

Strongly Multiplicative Labeling in Spider Web Network

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Abstract

A graph $G = (V(G), E(G))$ with p vertices is said to be strongly multiplicative if the vertices of G can be labeled with p distinct integers $1, 2, 3, \dots, p$ such that the labels induced on the edges by the product of labels of the end vertices are all distinct. In this paper we prove that the Spider web network $SW(m, n)$ is strongly multiplicative when m, n are even.

AMS Subject Classification: 05C

Key Words and Phrases: graph labeling, strongly multiplicative graph, spiderweb graph.

1 Introduction

Graph labeling is a mathematical discipline of graph theory which is closely related to the field of computer science. It concerns the assigning of values, usually represented by integers, to the edges and/or vertices of a graph. Labeling of graphs plays an important role in application of graph theory in Neural Networks, Coding Theory, Circuit Analysis etc .[4]. Labeled graph have variety of applications in coding theory, particularly for missile guidance codes, design of good radar type codes and convolution codes with optimal autocorrelation properties and also plays vital role in the study of X-ray crystallography, communication network and to determine

optimal circuit layouts. According to Beineke L., graph labeling serves as a frontier between number theory and structure of graphs [1]. A systematic study on applications of graph labeling is reported in [3]. Beineke and Hedge [2] defined a graph $G = (V(G), E(G))$ with p vertices as strongly multiplicative if the vertices of G can be labeled with p distinct integers $1, 2, 3, \dots, p$ such that the label induced on the edges by the product of labels of the end vertices are all distinct.

In this paper we prove that the Spider web network $SW(m, n)$ is strongly multiplicative when m, n are even.

2 Strongly Multiplicative Labeling of Spider Web Network $SW(m, n)$

Definition 2.1. [5] A graph $G = (V(G), E(G))$ with p vertices is said to be strongly multiplicative if the vertices of G can be labeled with p distinct integers $1, 2, 3, \dots, p$ such that the labels induced on the edges by the product of labels of the end vertices are all distinct.

Definition 2.2. [6]: **Generalized Spider web Network** $SW(m, n)$

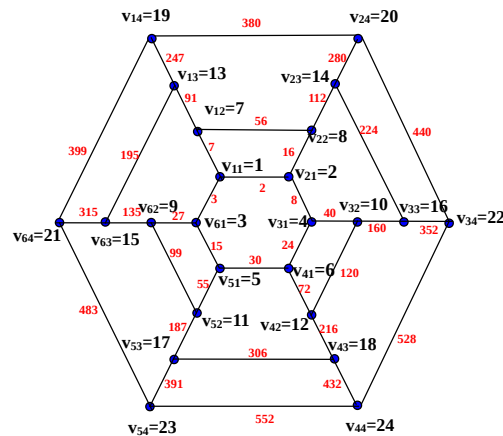
A Spider web network $SW(m, n)$ is the graph with the vertex set $\{(i, j) \text{ where } 0 \leq i \leq m, 0 \leq j \leq n\}$ such that (i, j) and (k, l) are adjacent if they satisfy one of the following conditions:

1. $j = k$ and $j = l \pm 1$
2. $j = l$ and $k = [i + 1]_m$ if $i + j$ is odd or $j = n - 1$
3. $j = l$ and $k = [i - 1]_m$ if $i + j$ is even or $j = 0$.

Remark 2.3. For convenience we denote the vertex set $\{(0, 0), (0, 1), \dots, (0, n), (1, 0), (1, 1), \dots, (1, n), \dots, (m, 0), (m, 1), \dots, (m, n)\}$ as $\{v_{11}, v_{21}, \dots, v_{m1}, v_{12}, \dots, v_{m2}, \dots, v_{1n}, v_{2n}, \dots, v_{mn}\}$ respectively.

Theorem 2.4. The Spider web network $SW(m, n)$ is strongly multiplicative when m, n are even.

Proof. Let $SW(m, n)$ be a Spider web network with $V = \{v_{ij} / 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edgeset $E = E_1 \cup E_2 \cup E_3$ where $E_1 = \{e_{ij} = (v_{ij}, v_{i+1j}) \cup (v_{mj}, v_{1j}) / 1 \leq i \leq m - 1, j = 1, n\}$, $E_2 = \{\delta_{ij} = (v_{ij}, v_{ij+1}) / 1 \leq i \leq m, 1 \leq j \leq n - 1\}$ and $E_3 = \{\lambda_{ij} = (v_{2i-1, 2j}, v_{2i, 2j}) \cup (v_{2i, 2j+1}, v_{2i+1, 2j+1}) \cup (v_{m, 2j+1}, v_{1, 2j+1}), 1 \leq i \leq \frac{m}{2}, 1 \leq j \leq \frac{n-2}{2}\}$.

**SW(6,4)**

To prove that Spider web network $SW(m, n)$ is strongly multiplicative when m, n is even, we define a bijective mapping

$f : V \rightarrow N$ as follows:

$$f(v_{ij}) = \begin{cases} 1 + m(j-1) & i = 1, 1 \leq j \leq n \\ 2(i-1) + m(j-1) & 2 \leq i \leq \frac{m+2}{2}, 1 \leq j \leq n \\ 2(m-i) + 3 + m(j-1) & \frac{m+4}{2} \leq i \leq m, 1 \leq j \leq n \end{cases}$$

We shall prove that all the edge labelings in E_1, E_2, E_3, E_1 and E_2, E_2 and E_3 are distinct. We shall show that the labeling of edges within the edge set and among the edge sets are distinct.

For the edges in E_1 :

Define an edge induced function $g : E_1 \rightarrow N$ such that for all $e_{ij} \in E_1$ and $v_{ij}, v_{i+1j} \in V$

$$g(e_{ij}) = f(v_{ij})f(v_{i+1j}), 1 \leq i \leq m-1, j = 1, n.$$

$$g(e_{mj}) = f(v_{mj})f(v_{1j}), i = m, j = 1, n.$$

We claim that if e_{ij} and e_{pq} are distinct edges in E_1 then $g(e_{ij}) \neq g(e_{pq})$.

Case1: For $j = q$ and $i \neq p, 1 \leq i \leq m$, we have the following subcases

Subcase (a): If $j = 1, i = 1$ and $p < \frac{m+4}{2}$.

Assume that $g(e_{11}) = g(e_{p1})$

$$g(v_{11}, v_{21}) = g(v_{p1}, v_{p+11})$$

$$f(v_{11})f(v_{21}) = f(v_{p1})f(v_{p+11})$$

$$1.2 = 4(p-1)p \text{ This implies a contradiction for } p.$$

Hence $g(e_{11}) \neq g(e_{p1}), \forall p < \frac{m+4}{2}$

Simiarly we can prove that if $j = 1, i = 1$ and $p \geq \frac{m+4}{2}$,

$$g(e_{11}) \neq g(e_{p1}), \forall p > \frac{m+4}{2}.$$

If $j = n, j = q, i = 1$ and $p < \frac{m+4}{2}$

Assume that $g(e_{1n}) = g(e_{pn})$

$$g(v_{1n}, v_{2n}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{1n})f(v_{2n}) = f(v_{pn})f(v_{p+1n})$$

$$(1+x)(2+x) = (2(p-1)+x)(2p+x) \text{ where } x = m(n-1).$$

This implies a contradiction for m .

Hence $g(e_{1n}) \neq g(e_{pn}), \forall p < \frac{m+4}{2}$.

Simiarly we can prove that if $j = n, j = q, i = 1$ and

$$p \geq \frac{m+4}{2}, g(e_{1n}) \neq g(e_{pn}), \forall p \geq \frac{m+4}{2}$$

Subcase (b): If $i \neq p, j = 1, j = q$ and $i, p < \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{i1}) = g(e_{p1})$

$$g(v_{i1}, v_{i+11}) = g(v_{p1}, v_{p+11})$$

$$f(v_{i1})f(v_{i+11}) = f(v_{p1})f(v_{p+11})$$

$$4i(i-1) = 4(p-1)p \text{ This implies a contradiction}$$

for p .

Hence $g(e_{i1}) \neq g(e_{p1}), \forall p, i < \frac{m+4}{2}$.

Similarly we can prove that if $i \neq p, j = n$ and $i, p < \frac{m+4}{2}$,

$$2 \leq i, p \leq m, g(e_{in}) \neq g(e_{pn}), \forall i, p < \frac{m+4}{2}.$$

We can prove similarly that if $i \neq p, j = 1, j = q, j = n$ and $i, p \geq \frac{m+4}{2}, 2 \leq i, p \leq m$, then $g(e_{in}) \neq g(e_{pn}), \forall i, p \geq \frac{m+4}{2}$

Subcase(c): If $i \neq p, j = 1, j = q$ and $i < \frac{m+4}{2}, p \geq \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{i1}) = g(e_{p1})$

$$g(v_{i1}, v_{i+11}) = g(v_{p1}, v_{p+11})$$

$$f(v_{i1})f(v_{i+11}) = f(v_{p1})f(v_{p+11})$$

$$4i(i-1) = y(y-2), \text{ where } y = 2(m-p) + 3 \text{ This}$$

implies a contradiction for i .

Hence $g(e_{i1}) \neq g(e_{p1}), \forall i < \frac{m+4}{2}, p \geq \frac{m+4}{2}$.

If $i \neq p, j = n$ and $i < \frac{m+4}{2}, p \geq \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{in}) = g(e_{pn})$

$$g(v_{in}, v_{i+1n}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{in})f(v_{i+1n}) = f(v_{pn})f(v_{p+1n})$$

$$x(x-2) = y(y-2) \text{ where } x = 2i + m(n-1),$$

$$y = 2(m-p) + 3 + m(n-1)$$

This implies a contradiction for i .

Hence $g(e_{in}) \neq g(e_{pn}), \forall i < \frac{m+4}{2}, p \geq \frac{m+4}{2}$.

Subcase(d): If $i \neq p, j = 1, j = q, i = m, p < \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{m1}) = g(e_{p1})$

$$g(v_{m1}, v_{11}) = g(v_{p1}, v_{p+11})$$

$$f(v_{m1})f(v_{11}) = f(v_{p1})f(v_{p+11})$$

$$3.1 = 4(p-1)p, \text{ where } y = 2(m-p) + 3$$

This implies a contradiction for p .

Hence $g(e_{m1}) \neq g(e_{p1}), \forall i = m, p < \frac{m+4}{2}$.

If $i \neq p, j = n$ and $i = m, p < \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{mn}) = g(e_{pn})$

$$g(v_{mn}, v_{1n}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{in})f(v_{i+1n}) = f(v_{pn})f(v_{p+1n})$$

$$(1+x)(3+x) = y(y-2) \quad \text{where } x = m(n-1),$$

$$y = 2(i-1) + m(n-1)$$

This implies a contradiction for p .

Hence $g(e_{mn}) \neq g(e_{pn}), \forall i = m, p < \frac{m+4}{2}$.

Similarly we can prove that if $i \neq p, j = 1, j = q, i = m$ and $p \geq \frac{m+4}{2}, 2 \leq i, p \leq m$ and if $j = n$, then

$$g(e_{m1}) \neq g(e_{p1}), \forall i = m, p \geq \frac{m+4}{2}$$

$$\text{and } g(e_{mn}) \neq g(e_{pn}), \forall i = m, p \geq \frac{m+4}{2}.$$

Case 2: For $j \neq q$ and $i \neq p, 1 \leq i \leq m$, we have the following subcases.

Subcase (a): If $j = 1, q = n$ and $i < \frac{m+4}{2}, p \geq \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{i1}) = g(e_{pn})$

$$g(v_{i1}, v_{i+11}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{i1})f(v_{i+11}) = f(v_{pn})f(v_{p+1n})$$

$$4i(i-1) = y(y-2) \quad \text{where } y = 2(m-p) + 3 + m(n-1).$$

This implies a contradiction for i .

Hence $g(e_{i1}) \neq g(e_{pn}), \forall i < \frac{m+4}{2}, p \geq \frac{m+4}{2}$.

If $j = 1, q = n$ and $i = 1, p \geq \frac{m+4}{2}, 2 \leq p \leq m$

Assume that $g(e_{11}) = g(e_{pn})$

$$g(v_{11}, v_{i+11}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{11})f(v_{i+11}) = f(v_{pn})f(v_{p+1n})$$

$$2.1 = y(y-2) \quad \text{where } y = 2(m-p) + 3 + m(n-1).$$

This implies a contradiction for p .

Hence $g(e_{11}) \neq g(e_{pn}), \forall i < \frac{m+4}{2}, p \geq \frac{m+4}{2}$.

Subcase(b): If $i \neq p, j = 1, q = n, i, p < \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{i1}) = g(e_{pn})$

$$g(v_{i1}, v_{i+11}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{i1})f(v_{i+11}) = f(v_{pn})f(v_{p+1n})$$

$$4i(i-1) = y(y-2), \text{ where } y = 2p + m(n-1)$$

This implies a contradiction for i .

Hence $g(e_{i1}) \neq g(e_{pn}), \forall i, p < \frac{m+4}{2}$.

We can prove similarly that if $i \neq p, j = 1, q = n, i, p \geq \frac{m+4}{2}, 2 \leq i, p \leq m$ then $g(e_{i1}) \neq g(e_{pn}), \forall i, p \geq \frac{m+4}{2}$.

Subcase(c): If $i \neq p, j = 1, q = n, i = m, p < \frac{m+4}{2}, 2 \leq i, p \leq m$

Assume that $g(e_{m1}) = g(e_{pn})$

$$g(v_{m1}, v_{11}) = g(v_{pn}, v_{p+1n})$$

$$f(v_{m1})f(v_{11}) = f(v_{pn})f(v_{p+1n})$$

$$3.1 = y(y-2), \text{ where } y = 2p + m(n-1)$$

This implies a contradiction for p .

Hence $g(e_{m1}) \neq g(e_{pn}), \forall i = m, p \geq \frac{m+4}{2}$.

Similarly we can prove the subcase when $j = n$ and $p = 1$.

Hence the edge labelings in E_1 are distinct.

For the edges in E_2 : Define an edge induced function $g : E_2 \rightarrow N$ such that for all $\delta_{ij} \in E_2$ and $v_{ij}, v_{i+1j} \in V$,

$$g(\delta_{ij}) = f(v_{ij})f(v_{i+1j}), 1 \leq i \leq m, 1 \leq j \leq n-1.$$

If δ_{ij}, δ_{pq} are two distinct edges in E_2 , we can prove that edge labelings are distinct for all edges in E_2 as in the case of E_1 .

For the edges in E_3 :

Define an edge induced function $g : E_3 \rightarrow N$ such that for all $\lambda_{ij} \in E_3$ and $v_{ij}, v_{i+1j} \in V$,

$$g(\lambda_{ij}) = f(v_{ij})f(v_{i+1j}), 1 \leq i \leq m, 1 \leq j \leq n-1.$$

If $\lambda_{ij}, \lambda_{pq}$ are two distinct edges in E_3 , we claim that $g(\lambda_{ij}) \neq g(\lambda_{pq})$.

Case1: If $j = q, i$ is odd and j is even then $\lambda_{ij} = (v_{2i-1}, v_{2i})$ where $1 \leq i \leq \frac{m}{2}, 1 \leq j \leq \frac{n-2}{2}$, we have the following subcases.

Subcase(a): If $i = 1, p < \frac{m+4}{2}$ and p is odd

Assume that $g(\lambda_{1j}) = g(\lambda_{pj})$

$$g(v_{12j}, v_{22j}) = g(v_{2p-12j}, v_{2p2j})$$

$$f(v_{12j})f(v_{22j}) = f(v_{2p-12j})f(v_{2p2j})$$

$$(1+x)(2+x) = (4p-4+x)(4p-2+x) \text{ where } x = m(2j-1).$$

This implies a contradiction for j .

Hence $g(\lambda_{1j}) \neq g(\lambda_{pj}), \forall i = 1, p < \frac{m+4}{2}$.

If $i, p < \frac{m+4}{2}$ and p is odd

Assume that $g(\lambda_{ij}) = g(\lambda_{pj})$

$$g(v_{2i-12j}, v_{2i2j}) = g(v_{2p-12j}, v_{2p2j})$$

$f(v_{2i-1} 2j) f(v_{2i} 2j) = f(v_{2p-1} 2j) f(v_{2p} 2j)$
 $(4i - 4 + x)(4i - 2 + x) = (4p - 4 + x)(4p - 2 + x)$ where
 $x = m(2j - 1)$. This implies a contradiction for i .

Hence $g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i, p < \frac{m+4}{2}$ and p is odd.

We can prove similarly that if $i, p \geq \frac{m+4}{2}$ and p is odd, then
 $g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i, p \geq \frac{m+4}{2}$.

Subcase(b): If $i < \frac{m+4}{2}, p \geq \frac{m+4}{2}$ and p is odd

Assume that $g(\lambda_{ij}) = g(\lambda_{pj})$

$g(v_{2i-1} 2j v_{2i} 2j) = g(v_{2p-1} 2j, v_{2p} 2j)$
 $f(v_{2i-1} 2j) f(v_{2i} 2j) = f(v_{2p-1} 2j) f(v_{2p} 2j)$
 $(4i - 4 + x)(4i - 2 + x) = (2m - 4p + 5 + x)(2m - 4p + 3 + x)$
 where $x = m(2j - 1)$. This implies a contradiction for j .

Hence $g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i < \frac{m+4}{2}, p \geq \frac{m+4}{2}$.

Case 2: If $j = q, i$ is even and j is odd then $\lambda_{ij} = (v_{2i} 2j+1, v_{2i+1} 2j+1)$ where $1 \leq i \leq \frac{m}{2}, 1 \leq j \leq \frac{n-2}{2}$, we have the following subcases.

Subcase(a): If $i, p < \frac{m+4}{2}$ and p is even

Assume that $g(\lambda_{ij}) = g(\lambda_{pj})$

$g(v_{2i} 2j+1 v_{2i+1} 2j+1) = g(v_{2p} 2j+1, v_{2p+1} 2j+1)$
 $f(v_{2i} 2j+1) f(v_{2i+1} 2j+1) = f(v_{2p} 2j+1) f(v_{2p+1} 2j+1)$
 $x + y = 2$ where $x = 4i + m2j, y = 4p + m2j$. This implies a
 contradiction for i .

Hence $g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i, p < \frac{m+4}{2}$.

We can prove similarly that if $i, p \geq \frac{m+4}{2}$ and p is even, then
 $g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i, p \geq \frac{m+4}{2}$.

Similarly we can prove for $i < \frac{m+4}{2}, p \geq \frac{m+4}{2}, g(\lambda_{ij}) \neq g(\lambda_{pj})$.

Subcase(b): If $i = m, p < \frac{m+4}{2}$

Assume that $g(\lambda_{ij}) = g(\lambda_{pj})$

$g(v_{m} 2j+1 v_1 2j+1) = g(v_{2p} 2j+1, v_{2p+1} 2j+1)$
 $f(v_{2i} 2j) f(v_{2i+1} 2j) = f(v_{2p} 2j) f(v_{2p+1} 2j)$
 $(3 + x)(1 + x) = y(y - 2)$ where $x = 1 + 2mj, y = (4p + 2mj)$.
 This implies a contradiction for j .

Hence $g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i = m, p < \frac{m+4}{2}$.

Case 3: If $j \neq q, i$ is odd and j is even then $\lambda_{ij} = (v_{2i-1} 2j, v_{2i} 2j)$
 where $1 \leq i \leq \frac{m}{2}, 1 \leq j \leq \frac{n-2}{2}$, we have the following subcases.

Subcases(a): If $i, p < \frac{m+4}{2}$ and p is odd

Assume that $g(\lambda_{ij}) = g(\lambda_{pj})$

$g(v_{2i-1} 2j v_{2i} 2j) = g(v_{2p-1} 2q, v_{2p} 2q)$
 $f(v_{2i-1} 2j) f(v_{2i} 2j) = f(v_{2p-1} 2q) f(v_{2p} 2q)$

$$(x + y) = 2 \text{ where } x = 4i - 2 + m(2j - 1), y = 4p - 2 + m(2q - 1).$$

This implies a contradiction for i .

$$\text{Hence } g(\lambda_{ij}) \neq g(\lambda_{pj}), \forall i, p < \frac{m+4}{2}.$$

$$\text{Similarly we can prove for } i < \frac{m+4}{2}, p \geq \frac{m+4}{2}, g(\lambda_{ij}) \neq g(\lambda_{pj}).$$

Case 4: If $j \neq q$, i is even and j is odd then $\lambda_{ij} = (v_{2i-2j+1}, v_{2i+1-2j+1})$ where $1 \leq i \leq \frac{m}{2}, 1 \leq j \leq \frac{n-2}{2}$, we have the following subcases.

Subcase(a): If $i, p < \frac{m+4}{2}$ and p is even

$$\text{Assume that } g(\lambda_{ij}) = g(\lambda_{pq})$$

$$g(v_{2i-2j+1}v_{2i+1-2j+1}) = g(v_{2p-2q+1}, v_{2p+1-2q+1})$$

$$f(v_{2i-2j+1})f(v_{2i+1-2j+1}) = f(v_{2p-2q+1})f(v_{2p+1-2q+1})$$

$x + y = 2$ where $x = 4i + m2j, y = 4p + m2q$. This implies a contradiction for i .

$$\text{Hence } g(\lambda_{ij}) \neq g(\lambda_{pq}), \forall i, p < \frac{m+4}{2}.$$

Subcase(b): If $i, p \geq \frac{m+4}{2}$ and p is even

$$\text{Assume that } g(\lambda_{ij}) = g(\lambda_{pq})$$

$$g(v_{2i-2j+1}v_{2i+1-2j+1}) = g(v_{2p-2q+1}, v_{2p+1-2q+1})$$

$$f(v_{2i-2j+1})f(v_{2i+1-2j+1}) = f(v_{2p-2q+1})f(v_{2p+1-2q+1})$$

$$x + y = 2 \text{ where } x = (2m - 4i + 3 + 2mj), y = (2m - 4p + 3 + 2mq).$$

This implies a contradiction for i .

$$\text{Hence } g(\lambda_{ij}) \neq g(\lambda_{pq}), \forall i, p \geq \frac{m+4}{2}.$$

$$\text{Similarly we can for } i < \frac{m+4}{2}, p \geq \frac{m+4}{2}, g(\lambda_{ij}) \neq g(\lambda_{pq}).$$

Subcase(c): If $i = m, p < \frac{m+4}{2}$

$$\text{Assume that } g(\lambda_{mj}) = g(\lambda_{pq})$$

$$g(v_{m-2j+1}v_{1-2j+1}) = g(v_{2p-2q+1}, v_{2p+1-2q+1})$$

$$f(v_{m-2j+1})f(v_{1-2j+1}) = f(v_{2p-2q+1})f(v_{2p+1-2q+1})$$

$$(3 + x)(1 + x) = y(y - 2) \text{ where } x = 1 + 2mj, y = (4p + 2mq).$$

This implies a contradiction for j .

$$\text{Hence } g(\lambda_{ij}) \neq g(\lambda_{pq}), \forall i = m, p < \frac{m+4}{2}.$$

Hence the edge labelings in E_3 are distinct.

Similarly we can prove that edge labelings in E_1 and E_2 are distinct and edge labeling in E_1 and E_3 are distinct.

Hence the spider web graph $SW(m, n)$, for all m, n even is strongly multiplicative. $\square$

3 Conclusion

In this paper we have proved that the spider web graph $SW(m, n)$, for all m, n even is strongly multiplicative.. Finding strongly multi-

plicative labeling for other interconnection networks is quite challenging.

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