

# Local Metric Dimension of Cube Connected Cycle Derived Networks

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## Abstract

Let  $G(V, E)$  be a graph and let  $W \subseteq V$ . Then  $W$  is said to be a local metric basis of  $G$ , if for any two adjacent vertices  $u, v \in V \setminus W$ , there exists a vertex  $w \in W$  such that  $d(u, w) \neq d(v, w)$ . The minimum cardinality of local metric basis is called the local metric dimension ( $lmd$ ) of graph  $G$ . In this paper we investigate the local metric dimension of  $n$ -dimensional Cube Connected Complete Graph  $CCCP(n)$ .

**Key Words:** Local metric basis, Local metric dimension and  $n$ -dimensional Cube Connected Complete Graph.

## 1 Introduction

The metric dimension problem is an application to network discovery and verification in the area of the telecommunication. This problem was introduced by Slater [7] where the metric generators were called locating set and by Harary & Metler [2], where metric generators received the name of resolving sets. Applications of this invariant to the navigation of robots in networks are discussed in [8]. A more common problem in graph theory concerns distinguishing every two neighbors in a graph  $G$  by means of some coloring rather than distinguishing all the vertices of a connected graph  $G$  has been studied with the aid of distances in  $G$ . This suggests the topic of using distances to distinguish the two vertices in each pair of neighbors only and thus Okamoto *et.al* [3] introduced the local metric dimension problem.

Let  $G(V, E)$  be a graph with vertex set  $V$  and edge set  $E$ . Let  $W \subseteq V$  then  $W$  is said to be a *metric basis* of  $G$ , if for every pair of vertices  $u, v \in V \setminus W$ , there exists a vertex  $w \in W$  such that  $d(u, w) \neq d(v, w)$ , whereas  $W$  is said to be a *local metric basis* of  $G$ , if for any two adjacent vertices  $u, v \in V \setminus W$ , there exists a vertex  $w \in W$  such that  $d(u, w) \neq d(v, w)$ . The minimum cardinality of metric basis is called the *metric dimension* ( $\beta(G)$ ) of graph  $G$  and that of local metric basis is called the *local metric dimension* ( $lmd$ ) of graph  $G$ .

In view of the above definition, the problem of finding the local metric dimension of a graph is equivalent of finding the cardinality of a minimum set  $W \subseteq V$  say  $W = \{w_1, \dots, w_k\}$  so that  $(d(u, w_1), \dots, d(u, w_k)) \neq (d(v, w_1), \dots, d(v, w_k))$  for each pair of adjacent vertices  $u, v \in V$ .

Interconnection networks have been extensively applied in several areas, such as workstation networks, parallel computing and so on. According to D.F. Hsu [1], "An interconnection network consists of hardware and software entities that are interconnected to facilitate efficient computation and communication; these entities can be in the form of processor, processes, memory modules or computer systems".

Interconnection networks provides communication among the processors and the memories. It also play a key role in the design and implementation of communication networks and recent advent of optic technology adds more design problems. They are often modeled by finite graphs. The vertices of the graph represent the nodes of the network, (ie) processing elements, memory modules or switches and the edges corresponds to communication lines [6].

The hypercube, suggested by Sullivan and Bashkow [4], is one of the most popular, versatile and efficient topological structures of interconnection networks. The hypercube  $Q_n$  has many excellent features, and thus becomes the first choice for the topological structure of parallel processing and computing systems. In [5] Juan Liu and Xindong Zhang introduced a new interconnection network called  $n$ -dimensional cube-connected complete graph. They have also proved that  $CCCP(n)$  is Cayley graph, and studied the basic properties of  $CCCP(n)$ , including spectra, connectivity, Hamiltonian, diameter etc.

The  $n$ -dimensional cube-connected complete graph, denoted by

$CCCP(n)$ , is constructed from the  $n$ -dimensional hypercube  $Q_n$  by replacing each vertex of  $Q_n$  with a complete graph of order  $n$ . By modifying the labeling scheme of  $Q_n$ , we can represent each vertex of  $CCCP(n)$  by a pair  $(\bar{x}; i)$  where  $i (1 \leq i \leq n)$  is a position of the vertex within its complete graph and  $\bar{x}$  (any  $n$ -bit binary string ie.  $\bar{x} = x_1x_2 \dots x_n$ ) is the label of the vertex in  $Q_n$  that corresponds to the complete graph. Precisely, the vertex set of  $CCCP(n)$  is  $V = \{(\bar{x}; i) : \bar{x} \in V(Q_n); 1 \leq i \leq n\}$ .

Two vertices  $(\bar{x}; i)$  and  $(\bar{y}; j)$  are linked by an edge in the  $CCCP(n)$  if and only if either

- i)  $\bar{x} = \bar{y}$  and  $|i - j| \equiv s \pmod{n}, s \in \{1, 2, \dots, \lfloor \frac{n}{2} \rfloor\}$  or
- ii)  $i = j$  and  $\bar{x}$  differs from  $\bar{y}$  in precisely the  $i$ th bit.

Edges of first type are called complete edges, while edges of the second type are referred to as hypercube edges. A vertex  $(\bar{x}; i) = (x_1x_2 \dots x_n; i)$  where  $i = 1, 2, \dots, n$  is defined as level- $n$  vertex if  $\sum_{i=1}^n x_i = n$ .

## 2 Literature survey

Okamoto *et al.* [3] characterize all nontrivial connected graphs of order  $n$  having local metric dimension 1,  $n - 2$  or  $n - 1$  and establish sharp bounds for the local metric dimension of a graph. Each resolving set of  $G$  is vertex distinguishing and each local resolving set is neighbour-distinguishing. Thus every resolving set is also a local resolving set of  $G$ , so if  $G$  is a non trivial connected graph of order  $n$ , then  $1 \leq lmd(G) \leq \beta(G) \leq n - 1$ .

J.A. Cynthia *et al.* investigated the lmd of Cyclic Split Graph  $C_nK_r^k$  [9], mesh related networks [10, 11] and Bloom Graph [12]. In [14, 15, 16, 13], they have also investigated local metric dimension of families of generalized Petersen graphs, de Bruijn digraphs, Circulant graphs, Benes and Butterfly networks.

## 3 Local metric dimension of CCCP(n)

In this section we provide the bounds for lmd of  $CCCP(n)$ .

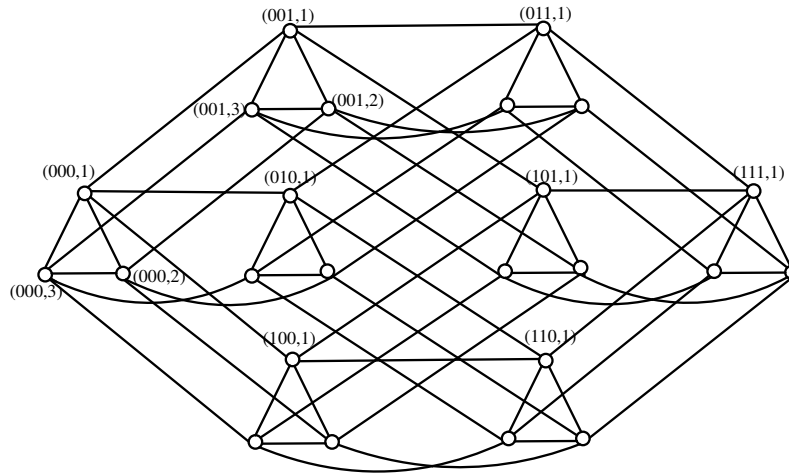


Figure 1: CCCP(3)

**Lemma 3.1.**  $lmd(CCCP(n)) \geq n - 1$

*Proof.* Consider the vertices of the complete graph in level-0 of  $CCCP(n)$ . Since all the vertices are adjacent to each other, any  $n - 1$  vertices among  $n$  vertices should be member of any local metric basis. Thus  $lmd(CCCP(n)) \geq n - 1$ .  $\square$

**Lemma 3.2.**  $lmd(CCCP(n)) \leq n - 1$

*Proof.* Let  $W = \{(\bar{0}; 1), (\bar{0}; 2), \dots, (\bar{0}; n - 1)\}$  be the set of  $n - 1$  vertices of a local metric basis.

**Case 1:** Consider the pairs of adjacent vertices  $(\bar{x}; j)$  and  $(\bar{x}; j')$ ,  $1 \leq j, j' \leq n$  of any level- $k$ ,  $1 \leq k \leq n$ . Then for  $i \in \{1, \dots, n - 1\}$

$$d((\bar{0}; i), (\bar{x}; j)) = \begin{cases} k & i = j \\ k + 1 & i \neq j \end{cases}$$

Hence for any two adjacent vertices  $(\bar{x}; j)$  and  $(\bar{x}; j')$ ,  $1 \leq j, j' \leq n$  belonging to the vertices of level- $k$  we have

$$(d((\bar{0}; 1), (\bar{x}; j)), \dots, d((\bar{0}; n - 1), (\bar{x}; j))) \neq (d((\bar{0}; 1), (\bar{x}; j')), \dots, d((\bar{0}; n - 1), (\bar{x}; j'))).$$

Thus the set  $W$  resolves the pairs of adjacent vertices in the complete graph at an level  $k \geq 1$ .

**Case 2:** Consider the pairs of adjacent vertices  $(\bar{x}; 1)$  of level- $k$  and  $(\bar{y}; 1)$  of level- $(k + 1)$ ,  $1 \leq k \leq n - 1$ .

Then

$$\begin{aligned} d((\bar{0}; 1), (\bar{x}; 1)) &= k \\ d((\bar{0}; 1), (\bar{y}; 1)) &= k + 1. \end{aligned}$$

**Case 3:** Consider the pairs of adjacent vertices  $(\bar{x}; j)$  of level- $k$  and  $(\bar{y}; j)$  of level- $(k + 1)$ ,  $1 \leq j \leq n$  and  $1 \leq k \leq n - 1$  we have

$$\begin{aligned} d((\bar{0}; 1), (\bar{x}; j)) &= k + 1 \\ d((\bar{0}; 1), (\bar{y}; j)) &= k + 2. \end{aligned}$$

Hence the vertex  $(\bar{0}; 1)$  resolves the pairs of adjacent vertices  $\{u, v\}$  where  $u$  belonging to level- $k$  and  $v$  belonging to level- $(k + 1)$ .

Thus  $\text{lmd}(CCCP(n)) \leq n - 1$ .  $\square$

**Theorem 3.3.** *The local metric dimension of  $CCCP(n)$  is  $n - 1$ .*

Proof follows from Lemma 3.1 and 3.2.

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