

Cost Analysis On A Vector Objective Multi -Showroom And Multi-Item Displayed Inventory Model With Pentagonal Fuzzy Number Using Two Defuzzification Techniques

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Abstract

In this study, a vector objective, multi-showroom and multi-item displayed inventory model under display space constraint in crisp and fuzzy environment has been formulated. Here demand rate of an item is considered as a function of the displayed inventory level. The problem is formulated to maximize the average profit, average total cost for store and warehouse. In real life situation, the goals and inventory parameters may not precise. Such type of uncertainty may be characterized by Pentagonal fuzzy numbers. Then the model is defuzzified by two techniques namely (i) nearest interval approximation and (ii) ranking of circumcenter of the centroid of Pentagonal fuzzy number. The optimal order quantity and number of display quantity have been determined by non linear programming problem. Finally the numerical results in two techniques have been compared.

Keywords: Display inventory, Economic Order Quantity (EOQ), Pentagonal fuzzy number, Nearest interval ap-

proximation, Ranking of circumcenter of the centroid.

AMS Subject Classification (2010): 90B05

1 Introduction

Multi - item classical inventory models under various types of constraints such as capital investment, available storage area, number of orders and available set -up time are presented in well known books (Churchman, Ackoff and Arnoff [1], Hardley and Whitin [5], Silver and Peterson [4]; etc). While modeling an inventory problem, generally three types of demands are considered. These are (i) Constant demand (ii) Time dependent demand and (iii) Stock - dependent demand. Among the stock dependent, specially displayed inventory level demand has an effect on sales for many retail products. But all these inventory problems are solved with the assumptions that the co-efficient or cost parameters are specified in a precise way. In real life, there are many diverse situations due to uncertainty. Here inventory costs are imprecise. That is, fuzzy in nature.

A multi item displayed inventory model using pentagonal fuzzy number with alternative power supply cost (generator cost) is presented in [2]. A single item inventory model using ranking of circumcenter of the centroid of pentagonal fuzzy number with alternative power supply cost (generator cost) is presented in [3]. The extension of [2] & [3] is given in this research.

Nowadays the decision makers are giving importance to the retail showrooms and setting showrooms in different places (multi-showroom) to increase the profit of the business. In this context a multi -showroom and multi- item displayed inventory model under display space constraint by using pentagonal fuzzy number with generator cost is considered. Finally a numerical example is given to illustrate the model.

2 Assumptions and Notations

Our proposed model is formulated under the following assumptions and notations.

2.1 Assumptions

1. The unit cost of the item is independent of Q .
2. The display cost is not depends on the length of cycle time T .
3. There is never more than a single order outstanding.
4. Lead time is zero.
5. Shortages are not allowed.
6. Demand rate is depend on display inventory for i^{th} item and k^{th} showroom
 $D_{ki} = d_{ki} S_{ki}^{d'_{ki}} \quad (d_{ki} > 0, \quad 0 < d'_{ki} < 1)$
 Here d_{ki} and d'_{ki} ($i=1, 2, \dots, n$ and $k=1, 2, \dots, N$) are scale and shape parameters of the demand function.
7. Full-shelf merchandising policy, i.e. The display area is always kept fully stocked, so the inventory is replenished as soon as the backroom inventory reaches zero. The displayed inventory will always be at its maximum value. The inventory level decreases at a constant rate.
8. Generator cost is allowed.

2.2 Notations

N - number of showrooms,

n - number of items,

The following are for i^{th} item and k^{th} showroom,

S_{ki} - display inventory level (decision variable),

$(S \equiv [S_{11}, S_{12}, \dots, S_{1n}, S_{21}, S_{22}, \dots, S_{2n}, \dots, S_{N1}, S_{N2}, \dots, S_{Nn}]^T)$,

Q_{ki} - number of order quantity (decision variable),

$(Q \equiv [Q_{11}, Q_{12}, \dots, Q_{1n}, \dots, Q_{N1}, Q_{N2}, \dots, Q_{Nn}]^T)$,

θ_{ki} - instantaneous inventory level of the entire system including

both the back room storage and the displayed Inventory.(net inventory)

p_{ki} - selling price per unit for all showrooms

C_{ki} - purchasing price per unit for all showrooms

C_{1ki} - total holding cost per unit per unit time

C_{2ki} - display shelf cost per unit per unit time

C_{3ki} - set up cost per cycle

D_{ki} - demand rate

P_{ki} - replenishment rate

g_{ki} - total generator cost per unit per unit time

PF - fuzzy Profit function

W_k - total display - Shelf space

w_{Ki} - space per unit

C_{w1ki} - holding cost per unit per unit time for warehouse

C_{s1ki} - holding cost per unit per unit time for store

g_{wki} - generator cost per unit per unit time for warehouse

g_{ski} - generator cost per unit per unit time for store

$\widehat{TC}_S$ - fuzzy total cost for store

$\widehat{TC}_W$ - fuzzy total cost for warehouse

3 Mathematical Model in Crisp Environment

Our proposed model is to maximize the average profit and to minimize the average total cost for store and warehouse.

$$Max PF(S, Q) = \sum_{k=1}^N \sum_{i=1}^n \left[\begin{aligned} & d_{ki} S_{ki}^{d'_{ki}} (p_{ki} - C_{ki}) - \frac{C_{3ki}}{Q_{ki}} d_{ki} S_{ki}^{d'_{ki}} \\ & - \frac{(g_{ki} + C_{1ki}) Q_{ki}}{2} \\ & + \frac{d_{ki} S_{ki}^{d'_{ki}} (C_{1ki} + g_{ki}) Q_{ki}}{2 P_{ki}} \\ & - (g_{ki} + C_{1ki} + C_{2ki}) S_{ki} \\ & + \frac{d_{ki} S_{ki}^{d'_{ki}+1} (C_{1ki} + g_{ki})}{P_{ki}} \end{aligned} \right] \quad (1)$$

$$Min TC_S(S, Q) = \sum_{k=1}^N \sum_{i=1}^n \left[\begin{aligned} & \frac{C_{3ki}}{Q_{ki}} d_{ki} S_{ki}^{d'_{ki}} + \frac{(g_{ski} + C_{s1ki}) Q_{ki}}{2} \\ & - \frac{d_{ki} S_{ki}^{d'_{ki}} (C_{s1ki} + g_{ski}) Q_{ki}}{2 P_{ki}} \\ & + (g_{ski} + C_{s1ki} + C_{2ki}) S_{ki} \\ & - \frac{d_{ki} S_{ki}^{d'_{ki}+1} (C_{s1ki} + g_{ski})}{P_{ki}} \end{aligned} \right] \quad (2)$$

$$\text{Min } TC_w(S, Q) = \sum_{k=1}^N \sum_{i=1}^n \left[\frac{(g_{wki} + C_{w1ki}) Q_{ki}}{2} - \frac{d_{ki} S_{ki}^{d'_{ki}} (C_{w1ki} + g_{wki}) Q_{ki}}{2P_{ki}} + \frac{t_{wki} d_{ki} S_{ki}^{d'_{ki}}}{Q_{ki}} \right] \quad (3)$$

Subject to $\sum_{i=1}^n w_{ki} S_{ki} \leq W_k$, $S_{ki}, Q_{ki} > 0$ and $k = 1, 2, 3, \dots, N$

Using LINGO software, values of Q^* and S^* are obtained.

4 THE PROPOSED INVENTORY MODEL IN FUZZY ENVIRONMENT

4.1 Method 1 [2]

Using nearest interval approximation, our proposed model is reduced to,

$$\begin{aligned} \text{Max } PF_C(S, Q) &= \sum_{k=1}^N \sum_{i=1}^n \left[d_{ki} S_{ki}^{d'_{ki}} (p_{kic} - C_{kic}) - \frac{C_{3kic}}{Q_{ki}} d_{ki} S_{ki}^{d'_{ki}} \right. \\ &\quad \left. - (g_{kic} + C_{1kic} + C_{2kic}) S_{ki} + \frac{d_{ki} S_{ki}^{d'_{ki}} (g_{kic} + C_{1kic}) Q_{ki}}{2P_{kic}} \right. \\ &\quad \left. + \frac{d_{ki} S_{ki}^{d'_{ki}+1} (g_{kic} + C_{1kic})}{P_{kic}} - \frac{(g_{kic} + C_{1kic}) Q_{ki}}{2} \right] \\ \text{Min } TC_{S_C}(S, Q) &= \sum_{k=1}^N \sum_{i=1}^n \left[\frac{C_{3kic}}{Q_{ki}} d_{ki} S_{ki}^{d'_{ki}} + \frac{(g_{skic} + C_{s1kic}) Q_{ki}}{2} \right. \\ &\quad \left. + (g_{skic} + C_{s1kic} + C_{2kic}) S_{ki} - \frac{d_{ki} S_{ki}^{d'_{ki}} (g_{skic} + C_{s1kic}) Q_{ki}}{2P_{kic}} \right. \\ &\quad \left. + \frac{d_{ki} S_{ki}^{d'_{ki}+1} (g_{skic} + C_{s1kic})}{P_{kic}} \right] \\ \text{Min } TC_{w_C}(S, Q) &= \sum_{k=1}^N \sum_{i=1}^n \left[\frac{(g_{wki} + C_{w1ki}) Q_{ki}}{2} - \frac{d_{ki} S_{ki}^{d'_{ki}} (C_{w1ki} + g_{wki}) Q_{ki}}{2P_{ki}} \right. \\ &\quad \left. + \frac{t_{wki} d_{ki} S_{ki}^{d'_{ki}}}{Q_{ki}} \right] \end{aligned}$$

Subject to $\sum_{i=1}^n w_{ki} S_{ki} \leq W_{kc}$, $S_{ki}, Q_{ki} > 0$ and $k = 1, 2, 3, \dots, N$

Using LINGO software, values of Q^* and S^* are obtained.

4.2 Method 2 [3]

Now using ranking fuzzy number, cost parameters and objective functions becomes

$$\begin{aligned}
 Max R_{\alpha}(PF(S, Q)) &= \sum_{k=1}^N \sum_{i=1}^n \left[\begin{aligned} &d_{ki} S_{ki}^{d'_{ki}} (R_{\alpha}(p_{ki}) - R_{\alpha}(C_{ki})) \\ &- \frac{R_{\alpha}(C_{3ki})}{Q_{ki}} d_{ki} S_{ki}^{d'_{ki}} - \\ &\frac{(R_{\alpha}(g_{ki}) + R_{\alpha}(C_{1ki}))}{2} Q_{ki} \\ &- (R_{\alpha}(g_{ki}) + R_{\alpha}(C_{1ki}) + R_{\alpha}(C_{2ki})) S_{ki} \\ &+ \frac{d_{ki} S_{ki}^{d'_{ki}} (R_{\alpha}(C_{1ki}) + R_{\alpha}(g_{ki})) Q_{ki}}{2R_{\alpha}(P_{ki})} \\ &+ \frac{d_{ki} S_{ki}^{d'_{ki}+1} (R_{\alpha}(C_{1ki}) + R_{\alpha}(g_{ki}))}{R_{\alpha}(P_{ki})} \end{aligned} \right] \\
 Min R_{\alpha}(TC_S(S, Q)) &= \sum_{k=1}^N \sum_{i=1}^n \left[\begin{aligned} &\frac{R_{\alpha}(C_{3ki})}{Q_{ki}} d_{ki} S_{ki}^{d'_{ki}} + \frac{(R_{\alpha}(g_{ski}) + R_{\alpha}(C_{s1ki}))}{2} Q_{ki} \\ &+ (R_{\alpha}(g_{ski}) + R_{\alpha}(C_{s1ki}) + R_{\alpha}(C_{2ki})) S_{ki} \\ &- \frac{d_{ki} S_{ki}^{d'_{ki}} (R_{\alpha}(g_{ski}) + R_{\alpha}(C_{s1ki})) Q_{ki}}{2R_{\alpha}(P_{ki})} \\ &+ \frac{d_{ki} S_{ki}^{d'_{ki}+1} (g_{ski} + C_{s1ki})}{R_{\alpha}(P_{ki})} \end{aligned} \right] \\
 Min R_{\alpha}(TC_w(S, Q)) &= \sum_{k=1}^N \sum_{i=1}^n \left[\begin{aligned} &\frac{(R_{\alpha}(g_{wki}) + R_{\alpha}(C_{w1ki})) Q_{ki}}{2} \\ &- \frac{d_{ki} S_{ki}^{d'_{ki}} (R_{\alpha}(C_{w1ki}) + R_{\alpha}(g_{wki})) Q_{ki}}{2P_{ki}} \\ &+ \frac{R_{\alpha}(t_{wki}) d_{ki} S_{ki}^{d'_{ki}}}{Q_{ki}} \end{aligned} \right]
 \end{aligned}$$

Subject to $\sum_{i=1}^n w_{ki} S_{ki} \leq R_{\alpha}(W_k)$, $S_{ki}, Q_{ki} > 0$ and $k = 1, 2, 3, \dots, N$

Using LINGO software, values of α^* , Q^* and S^* are obtained.

5 Numerical Example

A company had a boutique shop in three different places, in that shops they sale a two types of products, for this data's are given below:

$$\begin{aligned}
 w_1 &= 0.5 \forall k \quad w_2 = 0.6 \forall k \quad d_{11} = 20 \quad d_{12} = 25 \quad d_{21} = 30 \quad d_{22} = 15 \quad d_{31} = 40 \\
 d_{32} &= 50 \quad d'_{11} = 0.05 \quad d'_{12} = 0.006 \quad d'_{21} = 0.06 \quad d'_{22} = 0.009 \quad d'_{31} = 0.05 \\
 d'_{32} &= 0.01 \quad \tilde{p}_1 = [600 \ 800 \ 1000 \ 1200 \ 1400] \\
 \tilde{p}_2 &= [1000 \ 1500 \ 2000 \ 2500 \ 3000] \quad \tilde{C}_1 = [300 \ 400 \ 500 \ 600 \ 700] \quad \tilde{C}_2 =
 \end{aligned}$$

$$\begin{aligned}
& [100 \ 300 \ 500 \ 700 \ 900] \tilde{C}_{111} = [0.5 \ 1 \ 1.5 \ 2 \ 2.5] \\
& \tilde{C}_{112} = [1.5 \ 2.5 \ 3.5 \ 4.5 \ 5.5] \ \tilde{C}_{121} = [1.5 \ 2 \ 2.5 \ 3 \ 3.5] \ \tilde{C}_{122} = [0.6 \ 0.8 \ 1 \ 1.2 \ 1.4] \\
& \tilde{C}_{131} = [2.4 \ 2.5 \ 2.6 \ 2.7 \ 2.8] \ \tilde{C}_{132} = [2 \ 3 \ 4 \ 5 \ 6] \ \tilde{C}_{211} = [4 \ 4.5 \ 5 \ 5.5 \ 6] \\
& \tilde{C}_{231} = [5 \ 6 \ 7 \ 8 \ 9] \ \tilde{C}_{212} = [0.8 \ 0.9 \ 1 \ 1.1 \ 1.2] \ \tilde{C}_{221} = [4 \ 5 \ 6 \ 7 \ 8] \ \tilde{C}_{222} = \\
& [1 \ 1.5 \ 2 \ 2.5 \ 3] \ \tilde{C}_{232} = [2.8 \ 2.9 \ 3 \ 3.1 \ 3.2] \\
& \tilde{C}_{311} = [48 \ 49 \ 50 \ 51 \ 52] \ \tilde{C}_{312} = [80 \ 90 \ 100 \ 110 \ 120] \ \tilde{C}_{321} = [53 \ 54 \ 55 \ 56 \ 57] \\
& \tilde{C}_{322} = [130 \ 140 \ 150 \ 160 \ 170] \ \tilde{C}_{331} = [54 \ 55 \ 56 \ 57 \ 58] \ \tilde{C}_{332} = [100 \ 200 \ 300 \ 400 \ 500] \\
& \tilde{g}_{11} = [0.3 \ 0.4 \ 0.5 \ 0.6 \ 0.7] \ \tilde{g}_{12} = [1.3 \ 1.4 \ 1.5 \ 1.6 \ 1.7] \ \tilde{g}_{21} = [0.4 \ 0.5 \ 0.6 \ 0.7 \ 0.8] \\
& \tilde{g}_{22} = [1.2 \ 1.4 \ 1.6 \ 1.8 \ 2] \ \tilde{g}_{31} = [0.5 \ 0.6 \ 0.7 \ 0.8 \ 0.9] \ \tilde{g}_{32} = [1.3 \ 1.5 \ 1.7 \ 1.9 \ 2.1] \\
& \tilde{P}_{11} = [2000 \ 3000 \ 4000 \ 5000 \ 6000] \ \tilde{P}_{12} = [5300 \ 5400 \ 5500 \ 5600 \ 5700] \\
& \tilde{P}_{21} = [1500 \ 2500 \ 3500 \ 4500 \ 5000] \ \tilde{P}_{22} = [5800 \ 5900 \ 6000 \ 6100 \ 6200] \\
& \tilde{P}_{31} = [2300 \ 2400 \ 2500 \ 2600 \ 2700] \ \tilde{P}_{32} = [4800 \ 4900 \ 5000 \ 5100 \ 5200] \\
& \tilde{W}_1 = [3100 \ 3300 \ 3500 \ 3700 \ 3900] \ \tilde{W}_2 = [4300 \ 4400 \ 4500 \ 4600 \ 4700] \\
& \tilde{W}_3 = [2100 \ 2300 \ 2500 \ 2700 \ 2900] \\
& \tilde{P}F = [200000 \ 225000 \ 250000 \ 275000 \ 300000]
\end{aligned}$$

Table 1: Comparison table of crisp and fuzzy environment

Model	k	i	α	Q_{ki}	S_{ki}	PF	TC_S	TC_w
Crisp	1	1	-	35.66	108.29	2,04,503	4,45,890	2,78,900
		2	-	32.04	38.40			
	2	1	-	38.22	161.30			
		2	-	42.37	45.52			
	3	1	-	42.20	150.41			
		2	-	74.59	90.56			
Fuzzy Method 1: Nearest	1	1	-	35.65	116.24	2,14,331	3,45,890	1,49,428
		2	-	31.94	39.33			
	2	1	-	38.58	172.67			
		2	-	42.61	47.14			
	3	1	-	42.21	164.32			
		2	-	75.28	92.79			
Fuzzy Method 2: Ranking	1	1	0.989	35.65	108.2	2,49,759	3,25,990	1,28,678
		2		32.04	38.39			
	2	1		38.22	161.28			
		2		42.37	45.51			
	3	1		42.19	150.39			
		2		74.58	90.56			

Observation 1. 1. In table 1 the optimal values are given for the fuzzy models along with the crisp model.

2. In Method 1, the optimal value of the average profit is more

than that of crisp model.

3. In Method 2, the optimal value of the average profit is more, compared to that of Method 1 and crisp model.
4. Among the above Methods, Ranking method gives the best optimal solution.

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