s^3+51182s^2+482964s+194480}{11870s^4+27470s^3+37492s^2+28880s+9600}$	79.101	1.81× 10^{-2}	71.6827	262.8936
Routh stability method [17]	$\frac{173419.1s^3+322069s^2+439546.9s+194480}{6817.2s^4+14847.1s^3+31694s^2+25199s+9600}$	67.2759	1.54× 10^{-2}	61.7224	273.3306
Stability equation method[18]	$\frac{253150s^3+478540s^2+482964s+194480}{10308s^4+23868s^3+37083s^2+28880s+9600}$	40.2162	9.2× 10^{-3}	50.2674	175.5392
Stability equation and continued fraction method [46]	$\frac{253159+478501s^2+482964s+194480}{10308s^4+23868s^3+37083s^2+28880s+9600}$	40.1541	9.2× 10^{-3}	50.2317	175.4299
Differentiation method [58]	$\frac{13362048s^3+122834880s^2+347734080s+326726400}{284880s^4+3296400s^3+13497120s^2+24259200s+16128000}$	26.4528	6.0× 10^{-3}	23.638	34.9497
Improved pole clustering method [41]	$\frac{30.02s^3+235.6693s^2+382.1617s+247.7776}{s^4+8.5272s^3+21.1699s^2+25.2853s+12.2309}$	0.8788	2.01× 10^{-4}	5.1024	9.2287
Routh and Pade approximation [55], [56]	$\frac{31.6069s^3+75.1767s^2+75.4509s+30.3834}{s^4+3.5811s^3+5.6212s^2+4.5118s+1.4998}$	0.7775	1.77× 10^{-4}	6.164	18.5992
Balanced truncation method[20]	$\frac{34.96s^3+254.1s^2+403.1s+267.5}{s^4+9.194s^3+22.67s^2+26.8s+13.21}$	0.0361	8.23× 10^{-6}	5.8153	312.5173
Proposed Method	$\frac{33.6045s^3+151.6093s^2+219.8063s+121.55}{s^4+6s^3+13s^2+14s+6}$	0.0229	5.22× 10^{-6}	0.7115	0.9887

5. Conclusions and Future Scope

In this contribution a new model reduction method has been proposed for the higher order LTI systems, which is very simple compared to other existing popular methods. From table 3, it is also clear that the proposed method also gives better approximation compared to the some complicated optimization based model reduction methods. In this method, the denominator polynomial of the reduced model is obtained by model method and the numerator polynomial is calculated by a simple mathematical algorithm discussed in literature. The reduced models yielded by the proposed technique are guaranteed to be stable given that the original models are stable. The reduced models also guaranteed the retention of dominant poles, steady state and transient responses of the original models.

The accuracy and the superior performance of the proposed technique were evaluated by comparing the various performance error indices and tabulated in Tables 1 and 2. From these tables,it is found that the reduced model obtained by the proposed technique exhibited better performance and has least error indices.In Figures 1and 2, the response of lower order model obtained by proposed technique is closely matched to the response of original model compared to the responses of other reduced models obtained by existing popular and recentlyproposed model order reduction methods. Hence, the proposed technique is the closest approximation of the originalmodel. Fordemonstrating the effectiveness of the proposed technique, it has been applied on three standard numerical examples. It is also found that the proposed technique has least performance error indices and better approximation to the original model and thus it can be explored in the field of controller design and digital signal processing.

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