

Power Dominator Coloring of Corona Graphs

K Sathish Kumar¹, N Gnanamalar David¹
and K G Subramanian²

^{1,2}Department of Mathematics, Madras Christian College,
Tambaram, Chennai 600 059, India

¹{sathishkumar,ngdavid}@mcc.edu.in

²kgsmani1948@gmail.com

Abstract

A power dominator coloring of a graph G is a proper coloring of G in which each vertex of the graph power dominates every vertex of some color class. The power dominator chromatic number $\chi_{pd}(G)$ of a graph G is the minimum number of color classes in a power dominator coloring of G . In this paper, we obtain the power dominator chromatic number of several classes of corona graphs.

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1 Introduction

The graphs we consider here are finite, simple and undirected. For a graph G , the vertex set is denoted by $V(G)$ or simply V if G is understood. A path on n vertices is denoted by P_n , a cycle on n vertices is denoted by C_n and a complete graph on n vertices is denoted by K_n [1].

The concepts of vertex coloring [1] and domination [4] are well investigated concepts in graph theory. A proper coloring of a graph G is an assignment of colors to the vertices of the graph, such that

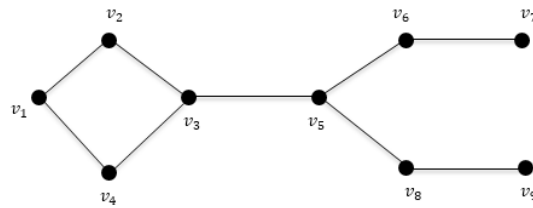
any two adjacent vertices have different colors, and the chromatic number $\chi(G)$ of G is the minimum number of colors needed in a proper coloring of the graph [1]. The notion of dominator coloring of a graph, which requires every vertex to dominate at least one color class, was introduced in [5]. On the other hand, motivated by the activity of monitoring the state of an electric power system, Haynes et al. [3] introduced the concept of power dominating set of vertices in a graph.

Combining the concepts of coloring and power domination, a new notion of coloring called power dominator coloring of a graph G was introduced in [7, 8]. We recall this notion but express the related details in a more precise form. For a vertex u in a graph G , we associate a monitoring set $M(u)$ [2] as follows:

1. *Step(i)* : $M(u) = N[u]$, the closed neighbourhood of u
2. *Step(ii)* : add a vertex w to $M(u)$, (which is originally not in $M(u)$) whenever w has a neighbour $v \in M(u)$ such that all the neighbours of v other than w , are already in $M(u)$
3. *Step(iii)* : repeat *Step(ii)* until no other vertex could be added to $M(u)$.

Then we say that u power dominates the vertices in $M(u)$. The power dominator coloring of G is a proper coloring of G such that every vertex of the vertex set V power dominates all vertices of at least one color class. The power dominator chromatic number $\chi_{pd}(G)$ is the minimum number of colors required for a power dominator coloring of G .

We illustrate with an example. Consider the graph in Fig. 1, for which $\chi_{pd}(G) = 3$. In fact color 1 can be assigned to the vertex v_5 while color 2 can be assigned to the vertices v_1, v_3, v_6, v_8 . The vertices v_2, v_4, v_7, v_9 can be given color 3. For the vertex v_1 , initially the monitoring set $M(u) = \{v_1, v_2, v_4\}$ using the *Step(i)* in the formation of monitoring set. Now the vertex v_3 is the only vertex, which is not in $M(v_1)$, but adjacent to v_2 and hence v_3 is added to $M(v_1)$ using *Step(ii)* in the formation of monitoring set. Continuing the process, the vertex v_5 is the only vertex adjacent to v_3 but not in $M(v_1)$ and hence is added to $M(v_1)$. No other vertex could be added to $M(v_1)$. Thus $M(v_1) = \{v_1, v_2, v_3, v_4, v_5\}$. In fact

Figure 1: A graph with $\chi_{pd}(G) = 3$

each of the vertices v_2, v_3, v_4 has the same monitoring set as that of v_1 . Each of the vertices v_1, v_2, v_3, v_4 power dominates the color class with color 1. Note that none of these vertices v_1, v_2, v_3, v_4 can power dominate any other color class. Likewise the vertices v_6, v_7, v_8, v_9 also power dominate the color class with color 1. The vertex v_5 power dominates its own color class. Hence each vertex power dominates all the vertices in at least one color class and so $\chi_{pd}(G) = 3$. Note that two colors are not enough to obtain a power dominator coloring in this graph. For example, if v_5 is given color 2, then no vertex can power dominate all the vertices in a color class. Note also that the dominator chromatic number $\chi_d(G)$ for this graph is 5.

The corona $G_1 o G_2$ [6] of two graphs G_1 and G_2 is the graph formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 such that the i th vertex of G_1 is adjacent to every vertex in the i th copy of G_2 . In this paper, we obtain the power dominator chromatic numbers of the corona graphs (i) $P_n o C_n, C_n o P_n$ of a path and a cycle of the same order, (ii) $K_n o P_n, P_n o K_n$ of a complete graph and a path of the same order, (iii) $K_n o C_n, C_n o K_n$ of a complete graph and a cycle of the same order, (iv) $P_n o K_{1,n}, K_{1,n} o P_n$ of a path of order n and a star graph of order $n + 1$.

2 Main Results

Theorem 1. *Let $n \geq 2$ be an integer. Then*

$$\chi_{pd}(P_n o C_n) = \begin{cases} n + 2, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even.} \end{cases}$$

Proof. Let C_n^i be the i th copy ($1 \leq i \leq n$), of the cycle C_n . Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(C_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ with $u_{i1}, u_{i2}, \dots, u_{in}$ in order so that $V(P_n o C_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$ where, by definition, each vertex v_i is adjacent to every vertex u_{ij} of C_n^i , the i th copy of C_n . For assigning colors to the vertices of $P_n o C_n$, we consider the following cases:

Case (i): n is even. Assign color 1 to the vertex u_{ij} if j is odd and color 2 if j is even, for all i , $1 \leq i \leq n$. Now assign a new color b_1 to the vertices v_1 and v_n and assign new colors b_i to v_i for $2 \leq i \leq n-1$ so that $n-1$ colors are used for the vertices v_i , $1 \leq i \leq n$.

Case (ii): n is odd. For all i , $1 \leq i \leq n$, assign to each vertex u_{ij} , color 1 for odd j , $1 \leq j \leq n-1$ and color 2 for even j . Assign color 3 to u_{in} for all i , $1 \leq i \leq n$.

Now assign a new color b_1 to the vertices v_1 and v_n and assign new colors b_i to v_i , for $2 \leq i \leq n-1$ so that $n-1$ are used for the vertices v_i , $1 \leq i \leq n$.

It can be seen that each vertex power dominates a color class in each of the cases. Thus $\chi_{pd}((P_n o C_n)) = 2 + n - 1 = n + 1$ in case (i) while $\chi_{pd}(P_n o C_n) = 3 + n - 1 = n + 2$ in case (ii). $\square$

Theorem 2. *Let $n \geq 2$ be an integer. Then*

$$\chi_{pd}(C_n o P_n) = n + 2$$

Proof. Let P_n^i be the i th copy ($1 \leq i \leq n$), of the path P_n . Let $V(C_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(P_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ with $u_{i1}, u_{i2}, \dots, u_{in}$ in order so that $V(C_n o P_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$ where, by definition, each vertex v_i is adjacent to every vertex u_{ij} of P_n^i , the i th copy of P_n . For a proper coloring to the vertices of $C_n o P_n$ assign to u_{ij} , $1 \leq i \leq n$, color 1 to u_{ij} for odd j and color 2 for even j . Assign new colors b_i to v_i , $1 \leq i \leq n$. Thus n colors are used

for the vertices v_i , $1 \leq i \leq n$. It is clear that every vertex power dominates at least one color class. Thus $\chi_{pd}(C_n o P_n) = n + 2$. $\square$

Theorem 3. Let $n \geq 2$ be an integer. Then

$$\chi_{pd}(P_n o K_n) = 2n - 1$$

Proof. Let K_n^i be the i th copy ($1 \leq i \leq n$), of the complete graph K_n . Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(K_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ so that $V(P_n o K_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$ where, by definition, each vertex v_i is adjacent to every vertex u_{ij} of K_n^i , the i th copy of K_n . For all i , $1 \leq i \leq n$, assign colors c_j to u_{ij} for $1 \leq j \leq n$ so that n colors are used. Assign a new color b_1 to v_1 and v_n and new colors b_i to v_i , $2 \leq i \leq n-1$ so that $n-1$ colors are used for these vertices. This yields a power dominator coloring. Thus $\chi_{pd}(P_n o K_n) = 2n - 1$.

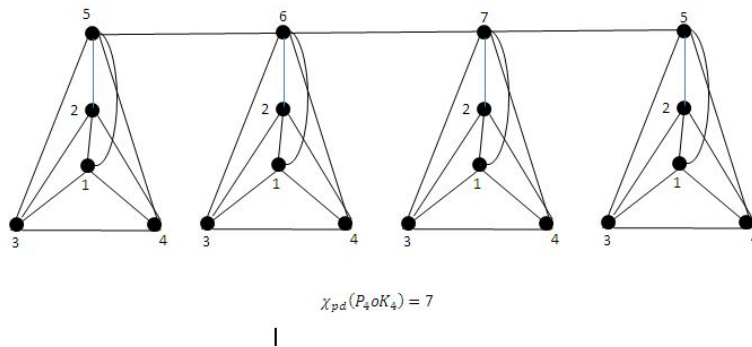


Figure 2: The corona product $P_4 o K_4$

$\square$

Theorem 4. Let $n \geq 2$ be an integer. Then

$$\chi_{pd}(K_n o P_n) = n + 2$$

Proof. Let P_n^i be the i th copy ($1 \leq i \leq n$), of the path P_n . Let $V(K_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(P_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ with $u_{i1}, u_{i2}, \dots, u_{in}$ in order so that

$V(K_n o P_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$. By definition, each vertex v_i is adjacent to every vertex u_{ij} of P_n^i , the i th copy of P_n . For $1 \leq i \leq n$ assign the color c_i to v_i . Assign color 1 to u_{ij} for odd j and color 2 to even j . Thus $\chi_{pd}(P_n o K_n) = n + 2$. $\square$

Theorem 5. *Let $n \geq 2$ be an integer. Then*

$$\chi_{pd}(C_n o K_n) = 2n$$

Proof. Let K_n^i be the i th copy ($1 \leq i \leq n$), of the complete graph K_n . Let $V(C_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(K_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ so that $V(C_n o K_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$ where, by definition, each vertex v_i is adjacent to every vertex u_{ij} of K_n^i , the i th copy of K_n . For all i , $1 \leq i \leq n$, assign colors c_j to u_{ij} for $1 \leq j \leq n$ so that n colors are used. Assign color $n + i$ for v_i , $1 \leq i \leq n$. This gives a power dominator coloring. Thus $\chi_{pd}(C_n o K_n) = 2n$. $\square$

Theorem 6. *Let $n \geq 2$ be an integer. Then*

$$\chi_{pd}(K_n o C_n) = \begin{cases} n + 2, & \text{if } n \text{ is even} \\ n + 3, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let C_n^i be the i th copy ($1 \leq i \leq n$), of the cycle C_n . Let $V(K_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(C_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ with $u_{i1}, u_{i2}, \dots, u_{in}$ in order so that $V(K_n o C_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n\}$. By definition, each vertex v_i is adjacent to every vertex u_{ij} of C_n^i , the i th copy of C_n . For $1 \leq i \leq n$ assign the color c_i to v_i . For assigning colors to other vertices of $K_n o C_n$, we consider the following cases:
Case(i): n is even. Assign color 1 to the vertex u_{ij} if j is odd and color 2 if j is even, for all i , $1 \leq i \leq n$. Thus $\chi_{pd}(K_n o C_n) = n + 2$.
Case(ii): n is odd. Assign color 1 to the vertex u_{ij} for odd j , $1 \leq j \leq n - 1$ and color 2 for all even j . Finally assign color 3 to u_{in} , $1 \leq i \leq n$. Thus $\chi_{pd}(K_n o C_n) = n + 3$. $\square$

Theorem 7. *Let $n \geq 2$ be an integer. Then*

$$\chi_{pd}(P_n o K_{1,n}) = n + 2$$

Proof. Let $K_{1,n}^i$ be the i th copy, ($1 \leq i \leq n$), of the star graph $K_{1,n}$. Let $V(P_n) = \{v_i \mid 1 \leq i \leq n\}$ and for each i , $1 \leq i \leq n$, let $V(K_{1,n}^i) = \{u_{ij} \mid 1 \leq j \leq n+1\}$ with the central vertex $u_{i,1}$ so that $V(C_n \circ K_n) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i \leq n; 1 \leq j \leq n+1\}$ where, by definition, each vertex v_i is adjacent to every vertex u_{ij} of $K_{1,n}^i$, the i th copy of $K_{1,n}$. Assign colors to the vertices in the following manner: For $1 \leq i \leq n$, assign the color c_i to v_i . This uses n colors. Then assign color 1 to the vertex $u_{i,1}$, $1 \leq i \leq n$ and color 2 to the remaining vertices u_{ij} , $2 \leq j \leq n+1$, $1 \leq i \leq n$. This is a proper coloring and every vertex dominates at least one color class. Thus $\chi_{pd}(P_n \circ K_{1,n}) = n+2$. $\square$

Theorem 8. Let $n \geq 2$ be an integer. Then

$$\chi_{pd}(K_{1,n} \circ P_n) = 4$$

Proof. Let P_n^i be the i th copy ($1 \leq i \leq n$), of the path P_n . Let $V(K_{1,n}) = \{v_1, v_2, \dots, v_{n+1}\}$ with central vertex v_1 . For each i , $1 \leq i \leq n+1$, let $V(P_n^i) = \{u_{ij} \mid 1 \leq j \leq n\}$ with $u_{i,1}, u_{i,2}, \dots, u_{i,n}$ in order so that $V(K_{1,n} \circ P_n) = \{v_i \mid 1 \leq i \leq n+1\} \cup \{u_{ij} : 1 \leq i \leq n+1; 1 \leq j \leq n\}$. By definition, each vertex v_i is adjacent to every vertex u_{ij} of P_n^i , the i th copy of P_n . For proper coloring, assign color 1 to the vertex v_1 and color 2 to v_i , $2 \leq i \leq n+1$. Then assign color 3 to u_{ij} for odd j and color 4 to u_{ij} , for even j $1 \leq j \leq n$. Hence only four colours are needed, for power dominator coloring of the vertices. Thus $\chi_{pd}(K_{1,n} \circ P_n) = 4$. $\square$

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