

EDGE-DISJOINT CYCLE COVER OF BENES NETWORKS

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Abstract

Cycles in interconnection networks are useful in many applications such as indexing, embedding line arrays and rings, computing FFT. Benes networks consists of back-to-back butterflies which are most popular bounded-degree derivative of the hypercube network. Benes networks are important in multistage interconnection networks, which possess attractive topologies for communication networks. In this paper, we obtain the minimum edge-disjoint cycle cover number of Benes networks.

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Key Words and Phrases: Benes networks, Binding diamonds, Binding vertices, Edge-disjoint cycle cover.

1 Introduction

Graph theory is one of the most powerful mathematical tools for designing and analyzing topological structures of interconnection networks. It is applied in diverse areas such as social sciences, linguistics, physical sciences, communication engineering and so on. It plays an important role in several areas of computer science, such as switching theory and logic design, parallel computing, artificial intelligence, formal languages, computer

graphics, operating systems, compiler writing and information organization. Graphs are extremely important to represent many problems in computer science that are otherwise abstract.

Finding a way to represent the solution to a problem as a graph can present new approaches to solve the problem or even lead directly to a solution derived from graph theory. For many problems, solutions obtained using computers involve representing relationships between objects, places, or concepts. Because graphs can be either directed or undirected, they are a flexible method of showing connections. We represent networks as undirected graphs whose nodes represent processors and whose edges represent inter-processor communication links.

A multistage network consists of more than one stage of switching elements and is usually capable of connecting an arbitrary input terminal to an arbitrary output terminal. Multistage networks are classified into blocking, rearrangeable or nonblocking network. A network which can handle all possible permutations without rearranging connections is a nonblocking networks. A network is a rearrangeable nonblocking network if it can realize all possible permutations between inputs and outputs. The Benes network[1], which is a special insistence of CLOS networks[2], is an excellent example of a rearrangeable network.

Benes network is the most popular bounded-degree derivative of the hypercube network which consists of back-to-back butterfly networks known for permutation routing which is used in parallel computing systems and in the internal structures of optical couplers. Benes network is very similar to the butterfly in terms of both its computational power and its network structure. An r -dimensional Benes network denoted by $BB(r)$ has $(2r + 1)^{2r}$ vertices and $r2^{r+2}$ edges and $(2r + 1)$ levels, each level with $2r$ nodes. Overall, the r dimensional Benes network has $2r - 1$ level of switches, with 2^{r-1} switches in each level.

2 Edge-disjoint cycle cover

An r -dimensional Benes network is denoted by $BB(r)$. The r -dimensional Benes has $(2r + 1)2^r$ nodes and $r2^{r+2}$ edges. An r -dimensional Benes network $2r + 1$ levels, each level with 2^r nodes.

The level 0 to level r nodes in the network form an r -dimensional butterfly. The Benes network consists of back-to-back butterflies. The middle level of Benes network is shared by these butterflies[7]. An r -dimensional Benes network is denoted by $BB(r)$ [7].

An edge-disjoint cycle cover of a graph $G(V, E)$ where $V(G)$ and $E(G)$ denote the vertex set and edge set of the graph G , is a partition $\{E_1, E_2, \dots, E_k\}$ of the edge set $E(G)$ into subsets of $E(G)$ such that each E_i , $1 \leq i \leq k$ induces a cycle. The edge-disjoint cycle cover problem is to minimize k [3]. We denote this by $\eta'(G)$. The edge-disjoint cycle problem is NP-complete[3].

Maneul et al [4] have identified new topological representations for Benes network as diamond representation and proved that normal and diamond representation of Benes network are isomorphic. Normal and diamond representation of $BB(r)$ is given below

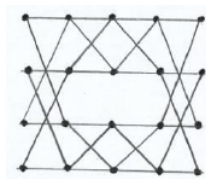


Figure 1: Normal $BB(2)$

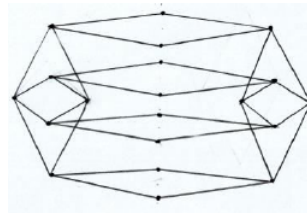


Figure 2: Diamond $BB(2)$

Lemma 1. Let G be a graph with an edge-disjoint cycle cover. Let Δ be the maximum degree in G . Then $\eta'(G) \geq (\Delta/2)$.

Theorem 2. Let $BB(r)$, $r \geq 1$ be a Benes network. Then $\eta'(BB(r)) = 2$.

Proof. $BB(r)$ contains 2^{r-1} horizontal binding diamonds $BB_i(1)$, $1 \leq i \leq 2^{r-1}$. Let v_i and v'_i , $1 \leq i \leq 2^{r-1}$ be 4 degree vertices in $BB_i(1)$, $1 \leq i \leq 2^{r-1}$, then the neighbourhood of v_i has at least one of the two vertices v_{j1} and v_{j2} of degree two.

Let $BB(r)_1^T$ and $BB(r)_1^B$, $1 \leq r \leq 2^{r-1}$ denote the topmost vertex and bottom most vertex of r -dimensional Benes network taken from the right.

Let u_i , $1 \leq i \leq 2^r$ represent the vertical binding diamonds of $BB(r)$. Let BF_i^{MT} , $1 \leq i \leq 2^r$ represent the vertices of the

topmost middle Butterfly network u_i . $BB(r)$ contains $2^{r-1}BB(1)$ and $4BF(r-1)$. Therefore, there are 2^r vertical diamonds in the middle, 2^{r-1} vertical diamonds on the left and right side. The total number of vertical diamonds are $2^r + 2^{r-1} + 2^{r-1} = 2^{r+1}$, $r \geq 3$.

Hence the neighbourhood of vertices in the vertical and horizontal binding diamonds of $BB(r)$ is $N(u_i) = \{u_i^{(1)}, u_i^{(2)}\}$ and $N(v_i) = \{v_i^{(1)}, v_i^{(2)}\}$, $1 \leq i \leq 2^{r-1}$.

In the following procedure, $u \rightarrow v$ denotes a traversal from vertex u to vertex v .

Procedure CYCLECOVER $BB(r)$

Input: An r -dimensional Benes network $BB(r)$.

Algorithm:

Step 1: Begin C_1 .

Let $1 \leq i \leq 2^r$, $1 \leq j \leq 2^r$ Initialize $i = 1$ and $j = 1$

Step 2: $u_i^{(2)} \rightarrow u_i^{(1)}$ traversing 2 edges through v_i .

$u_i^{(1)} \rightarrow BB_{i(1)}^{(1)}$ traversing $r-2$ edges.

$BB_{i(1)}^{(1)} \rightarrow v_i$ traversing one edge.

$v_i \rightarrow v_{j1}$ traversing one edge.

Step 3: Increment i in u_i and pass through u_i reaching $u_i^{(2)}$ traversing $2^{r-2} + 1$ edges.

Step 4: $u_j^{(2)} \rightarrow v_{j1}$ traversing one edge.

$v_{j1} \rightarrow v_j$ traversing one edge.

$v_j \rightarrow BB_{i(1)}^{(1)}$ traversing one edge.

Step 5: Increment or decrement i and j such that all edges traversing u_i and v_i and v_j are covered respectively.

Step 6: Repeat.

Output: A cycle C_1 .

□

Proof of correctness: $BB(r)$ contains 2^{r-1} horizontal and 2^{r-1} vertical binding diamonds. C_1 traverses two edges in each of the 2^{r-1} binding diamonds. The number of edges used in 2^r binding diamonds is 2^{r+1} . C_1 traverses $(r-2)$ edges in each of the butterflies of smaller dimension, namely $BB(r)_1^T$ and $BB(r)_1^B$, $1 \leq r \leq 2^{r-1}$ and BF_i^{MT} , $1 \leq i \leq 2^r$ while moving from u_i , $1 \leq i \leq 2^r$ to v_j , $1 \leq j \leq 2^{r-1}$ and vice-versa.

To proceed we prove that the complement of C_1 in $BB(r)$ is another cycle C_2 isomorphic to C_1 . By procedure CYCLECOVER $BB(r)$, which generates a cycle C_1 , the subgraph of C_1 in any 4-cycle C^* in $BB(r)$ is isomorphic to its complement in C^* . Thus the sub graph induced by the complementary sub graphs of all the four cycles in $BB(r)$ induce a cycle C_2 isomorphic to C_1 . Hence $|E(C_1)| = |E(C_2)| = r2^{r+1}$.

Therefore, $|E(C_1)| + |E(C_2)| = r2^{r+1} = |E(BB(r))|$. The procedure is illustrated in Figure 3.

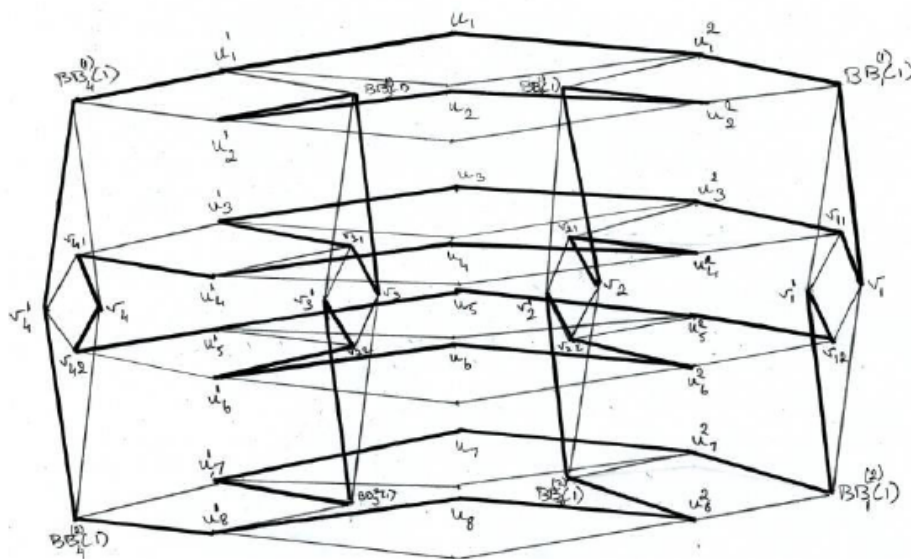


Figure 3: Two edge-disjoint cycles of $BB(3)$

3 Conclusion

In this paper, minimum edge-disjoint cycle cover number for r dimensional Benes network $BB(r)$, $r \geq 1$ is obtained.

References

- [1] V. Benes, Permutation groups, complexes, and rearrangeable multistage connecting networks, *Bell System Technical Journal*, **43** (1964), 1619-1640.
- [2] C. Clos, A study of non blocking switching networks, *IEEE Computer*, **14**(12) (1981), 12-27.
- [3] M.R. Garey, D.S. Johnson, *Computers and Intractability: A guide to the theory of NP Completeness*, Freeman and Company (1979).
- [4] P. Manuel, M.I. Abd-El-Barr, I. Rajasingh, B. Rajan, An Efficient representation of Benes Network and its Applications, *Journal of Discrete Algorithms*, **6** (2008), 11-19.
- [5] A. William, A. Shanthakumari, Bound on the minimum cycle cover number of a graph, *J. Comp. & Math. Sci.*, **2** (2011), 842-845.
- [6] J. Xu, *Topological Structure and Analysis of Interconnection Networks*, Kluwer Academic Publishers (2001).
- [7] T.F. Leighton, *Introduction to parallel algorithms and architecture: Arrays, Trees, Hypercubes, Morgan*, Kaufmann Publishers (1992).

