The Development of Economic And Mathematical Model of State Support to Small and Medium-Sized Businesses with the Use of the Duality Theory

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Abstract

The article investigates the need to develop an effective mechanism of state support for small and medium-sized businesses that would take into account regional peculiarities and geographical differentiation in the development of individual regions, the modeling of such mechanism from the economic and mathematical point of view. The research tasks included the evaluation of the Federal target programs for supporting small and medium-sized businesses, which should take into account those characteristics of external environment that create the most favorable conditions for its functioning. The toolkit for economic and mathematical modeling of interaction between the state and SMEs has been developed. It includes an economic and mathematical model of state support for SMEs with the use of the duality theory.

Key Words: state support, linear programming, duality theory, transaction costs.

JEL Classification:C 000.
1 Introduction

The need to develop an effective mechanism of state support for small and medium-sized businesses that would take into account regional peculiarities and geographical differentiation in the development of individual regions involves the modeling of such mechanism from the economic and mathematical point of view (Akhmetshin & Vasilev, 2016; Latyshev & Akhmetshin, 2015; Lukyanova & Soldatova, 2013; Akhmetshin & Osadchy, 2015).

As a starting material for modeling it is possible to consider the content of the existing state support program for SMEs implemented at the federal level, the existing legal framework and expert opinions. The problems of linear programming are quite a simple class of tasks from the mathematical point of view, but they are also the most accessible for interpretation from economic point of view. Therefore, they are often used in the economic and mathematical modeling. Of course, one cannot say that all economic dependencies (factors, indicators) are linear. It is the other way around in most cases (Von Neumann, & Morgenstern, 1970). But, formulating the problem of vector optimization containing several target functions with the corresponding systems of restrictions one has to admit that from the mathematical point of view it is still necessary to recognize that its solution, as a rule, will be in choosing the priority between the target functions and its consistent solution in this order. Therefore, in our opinion, a solution of particular problems, including the allocation of resources, should begin with the formulation of the task of linear programming making the maximum use of its capabilities.

2 Methodological Framework

The duality theory of linear programming (LP) makes it possible to compare the original task with the dual task that will be linked to the original one. The conditions of the original and dual tasks will contain the same variables, only in the different order.

For example, a general LP task is to find the maximum value of the function:

\[ F = c_1x_1 + c_2x_2 + \cdots + c_jx_j + \cdots + c_nx_n \rightarrow \max, \quad (1) \]
with restrictions:

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1j}x_j + \cdots a_{1n}x_n & \leq b_1, \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2j}x_j + \cdots a_{2n}x_n & \leq b_2, \\
    \vdots & \quad \vdots \\
a_{11}x_1 + a_{22}x_2 + \cdots + a_{ij}x_j + \cdots a_{in}x_n & \leq b_1, \\
    \vdots & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mj}x_j + \cdots a_{mn}x_n & \leq b_m
\end{align*}
\] (2)

\[x_j \geq 0, (j = 1, n)\] (3)

A task dual to the initial LP task will be the task with a minimum target function:

\[F^* = b_1y_1 + b_2y_2 + \cdots + b_iy_i + \cdots + b_my_m \rightarrow min,\] (4)

which will constitute a dual pair of the original task.

\[
\begin{align*}
    a_{11}y_1 + a_{12}y_2 + \cdots + a_{1j}y_j + \cdots a_{1m}y_m & \leq c_1, \\
a_{21}y_1 + a_{22}y_2 + \cdots + a_{2j}y_j + \cdots a_{2m}y_m & \leq c_2, \\
    \vdots & \quad \vdots \\
a_{11}y_1 + a_{22}y_2 + \cdots + a_{ij}y_i + \cdots a_{mn}y_m & \leq c_j, \\
    \vdots & \quad \vdots \\
a_{m1}y_1 + a_{m2}y_2 + \cdots + a_{mn}y_m & \leq c_m
\end{align*}
\] (5)

\[y_i \geq 0, (i = 1, m)\] (6)

Task (1, 3) is dual to task (4, 6). This pair of tasks in linear programming is called a dual pair. It can be seen that it is possible to formulate some general rules for compiling a dual pair:

1) target functions of both tasks have opposite directions, for example, the target function of the original task (1) tends to maximum, while the target function of the dual task (4) tends to minimum;

2) matrix

\[
A = \begin{pmatrix}
    a_{11} & a_{1j} & a_{1n} \\
    a_{i1} & a_{ij} & a_{in} \\
    a_{m1} & a_{mj} & a_{mn}
\end{pmatrix}
\] (7)

containing coefficients with unknown variables of the original task through the transposition (rows are replaced by columns) becomes matrix

3
\[ A^T = \begin{pmatrix} a_{11} & a_{i1} & a_{m1} \\ a_{1i} & a_{ji} & a_{mi} \\ a_{1n} & a_{jn} & a_{mn} \end{pmatrix} \]

Which includes the coefficients with unknown variables of the dual task:

3) the number of restrictions of the dual task is equal to the number of variables of the original task, and vice versa;

4) free terms of the original task are unknown coefficients with unknown target function of the dual task (Stepanov, n./d.).

There are also conditions concerning non-negativity of the variables and symmetry or asymmetry of the initial and dual tasks, but their detailed consideration is not included in the research objective and, if necessary, can be carried out by using a specialized literature. It can be noted that variables \( x_j \) of the original task can assume only positive values (based on the problem statement). Respectively, the restrictions of the dual task can only be inequalities of the type \( = \). Regarding the symmetry, it should be noted that variables of the dual task \( y_i \) can only assume positive values. Respectively, the dual pair of tasks will be symmetric, and the restrictions of the original task will be inequalities of the type \( \neq \).

3 Results and Discussion

Before presenting the interpretation of the original task of linear programming it is necessary to make some digression from the topic of the research.

The goal of the study is to assess the Federal target programs for supporting small and medium-sized businesses (SMEs), which should take into account those characteristics of the external environment that create the most favorable conditions for its functioning (Figure 1).
The basis for the formation of criteria for the evaluation of the Federal Target Programs is an assumption that, once implemented, the existing position of SMEs, including their results and potential, will improve. This is a model of the process of ensuring the transition of the values of SMEs performance indicators from the real to the desired ones. It is proposed to carry out a number of activities the implementation of which will make such transition possible (Dadalko, Mikhalko&Savchuk, 2010; Dareev, 2012; Kolesnikov&Vidyakina, 2013).

If we summarize the results of the analysis of insufficiently effective implementation of the Federal target programs for supporting small and medium-sized businesses, the reasons for this phenomenon can be presented as follows (Figure 2).

The deficiencies of the developed SME support programs include (Figure 3).
The analysis of the previously developed Federal programs of support and development of small businesses shows that they have a number of significant shortcomings. They include:

It is known that the need to develop SME support programs arises if there are actual problems in this sector of the economy, i.e. there are significant discrepancies between what already exists and what is desirable in terms of socio-economic interests of the country.

The justification for the allocation of funds among regions and areas of support should answer the questions of the need for allocating funds precisely in those areas in which problems are identified. The factors that interfere with the development as well as the factors contributing to it should be identified. Both of them should have a quantitative and a qualitative assessment.

There is another aspect in the formation of state target programs of support to SMEs at the regional level: the effectiveness of the regional authorities (the effectiveness of monitoring of the condition of SMEs in the region, a timely response to changing performance indicators, the speed of documentation and overcoming of administrative barriers, the ability to provide information about the regions needs on the federal level, etc.).

In other words, it is necessary to assess the transaction costs of regional authorities incurred by them to ensure the allocation of funds for a particular federal program to support SMEs in the region. This is a so-called price of resources determined by the
federal target program to support SMEs in the region, which can be interpreted with the help of the aforementioned linear programming duality theory. It is possible to use indicators C1j or 3j as gij depending on the research objectives, or an indicator calculated by using any authors method, which takes into account the distribution of subsidies in accordance with the previously proposed indicators of the "triple helix" - the unity of the state, business and science.

Then the original task can look as follows. It is necessary to maximize the amount of state support (subsidies) to SMEs in the regions (9) in such a way that the indices of the values of the target program indicators (the number of SMEs in the region, the number of employees, etc.) increase within the scope of the possible subsidies for each program (10):

![Figure 3. The deficiencies of the previously developed Federal program of support and development of SMEs.](image)

<table>
<thead>
<tr>
<th>Table 1. The interpretation of the original task of linear programming.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The program of financing (housing construction, road construction, etc.), the volume of funds, million</td>
</tr>
<tr>
<td>$p_1$</td>
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<td>...</td>
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<tr>
<td>$p_2$</td>
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<td>...</td>
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<tr>
<td>$p_k$</td>
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The volume of financing of SMEs in the regions, million.
Then the original task can look as follows. It is necessary to maximize the amount of state support (subsidies) to SMEs in the regions (9) in such a way that the indices of the values of the target program indicators (the number of SMEs in the region, the number of employees, etc.) increase within the scope of the possible subsidies for each program (10):

\[ F = v_1 x_1 + v_2 x_2 + \cdots + v_j x_j + \cdots + v_n x_n \rightarrow \text{max}, \]  

\[
\begin{align*}
  g_{11} x_1 + g_{12} x_2 + \cdots + g_{1j} x_j + \cdots + g_{1n} x_n & \leq p_1, \\
  g_{21} x_1 + g_{22} x_2 + \cdots + g_{2j} x_j + \cdots + g_{2n} x_n & \leq p_2, \\
  \vdots & \quad \vdots \\
  g_{m1} x_1 + g_{m2} x_2 + \cdots + g_{mj} x_j + \cdots + g_{mn} x_n & \leq p_m \\
\end{align*}
\]  

\[ y_j \geq 0, (j = 1, n) \]  

Figure 4. Calculation of the composition of the total volume of subsidies allocated from the federal budget for the next fiscal year on the state support of SMEs.

Then the dual task of LP will be to minimize the state budget funds to support SMEs in the regions (12) in the volumes sufficient to ensure the growth of indicators of the target federal SME support programs (13). In the target function of the dual task the indicators of the cost of financing \( y_j \) will be unknown, which, in this case, can be interpreted as transaction costs to provide financing for the region, or, more accurately, the growth rate of these costs. It should be noted that a conventional classification of transaction costs has not
evolved. Each of the researchers drew attention to the most interesting, from his point of view, elements.

In the economic literature there are two approaches to the possibility of a quantitative assessment of transaction costs: ordinal and cardinal. In the new institutional economic theory researchers use the ordinal approach explaining the change in the structure of transactions in the economy or in the industry, substitution of intra-firm transactions by market transaction and vice versa, the emergence of hybrid forms of institutional arrangements by changes in relative transaction costs.

At the same time, many attempts were made to quantify transaction costs in the cardinal version, that is, to obtain such quantitative data that would show the value of transaction costs or their share in the gross national or gross domestic product, the share in the transaction price or as the amount of money (including the monetary estimate of time) necessary to complete the transaction. D. North and J. Wallis (1986) made an attempt to systematically assess transaction costs in the economy as a whole.

In Russia it is a common practice that deals are made with the proven and well-known partners. This form of personalized exchange involves, in some cases, high transaction costs due to the reduction in the choice of participants and the opportunity to conclude a better deal, opportunities for further expansion of the field of activity, etc., and, in other cases, their savings on transaction costs due to opportunistic behavior.

Table 2 Interpretation of the dual task of linear programming

<table>
<thead>
<tr>
<th>The volume of financing of SBUs in region</th>
<th>The index of transaction costs of region 1 to achieve financing of the type (2 = 1)</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>The program of financing (housing construction, road construction, etc.), the volume of funds, millions.</td>
<td></td>
<td>P1</td>
<td>P2</td>
<td>P3</td>
<td>P4</td>
</tr>
</tbody>
</table>
Figure 5. The distribution of subsidies among the constituent entities of the Russian Federation for co-financing of the activities specified in subparagraphs “a” - “d” of paragraph 2 of the Rules.

\[ F = p_1 y_1 + p_2 y_2 + \cdots + p_i y_i + \cdots + p_m y_m \rightarrow \min, \quad (12) \]

\[
\begin{align*}
    g_{11} y_1 + g_{12} y_2 + \cdots + g_{1i} y_i + \cdots + g_{1m} y_m &\geq v_1, \\
    g_{21} y_1 + g_{22} y_2 + \cdots + g_{2i} y_i + \cdots + g_{2m} y_m &\geq v_2, \\
    \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
    g_{i1} y_1 + g_{i2} y_2 + \cdots + g_{ji} y_i + \cdots + g_{im} y_m &\geq v_i, \\
    \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
    g_{m1} y_1 + g_{m2} y_2 + \cdots + g_{mi} y_i + \cdots + g_{mn} y_m &\geq v_n
\end{align*}
\]

\[ y_i \geq 0, (i = 1, m) \quad (14) \]

Restriction (13) expresses the condition of sufficiency of financing. It should be sufficient to provide the desired amount of financial support to SMEs.

The abridged version can be represented as follows:

\[ F = \sum_{j=1}^{n} v_j x_j \rightarrow \max, \quad (15) \]
\[
\begin{align*}
\sum_{j=1}^{n} g_{1j} x_j & \leq p_1, \\
\sum_{j=1}^{n} g_{mj} x_j & \leq p_m, \\
x_j & \geq 0, \ (j = 1, n) \\
\end{align*}
\]

\[F^* = \sum_{i=1}^{m} p_i y_i \rightarrow \text{min},\]

\[
\begin{align*}
\sum_{j=1}^{n} g_{i1} y_i & \geq v_1, \\
\sum_{j=1}^{n} g_{in} y_i & \geq v_n, \\
y_j & \geq 0, \ (i = 1, m) \\
\end{align*}
\]

4 Conclusion

We have developed a toolkit for economic and mathematical modeling of interaction between the state and SMEs, including the economic and mathematical model of state support for SMEs with the use of the duality theory.

References


