International Journal of Pure and Applied Mathematics
Volume 118 No. 23 2018, 353-361
ISSN: 1314-3395 (on-line version)
url: http://acadpubl.eu/hub
Special Issue
ijpam.eu

SIGNED PRODUCT CORDIALITY OF CIRCULANT NETWORK

V.Jude Annie Cynthia[1] and E. Padmavathy[1]
[1] Department of Mathematics, Stella Maris College, Chennai, India
padmavathyelu@gmail.com

Abstract

A vertex labeling of a graph G, $f : V(G) \rightarrow \{-1, +1\}$ with induced edge labeling $f^* : E(G) \rightarrow \{+1, -1\}$ defined by $f^*(uv) = f(u) f(v)$ is signed product cordial labeling if $|v_f(1) - v_f(-1)| \leq 1$ and $|e_f{}^*(1) - e_f{}^*(-1)| \leq 1$, where $v_f(i)$ and $e_f{}^*(j)$ are respectively the number of vertices's labeled with i and the number of edges labeled with j; $i, j \in \{+1, -1\}$. A graph G is signed product cordial if it admits signed product cordial labeling. In this paper we prove that the circulant network and splitting graph of circulant network admit signed product cordial labeling.
Key Words : Signed product cordial, Circulant network, Splitting graph

1 Introduction

Most graph labeling methods trace their origin to one introduced by Rosa in 1967. Labeled graph are becoming an increasingly useful family of mathematical models for a broad range of application. According to Beineks and Hegde[9] graph labeling serves as a frontier between number theory and structure of graph. A detail study of variety of applications of graph labeling is given by Bloom and Golomb [3]. A dynamic survey of graph labeling is published and updated every year by Gallian [6]. Harary introduced S-Cordiality with the first letter of Signed Cordiality.
A *graph labeling* is an assignment of integers to the vertices or edges,

or both, subject to certain conditions.
A graph $G = (V, E)$ is called *signed cordial* if it is possible to label the edges with the number from the set $N = \{+1, -1\}$ in such a way that at each vertex v, the algebraic product of the labels of the edges incident with v is either $+1$ or -1 and the inequalities $|v_f(+1) - v_f(-1)| \leq 1$ and $|e_{f^*}(+1) - e_{f^*}(-1)| \leq 1$ are also satisfied, where $v_f(i)$, $i \in \{+1, -1\}$ and $e_f(j)$, $j \in \{+1, -1\}$ are respectively the number of vertices labeled with i and the number of edges labeled with j. A graph is called *signed-cordial* if it admits a signed-cordial labeling.
A vertex labeling of graph G , $f : V(G) \to \{-1, +1\}$ with induced edge labeling $f^* : E(G) \to \{+1, -1\}$ defined by $f^*(uv) = f(u) f(v)$ is *signed product cordial labeling* if $|v_f(1) - v_f(-1)| \leq 1$ and $|e_f{}^*(1) - e_f{}^*(-1)| \leq 1$, where $v_f(i)$ and $e_f{}^*(j)$ are respectively the number of vertices labeled with i and the number of edges labeled with j. A graph G is *signed product cordial* if it admits signed product cordial labeling.
For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph $S(G)$ thus obtained is called *splitting graph* of G.
An undirected *circulant graph* denoted by $G(n; \pm\{1, 2, \ldots, j\})$, $1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor$, $n \geq 3$ is defined as a graph consisting of the vertex set $V = \{0, 1, 2, \ldots n-1\}$ and the edge set $E = \{(i, j) : |i - j| \equiv s\,(mod\ n)\}$, $s \in \{1, 2, \ldots, j\}$. It is clear that $G(n, \pm 1)$ is the undirected cycle C_n and $G\left(n, \pm\left\{1, 2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor\right\}\right)$ is the complete graph K_n[7].

2 Literature Survey

The concept of cordial graph was introduced by Cahit[4] . Harary introduced S-Cordiality with the first letter of Signed Cordiality. Devaraj et al.[5] proved that the Petersen graph, complete graph, book graph, Jahangir graph and flower graph are signed cordial.
The concept of signed product cordial labeling was introduced by Baskar Babujee [8]. P.Lawrence et al.[13] proved that the arbitrary super subdivision of some graphs is signed product cordial. Santhi et al. [10],[11],[12] proved that flower graph, Binary tree, k-square graphs, cycle related graphs, some star and bistar related graphs are signed product cordial. They have also proved that the every

signed product cordial labeling is a total signed product cordial labeling.
The circulant network is a natural generalization of a double loop network, which was first considered by Wong and Coppersmith [2]. Circulant graphs have been used for decades in the design of computer and telecommunication networks due to their optimal fault-tolerance and routing capabilities [1].

3 Properties of Circulant Network

3.1 The number of edges of a circulant network $G\left(n, \pm j\right),$ is $n.$

3.2 The number of edges of a circulant network $G\left(n, \pm\{1, 2, \ldots j\right),$ $0 \leq j \leq \frac{n}{2} - 1,$ n even is $nj.$

3.3 The number of edges of a circulant network $G\left(n, \pm\{1, 2, \ldots j\right),$ $0 \leq j \leq \frac{n}{2},$ n odd is $nj.$

4 Signed Product Cordial labeling of Circulant Network

Theorem 1. *The Circulant network* $G\left(n, \pm\left\{1, 2, \ldots, j\right\}\right),$ $1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor - 1,$ $n \equiv 2\left(mod\ 4\right),$ $n\ \geq 6$ *admits signed product cordial labeling.*

Proof. Let G be a circulant graph on n vertices where $n\ \geq 6$ $, n \equiv 2\left(mod\ 4\right),$ n even. Let $v_0, v_1 \ldots v_{n-1}$ be the vertices of the graph G.
The labels are assigned for the vertices as follows:

$$f\left(v_l\right) = \begin{cases} +1 & l\ is\ even \\ -1 & l\ is\ odd \end{cases}$$

where $0 \leq l \leq n-1$. Hence $\frac{n}{2}$ vertices are labeled with $+1$ and the remaining $\frac{n}{2}$ vertices are labeled with -1. Thus $v_f\left(+1\right) = v_f\left(-1\right).$ Hence it satisfies the condition $\left|v_f\left(+1\right) - v_f\left(-1\right)\right| \leq 1.$

Thus we get the induced edge labeling to be

$$f^{*}(v_l, v_{l+k}) = \begin{cases} +1 & k \text{ is even} \\ -1 & k \text{ is odd} \end{cases}$$

where $0 \leq l \leq n-1$.
since $n \equiv 2\,(mod 4)$, $\frac{n}{2}$ is odd ; which implies that $\frac{n}{2} - 1$ is even, thus the atmost value of j is even.
Hence by property **3.2**, the graph G has $\frac{n(n-2)}{2}$ edges. Among which $\frac{n(n-2)}{4}$ edges have the induced label $+1$ and the remaining $\frac{n(n-2)}{4}$ edges are labeled with -1, as for each odd value of j the induced edge label will be -1 and for each even value of j the induced edge label will be $+1$. Thus $e_{f^*}(+1) = e_{f^*}(-1)$. Hence it satisfies the condition $|e_{f^*}(+1) - e_{f^*}(-1)| \leq 1$. Thus the graph G admits signed product cordial labeling. □

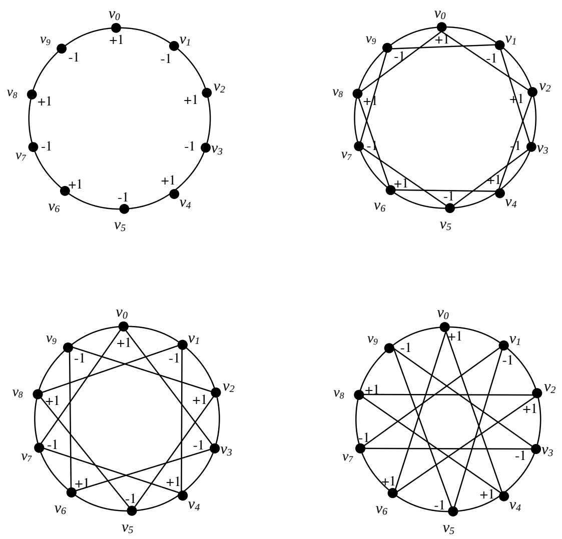

Figure 1: Signed Product cordial Labeling of $G(n, \pm\{1,2,3,4\})$

Theorem 2. *The Circulant network* $G(n; \pm\{1, 2, \ldots j\}), 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor - 2, n \equiv 0\,(mod\ 4), n \geq 8,$ *admits signed product cordial labeling.*

Proof. Let G be a circulant graph on n vertices where $n \geq 8$, $n \equiv 0\,(mod\ 4)$. Let $v_0, v_1 \ldots v_{n-1}$ be the vertices of the graph G. The labels are assigned for the vertices as follows:

$$f\,(v_l) = \begin{cases} +1 & l \text{ is even} \\ -1 & l \text{ is odd} \end{cases}$$

where $0 \leq l \leq n-1$.
Hence $\frac{n}{2}$ vertices are labeled with $+1$ and the remaining $\frac{n}{2}$ vertices are labeled with -1. Thus $v_f\,(+1) = v_f\,(-1)$. Hence it satisfies the condition $|v_f\,(+1) - v_f\,(-1)| \leq 1$.
Thus we get the induced edge labeling to be

$$f^*\,(v_l, v_{l+k}) = \begin{cases} +1 & k \text{ is even} \\ -1 & k \text{ is odd} \end{cases}$$

where $0 \leq l \leq n-1$.
Since $n \equiv 0\,(mod4)$, it implies that $\frac{n}{2} - 2$ is even, thus the atmost value of j is even.
Hence by the property **3.2**, the graph G has $\frac{n(n-4)}{2}$ edges. Since for each odd value of j the induced edge label will be -1 and for each even value of j the induced edge label will be $+1$, $\frac{n(n-4)}{4}$ edges have the induced label $+1$ and $\frac{n(n-4)}{4}$ edges are labeled with -1 Thus $e_{f^*}\,(+1) = e_{f^*}\,(-1)$.
Hence it satisfies the condition $|e_{f^*}\,(+1) - e_{f^*}\,(-1)| \leq 1$.
Thus the graph G admits signed product cordial labeling. □

5 Signed Product Cordiality of Splitting graph of Circulant Network

Theorem 3. *The splitting graph of circulant network* $S(G\,(n; \pm\{1,2\})$, *n even, admits signed product cordial labeling.*

Proof. Let $S\,(G)$ be the splitting graph of circulant network $G\,(n; \pm\{1,2\})$. Let $v_0, v_1 \ldots v_{n-1}$ be the vertices of the graph G and $v'_0, v'_1 \ldots v'_{n-1}$ are the newly added vertices. By the definition of splitting graph $S(G)$ contains $4n + 2n$ edges where $2n$ are the edges of $G\,(n; \pm\{1,2\})$ and $4n$ are the newly added edges. The vertex label are assigned as follows:

$$f(v_i) = f(v'_i) = \begin{cases} +1 & i \text{ is even} \\ -1 & i \text{ is odd} \end{cases}$$

where $0 \leq i \leq n-1$
Since n is even, $v_f(+1) = v_f(-1)$. Hence it satisfies the condition $|v_f(+1) - v_f(-1)| \leq 1$.
The edge set of $S(G)$ is given by

$$E(S(G)) = \left\{(v_i, v_{i+j}), (v'_i, v_{i+j}), \left(v'_{i+j}, v_i\right) / \ 0 \leq i \leq n-1\right\}$$

The indices are taken modulo n
If $j = 2$,
then the induced edge labeling is given by $f^*(e) = +1$.
If $j = 1$,
then the induced edge labeling is given by $f^*(e) = -1$.
Thus we have $3n$ edges with label $+1$ and $3n$ edges with label -1. Thus $e_{f^*}(+1) = e_{f^*}(-1)$. Hence it satisfies the condition $|e_{f^*}(+1) - e_{f^*}(-1)| \leq 1$.
Thus the graph $S(G)$ admits signed product cordial labeling. □

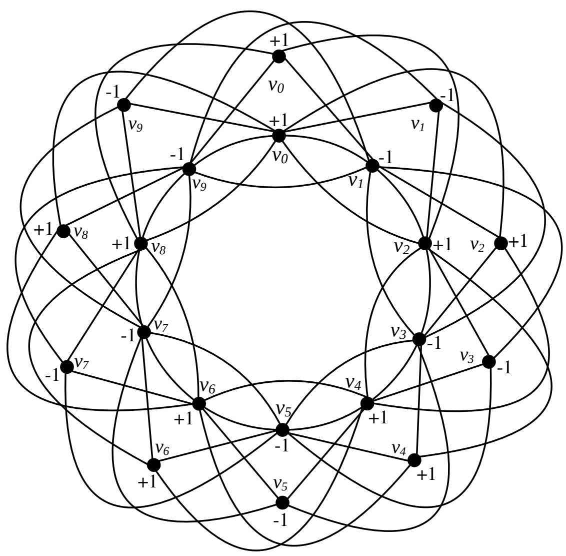

Figure 2: Signed Product Cordial Labeling of $S(G(1 \pm 1, 2))$

6 Remarks

Santhi et al. [11] proved that the every signed product cordial labeling is a Total signed product cordial labeling.

Corollary 4. *Thus Circulant network and splitting graph of circulant network are total signed product cordial.*

7 Conclusion

In this paper we have proved that circulant network and a related graph are signed product cordial. Further the signed and signed product cordiality of mesh derived architectures are under study.

References

[1] F.T. Boesch and J. Wang , *Reliable criculant networks with minimum transmission delay*, IEEE Transactions on Circuit and systems, **32** (1985), 1286-1291.

[2] G.K. Wong and D.A. Coppersmith, *A combinatorial problem related to multi module memory organization*, Journal of Association for Computing Machinery, **21** (1974), 392-401.

[3] G.S. Bloom and S.W Golomb, *Applications of numbered undirected graphs*, Proceedings of IEEE, (1977), 562-570.

[4] I. Cahit , *Cordial Graphs: A weaker version of graceful and Harmonic Graphs*, Ars Combinatoria, (1987), 201-207.

[5] J. Devaraj and P. Prem Delphy, *On Signed Cordial Graph*, International Journal of Mathematical sciences and application,**1** (2011).

[6] J.A. Gallian , *A dynamic survey of graph labeling,* The Electronics Journal of Combinatorics, **17** DS6, (2015).

[7] J.C. Bermond, F. Comellas and D.F. Hsu, *Distributed loop computers networks*, A survey journal of parallel distributed computing,**24** (1995), 2-10.

[8] Jayapal Baskar Babujee, Shobana Loganathan, *On Signed Product Cordial Labeling*, Scientific Research Journal On Applied Mathematics, **2** (2011), 1525-1530.

[9] L.W. Beineke and S.M. Hegde , *Strongly Multiplicative graphs* , Discuss. Math. Graph Theory, **21**, (2001), 63-75.

[10] M. Santhi and James Albert, *Signed Product cordial in Cycle Related graphs*, International Journal of Mathematics and Computer Application Research,**5**,(2016) 29-36.

[11] M. Santhi and James Albert, *Signed product cordial labeling and Signed total product cordial labeling for some new graphs*, International Journal of Research in Applied Natural and Social Science, **3** (2015), 133-138.

[12] M. Santhi and James Albert, *Some Star and Bistar Related Signed Product cordial graphs*, International Journal of Mathematical Archive-6 **10** (2015) , 232-236.

[13] P. Lawrence Rozario Raj and R.Lawrence Joseph Manoharan, *Signed Product Cordial graphs in the context of arbitrary supersubdivision*, Scientia Magna, **8**, (2012), 77-87.