PERISTALTIC FLOW OF A BINGHAM FLUID IN A NON-UNIFORM CHANNEL AND ITS EFFECTS ON HEAT TRANSFER, SLIP CONDITIONS AND WALL PROPERTIES

Dr.P.Vinod kumar*
* Department of Mathematics, JNTUH College of Engineering, Jagtial, Telangana, INDIA.
( pvk1420@gmail.com )
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Abstract

Wall slip conditions and heat transmission of Bingham fluid have been investigated by pumping the fluid through the elastic walled porous channel. This has been done at a long wavelength and low Reynolds number. By this experiment we observed temperature profile, velocity and heat transfer coefficient. Analysis of the effects of various physical parameters on temperature and velocity have been done. Its results are presented through graphs in details.

Key Words: Peristaltic flow, Slip, Wall properties, Heat transfer, Bingham fluid and Non-uniform channel.

1 Introduction:

Peristaltic pumping a mechanism by which fluid is transported through contracted or expanded area of distensible tube along with
the progressive waves. The peristaltic transportation mainly occurs in biological or biomedical systems. This system plays a vital role in transportation of physiological fluids through the body. This mechanism gets activated and begins to work while swallowing food through oesophagus. This will be seen while transporting urine from the kidneys to the urinary bladders through ureter. Peristaltic pumping is also observed in the movement of chime in the gastro-intestinal tract and in the transportation of spermatozoa in the ductus efferents of the male reproductive tract and in the cervical canal ect. This mechanism has multifarious applications in medicine and industry. Peristalsis literature occupies a vast area including physiology and industry. Some investigations [1-7] are presented in the references.

Peristalsis mechanism has recently become a very important subject of scientific research in both mechanical and physiological situations the first research work carried out by Latham [8] in this field.

In industry it has numerous applications. In sanitary fluid transportation and in transportation of corrosive fluid peristaltic mechanism plays an important role. The same mechanism can be found in nuclear industry for transportation of toxic liquid. After the first investigation of Latham a host of scientists and research scholars jumped upon it to carry out experiments Shapiro at al investigated a mathematical model for peristaltic flow. A great number of scholars take Shapiro’s [9] investigations as the base for their research in the area of peristaltic mechanism. Most of the studies and research work on peristaltic flow deals with Newtonian fluid. In the biological point of view this mechanism motivated investigations in different non-Newtonian fluid also.

There are a number of evidences regarding the scientific study of peristaltic mechanism.


Hence several researchers and investigators studied the flow behavior of non-Newtonian fluid in physiological system of living bod-
ies. Among them Bingham fluid explains velocity, temperature, stream function and heat transfer coefficient in detail. In this paper the graphical result are presented to discuss the physical behavior of various parameters of interest.

2 Mathematical formulation

Consider the flow of a Bingham fluid through a two-dimensional channel of uniform thickness. The walls of the channel are assumed to be flexible and are taken as stretched membranes, on which travelling sinusoidal waves of moderate amplitude are imposed. The geometry of the wall surface is given by

\[ \bar{\eta}(\bar{x}, \bar{t}) = d(x) - a \sin \frac{2\pi}{\lambda}(\bar{x} - c\bar{t}) \] (1)

where \( d(x) = d + \bar{m}x \), \( \bar{m} \ll 1 \)

The governing equations which describe the flow are

\[ \rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \mu \left( \tau_0 - \mu \frac{\partial \bar{u}}{\partial \bar{y}} \right) \] (2)

\[ \rho \left( \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \left( \mu \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) \] (3)

\[ \xi \left( \frac{\partial T}{\partial t} + \bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} \right) = \frac{k}{\bar{p}} \left( \frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} \right) + 2\nu \left[ (\frac{\partial \bar{u}}{\partial \bar{x}})^2 + (\frac{\partial \bar{v}}{\partial \bar{y}})^2 \right] + \left( \frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 \] (4)

where \( \bar{u}, \bar{v}, \rho, p, d, a, \lambda, \bar{m}, \xi, \nu, k, T \) and \( \tau_0 \) are the axial velocity, transverse velocity, fluid density, viscosity of the fluid, pressure, mean width of the channel, amplitude, wavelength, wave speed, dimensional non-uniformity of the channel, specific heat at constant volume, kinematic viscosity, thermal conductivity of the fluid, temperature and yield stress.

The equation of motion of the flexible wall is expressed as
$H^*(\bar{\eta}) = \bar{p} - \bar{p}_0$ \hspace{1cm} (6)

where $H^*$ is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

$$H^* = -\tau \frac{\partial^2}{\partial x^2} + m_1 \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} \hspace{1cm} (7)$$

where $\tau$ is the elastic tension in the membrane, $m_1$ is the mass per unit area, $C$ is the coefficient of viscous damping forces, $p_0$ is the pressure on the outer side surface of the wall due to the tension in the muscles and $\bar{\eta}$ is the dimensional slip parameter. We assumed $p_0=0$.

Continuity of stress at $\bar{y} = \pm (\bar{\eta})$ and using x- momentum equation, yield

$$\frac{\partial}{\partial t} = \frac{\partial p}{\partial \bar{x}} H^*(\bar{\eta}) = \mu \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \frac{\partial}{\partial \bar{y}} (\tau_0 - \mu \frac{\partial \bar{u}}{\partial \bar{y}}) - \rho \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \hspace{1cm} (8)$$

$$\bar{u} = -h \frac{\partial \bar{u}}{\partial \bar{y}} \bar{t} \bar{y} = \eta \hspace{1cm} (9)$$

$$\frac{\partial T}{\partial \bar{y}} = 0 \text{ on } y = y_0$$

$$T - T_1 \text{ on } y = \eta \hspace{1cm} (10)$$

Introducing $\Psi$ such that $u = \frac{\partial \Psi}{\partial \bar{y}}$ and $v = \frac{\partial \Psi}{\partial \bar{x}}$ and the following non-dimensional parameters are given by

$$\left\{\begin{array}{l}
x = \frac{\bar{x}}{\lambda}, \ \bar{y} = \frac{\bar{d}}{\lambda}, \ \bar{\eta} = \frac{\bar{\eta}}{\lambda}, \ \bar{p} = \frac{\partial^2}{\mu c \lambda}, \ t = \frac{c t}{\lambda}, \ K = \frac{k}{d^2}, \\
m = \frac{\lambda \bar{m}}{d}, \ \delta = \frac{\bar{d}}{\lambda}, \ \epsilon = \frac{\bar{a}}{\lambda}, \ \bar{\eta} = \frac{\lambda}{1 + \bar{m} x + \epsilon \sin 2\pi(x - \bar{t})}, \\
R = \frac{\rho c d}{\mu}, \ \theta = \frac{T - T_0}{T_1 - T_0}, \ P_r = \frac{\rho v \xi}{k}, \ E_c = \frac{c^2}{\lambda^2 \mu}, \ E_1 = \frac{-\tau d^3}{\lambda^3 \mu}, \ E_2 = \frac{m_1 c d^3}{\lambda^3 \mu}, \ E_3 = \frac{c d^3}{\lambda^3 \mu}, \ \beta = \frac{\bar{h}}{\lambda} \end{array}\right\} \hspace{1cm} (11)$$

where $R$ is the Reynolds number, $\delta$ and $\epsilon$ are the dimensionless geometric parameters, $P_r$ is the Prandtl number, $E_c$ is the Eckert
number, $E_1$, $E_2$ and $E_3$ are the dimensionless elasticity parameters is the non-uniform parameter and $\beta$ is the Knudsen number (slip parameter).

Using non-dimensional quantities the basic equations (1) - (10) reduce to

$$R\delta \left[ \frac{\partial^2 \Psi}{\partial t \partial y} + \frac{\partial \Psi}{\partial y} \frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} \right]$$

$$= - \frac{\partial p}{\partial x} + \delta^2 \frac{\partial^2 \Psi}{\partial x^2 \partial y} - \frac{\partial}{\partial y} (\tau_0 - \frac{\partial^2 \Psi}{\partial y^2}) \quad (12)$$

$$R\delta \left[ \frac{\partial^3 \Psi}{\partial t \partial y^2} + \frac{\partial \Psi}{\partial y} \frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} \right]$$

$$= - \frac{\partial p}{\partial y} + \delta^2 \left( \delta^3 \frac{\partial^3 \Psi}{\partial x^3} + \frac{\partial^3 \Psi}{\partial y \partial y^2} \right) \quad (13)$$

$$R\delta \left[ \frac{\partial \theta}{\partial t} + \frac{\partial \Psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \Psi}{\partial y} \frac{\partial \theta}{\partial y} \right] + \frac{1}{P_r} \left[ (\beta^2 \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}) \right]$$

$$+ E(4\delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2} \right)^2) \quad (14)$$

$$\frac{\partial \Psi}{\partial y} = \beta \frac{\partial^3 \Psi}{\partial y^2} \text{at} y = \eta \quad (15)$$

$$\delta^2 \frac{\partial^3 \Psi}{\partial x^3 \partial y} - \frac{\partial}{\partial y} (\tau_0 - \frac{\partial^2 \Psi}{\partial y^2})$$

$$- R\delta \left[ \frac{\partial^2 \Psi}{\partial t \partial y} + \frac{\partial \Psi}{\partial y} \frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial \Psi}{\partial x} \frac{\partial^2 \Psi}{\partial y^2} \right] \quad (16)$$

$$= \left[ E_1 \frac{\partial^3 \eta}{\partial x^3} + E_3 \frac{\partial^3 \eta}{\partial x \partial t^2} + E_3 \frac{\partial^2 \eta}{\partial x \partial t} \right]$$

Further, it is assumed that the zero value of the streamline at the line $y = 0$, i.e.

$$\Psi_p(0) = 0$$

$$\Psi_{yy}(0) = \tau_0 \text{at} y = 0 \quad (17)$$

$$\frac{\partial \eta}{\partial y} = 0 \text{on} y = y_0$$

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\[ \theta = 1 \eta = \eta \quad (18) \]

**Solution of the problem**

Applying the long wavelength and low Reynolds number approximation, the basic equations (12) - (18) reduce to

\[ 0 = -\frac{\partial p}{\partial x} - \frac{\partial}{\partial y}(\tau_0 - \frac{\partial^2 \Psi}{\partial y^2}) \quad (19) \]

\[ 0 = \frac{\partial p}{\partial y} \quad (20) \]

Equation (20) shows that \( P \) is not a function of \( y \)

\[ 0 = \frac{1}{P} \frac{\partial^2 \theta}{\partial y^2} + E \left( \frac{\partial \Psi}{\partial y} \right)^2 \quad (21) \]

By differentiating equation (19) with respect \( y \) we obtain

\[ \frac{\partial^2}{\partial y^2}(-\tau_0 + \frac{\partial^2 \Psi}{\partial y^2}) = 0 \quad (22) \]

From equation (16) we get

\[ \frac{\partial}{\partial y}(-\tau_0 + \frac{\partial^2 \Psi}{\partial y^2}) = [E_1 \frac{\partial^3 \eta}{\partial x^3} + E_3 \frac{\partial^3 \eta}{\partial x \partial t^2} + E_3 \frac{\partial^2 \eta}{\partial x \partial t}] \quad (23) \]

By solving equation (22) with boundary conditions (15), (17) and (23) we obtain the stream function in the plug flow region as

\[ \Psi_p = A \left[ (y_0^2 - \beta y_0 - \eta y_0) + \frac{1}{2} \left( y_0^2 - \eta^2 - 2\beta \eta \right) \right] y \quad (24) \]

and corresponding plug flow velocity is given by

\[ u_p = A \left[ (y_0^2 - \beta y_0 - \eta y_0) + \frac{1}{2} \left( y_0^2 - \eta^2 - 2\beta \eta \right) \right] \quad (25) \]

where

\[ y_0 = \frac{\tau_0}{A} \]

and

\[ A = -8 \xi \pi \left[ (E_1 + E_2) \cos 2\pi(x - t) - \frac{E_3}{2\pi} \sin 2\pi(x - t) \right] \]
and the stream function in the non-plug flow region as
\[ \Psi = \tau_0 \left( \frac{(y^2 - \eta_0^2)}{2} - \beta(y - \eta_0) - \eta(y - \eta_0) \right) \]
\[ + A\left( \frac{(y^3 - 8y_0^3)}{6} - \frac{\eta^2y - \eta_0^2}{2} - \frac{\beta\eta - y^2_0(\beta + \eta)}{2} \right) \]
(26)
The corresponding velocity in the non-plug flow region is given by
\[ u = \tau_0(y - \beta - \eta) + \frac{A}{2}(y^2 - \eta^2 - 2\beta\eta) \]
(27)
Using equation (26) in equation (21) subject to the condition (18) we obtain the temperature as
\[ \theta = -B_r\left( \frac{y^4}{6} + \frac{A^2y^4}{12} + \frac{\tau_0 Ay^3}{den} \right) + C_1y + C_2 \]
(28)
where
\[ C_1 = B_r\left[ \frac{\tau_0 y^2}{2} + \frac{A^2\eta_0^3}{3} + \tau_0 A\eta^2 \right] \]
\[ C_2 = 1 + B_r\left[ \frac{\tau_0 \eta^3}{6} + \frac{A^2\eta^4}{12} + \tau_0 A\eta^3 \right] \]
and \( B_r = E_cP_r \) is the Brinkman number. The coefficient of heat transfer at the wall is given by
\[ Nu = -\left( \frac{\partial \theta}{\partial y} \right)_{at y=\eta} \]
(29)

3 Results and Discussions

Equation (28) gives the expression for temperature as a function of \( y \). Temperature profiles are plotted from Figure 1 to Figure 4 to study the effects of different parameters such as non-uniform parameter \( m \), Brinkman number \( B_r \), amplitude ratio \( \epsilon \) and yield stress parameter \( \tau_0 \) on the temperature distribution. It is observed that the temperature profiles are almost parabolic. Fig 1 reveals that the temperature is higher diverging channel when \( m \) is positive compared with uniform when \( m \) is equal to zero and convergent channels when \( m \) is negative. Fig 2 and Fig 3 are plotted to study the effect of Brinkman number \( B_r \) and amplitude ratio \( \epsilon \). We notice that the temperature increases with increasing Brinkman number
Fig 4 shows that the temperature decreases with increasing yield stress parameter $\tau_0$.

Equation (27) gives the expression for velocity as a function of $y$. Velocity profiles are plotted from Figure 5 to Figure 8 to study the effects of different parameters such as slip parameter $\beta$, non-uniform parameter $m$, amplitude ratio $\epsilon$ and yield stress parameter $\tau_0$ on the velocity distribution. Fig 5 is plotted for different values of slip parameter $\beta$. It is observed that the velocity profiles are parabolic and the velocity increases with increasing $\beta$. Fig 6 depicts that the velocity for a divergent channel $m$ is positive is higher compared with uniform channel $m$ is equal to zero where as it is lower for a convergent channel $m$ is negative. From Fig 7 and Fig 8 we noticed that the velocity increases with increasing amplitude ratio $\epsilon$ and decreasing yield stress parameter $\tau_0$.

The rate of heat transfer ($Nu$) is calculated in equation (29). The variation in Nusselt number for different values of the interesting parameters Brinkman number $Br$, non-uniform parameter $m$, amplitude ratio $\epsilon$ and yield stress parameter $\tau_0$ can be examined through the figures 9-12. It is noticed that due to peristalsis, the rate of heat transfer shows oscillatory behaviour. From Fig 9 we observe that the Nusselt number increases with increasing Brinkman number $Br$. Figures Fig 10 to Fig.12 depict that the Nusselt number increases with increasing non-uniform parameter $m$, amplitude ratio $\epsilon$ and yield stress parameter $\tau_0$.

4 Trapping phenomenon

When the flow rate of streamlines is very high and occlusions are very large, the trapping presents an interesting phenomenon. Streamlines are plotted to study the consequences of slip parameter $\beta$ and non-uniform parameter $m$ on trapping through Fig.13(a-d). From Fig.13 (a-d) we observe that the number of trapped boluses increases with increasing slip parameter, reveals that the number of trapped boluses increases with increasing non-uniform parameter.
Fig. 1 Variation of dimensionless temperature distribution for various values of “m” with fixed $y_0 = 0.2$, $x = 0.2$, $\beta = 0.1$, $m = 0.1$, $B_r = 2$, $\epsilon = 0$, $E_1 = 0.5$, $E_2 = 0.3$, $E_3 = 0.2$

Fig. 2 Variation of dimensionless temperature distribution for various values of “Br” with fixed $y_0 = 0.2$, $x = 0.2$, $t = 0.1$, $m = 0.2$, $\epsilon = 0.1$, $E_1 = 0.5$, $E_2 = 0.3$, $E_3 = 0.2$
Fig.3 Variation of dimensionless temperature distribution for various values of “ε” with fixed $y_0 = 0.2, x = 0.2, t = 0.1, B_r = 2, m = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

Fig.4 Variation of dimensionless temperature distribution for various values of “τ₀” with fixed $x = 0.2, t = 0.1, B_r = 2, ε = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$
Fig. 5 Variation of velocity profile for various values of $\beta$ with fixed $y_0 = 0.2, x = 0.2, t = 0.1, B_r = 2, m = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$. 

Fig. 6 Variation of velocity profile for various values of $m$ with fixed $y_0 = 0.2, x = 0.2, t = 0.1, B_r = 2, \beta = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$. 

$\beta = 0.1$, $\beta = 0.2$, $\beta = 0.3$ 
$m = -0.1$, $m = 0$, $m = 0.1$
Fig. 6 Variation of velocity profile for various values of “m” with fixed $y_0 = 0.2, x = 0.2, \beta = 0.1, m = 0.1, B_r = 2, \epsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

Fig. 7 Variation of velocity profile for various values of “$\epsilon$” with fixed $y_0 = 0.2, x = 0.2, \beta = 0.1, m = 0.1, B_r = 2, t = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$
Fig. 8 Variation of velocity profile for various values of $\tau_0$ with fixed $x = 0.2$, $t = 0.1$, $m = 0.1$, $B_r = 2$, $\epsilon = 0.1$, $E_1 = 0.5$, $E_2 = 0.3$, $E_3 = 0.2$
Fig. 9 Heat transfer coefficient for various values of Brinkman number with fixed

\[ \tau_0 = 0.2, x = 0.2, t = 0.1, \epsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2 \]

Fig. 10 Heat transfer coefficient for various values of non-uniform parameter with fixed

\[ \tau_0 = 0.2, x = 0.2, t = 0.1, B_r = 2, \epsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2 \]
Fig. 11: Heat transfer coefficient for various values of amplitude ratio with fixed $\tau_0 = 0.2, x = 0.2, t = 0.1, B_r = 2, m = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$

Fig. 12: Heat transfer coefficient for various values of yield stress parameter with fixed $m = 0.1, x = 0.2, t = 0.1, B_r = 2, \epsilon = 0.1, E_1 = 0.5, E_2 = 0.3, E_3 = 0.2$
Fig. 13 Stream lines for (a) (b) & (c) (d) when
\[ m = 0.1, y_0 = 0.2, t = 0.5, E_1 = 0.6, E_2 = 0.2, \beta = 0.2, y_0 = 0.2, t = 0.5, \epsilon = 0.1, E_1 = 0.6, E_2 = 0.4, E_3 = 0.2 \]

References


