A Fuzzy Production Inventory Model in an Agricultural Field with Random Demand Rate

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Abstract

In this paper, a fuzzy inventory model with probabilistic production rate, demand rate for two items and constant deterioration rate have been proposed. The Lower-Upper (LU) bud fuzzy number is defined and its properties are given. The proposed model is formulated in fuzzy environment using LU-bud fuzzy number. i.e., the parameters involved in this model are represented by LU-bud fuzzy number. The agreement index of LU-bud is explained and using this technique the total cost is defuzzified. Maximum inventory level for the two items are determined. A numerical example is given to illustrate both the proposed crisp model and fuzzy model.

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Key Words and Phrases: Probabilistic production, probabilistic demand, LU-bud fuzzy number, agreement index technique.
1 Introduction

Inventory system is one of the main streams of the Operations Research which is essential in business enterprises and Industries. Deterioration cannot be avoided in business scenarios. Deterministic lot-size inventory model for deteriorating items with shortages and a declining market is discussed in [11].

In real life and global market situations randomness arises from the uncertainty in the production and demand capacity. In this situation, uncertainties are treated as randomness and are handled through probability theory is addressed in [1, 2, 9]. In some situations, the uncertainties are due to fuzziness. Deterministic and probabilistic models in inventory Control is given in [8]. A production inventory model for variable demand and production is explained in [4]. A continuous production control inventory model for deteriorating items is given in [5]. A Production Lot-size Inventory model for deteriorating item is given in [6, 10]. The production rate changes according to the number of working machines, economy, seasonal promotion forecasting, etc and demand rate changes due to weather, competition, customer status is discussed in [7, 12]. K. Dhanam and W. Jesintha discussed L-U bud fuzzy number and Agreement index method in [3].

In our proposed model is that the paddy is cultivated in the violin, after the harvesting 75% of paddy packages are stocking by our agriculture society bank, the reason of that is the society gives loan for the paddy packages. The remaining 25% of the paddy packages are in our supplying process. After completion of harvesting, some of the mineral nutrients are in the violin. so one of the millets is cultivated in the violin by the loan amount. In agricultural field, during the time of harvesting and supply the rate of deterioration is constant. Throughout the period, the rate of production and the rate of demand are uncertainty. so both are taken as a random variables for the first (paddy) item and the second (millet) item. The production rate depends demand rate and time. The demand rate for both the items follows pareto distribution. Hence a fuzzy inventory model for these two items with probabilistic production rate and demand rate is developed. The parameters are represented by L-U bud fuzzy number. This model is defuzzified by agreement index method. Finally a numerical example is provided to illustrate
both the proposed model.

**Assumptions:**
1. The inventory system pertains two items in a single period.
2. The production rate depends on the demand rate and time for first (paddy) and second (millet) item.
3. The demand rate is a random variable for first and second item (paddy), which follows pareto distribution.

**Notations:**
- \(I_{p1}\) - maximum inventory level of first (paddy) item at time \(t_2\).
- \(I_{p2}\) - maximum inventory level of second (millet) item at time \(t_5\).
- \(I_m\) - one fourth level of maximum inventory level of first (paddy) item at time \(t_2\).
- \(I_s\) - three fourth level of maximum inventory level of first (paddy) item at time \(t_2\).
- \(p_1, p_2\) - production rate (random variable) for first (paddy) and second (millet) item during the interval \(t_1 \leq t \leq t_2, t_4 \leq t \leq t_5\).
- \(d_1\) - demand rate (random variable) for first item (paddy) during the interval \(t_2 \leq t \leq t_5\).
- \(d_2\) - demand rate (random variable) for second item (millet) during the interval \(t_5 \leq t \leq t_6\).
- \(\theta\) - constant deterioration rate.
- \(s_c\) - setup cost per period.
- \(h_{c1}, h_{c2}\) - fuzzy holding cost for first and second item per unit per unit time.
- \(d_{c1}, d_{c2}\) - fuzzy deterioration cost for first (paddy) item per unit per unit time.

2 Mathematical Model in Crisp Environment

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost. The rate of change of the inventory during the following periods are governed by the following differential equations.
\[ \frac{dI_1(t)}{dt} + \theta I_1(t) = p_1 - d_1, \quad t_1 \leq t \leq t_2 \]  
\[ \frac{dI_2(t)}{dt} + \theta I_2(t) = -d_1, \quad t_2 \leq t \leq t_3 \]  
\[ \frac{dI_3(t)}{dt} + \theta I_3(t) = -d_1, \quad t_3 \leq t \leq t_5 \]  
\[ \frac{dI_4(t)}{dt} + \theta I_4(t) = p_2 - d_2, \quad t_4 \leq t \leq t_5 \]  
\[ \frac{dI_5(t)}{dt} + \theta I_5(t) = -d_2, \quad t_5 \leq t \leq t_6 \]

with the boundary conditions
\[ I_1(t_1) = 0, \quad I_1(t_2) = I_{p1}, \quad I_2(t_2) = I_{p1}, \]
\[ I_2(t_2) = I_m, \quad I_2(t_3) = 0, \quad I_3(t_3) = I_s, \quad I_3(t_5) = 0, \quad I_4(t_4) = 0, \quad I_4(t_5) = I_{p2}, \quad I_5(t_6) = 0. \]

From equation (1),(2)

\[ I_{p1} = \left( \frac{p_1 - d_1}{\theta} \right) \left( 1 - e^{\theta(t_1 - t_2)} \right), \quad I_m = \frac{d_1}{\theta} \left( e^{\theta(t_3 - t_2)} - 1 \right) \]

From equation (3),(4)

\[ I_s = \frac{d_1}{\theta} \left( e^{\theta(t_5 - t_3)} - 1 \right), \quad I_{p2} = \left( \frac{p_2 - d_2}{\theta} \right) \left( 1 - e^{\theta(t_4 - t_5)} \right) \]

From equation (5),

\[ I_4(t) = -\frac{d_2}{\theta} + \frac{d_2}{\theta} e^{\theta(t_6 - t)} \]

Expected value of production rate for both the items (paddy & millet)
\[ E(p_1) = E(d_1) \left( \frac{t_2 - t_1}{t_2 - t_1} \right), \quad E(p_2) = E(d_2) \left( \frac{t_5 - t_4}{t_5 - t_4} \right). \]

Expected value of demand rate for the first (paddy) item
\[ E(d_1) = \frac{a}{\alpha - 1}, \quad a > 0, \alpha > 1, \]

Expected value of demand rate for the second (millet) item
\[ E(d_2) = \frac{a}{\alpha - 1}, \quad a > 0, \alpha > 1. \]

Setup cost = \( s \)

Expected holding cost
\begin{align*}
\text{Expected deteriorating cost} & = d_{c1} [(p_1 - d_1)(t_2 - t_1) - d_1(t_4 - t_2)] + \\
& + d_{c2} [(p_2 - d_2)(t_5 - t_4) - d_2(t_6 - t_5)]
\end{align*}

The optimal value of \( t_1, t_2, t_3, t_4, t_5, \) and \( t_6 \) have been obtained from the expected total cost \( E(\text{TC}(t_1, t_2, t_3, t_4, t_5, t_6)) \) by using MATLAB Software.

## 3 LU-bud Fuzzy Number and its agreement index method

### Definition: (L-U bud fuzzy number)

A L-U bud fuzzy number \( \tilde{A} \) described as a normalized convex fuzzy subset on the real line \( \mathbb{R} \) whose membership function \( \mu_{\tilde{A}}(x) \) is defined as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{1}{4} \left( 1 - \frac{x-a}{b-a} \right)^{n+1} & \text{at } x = a, c \\
\frac{1}{4} \left( 1 - \frac{x-a}{b-a} \right)^{n+1} & \text{at } a \leq x \leq b \\
\frac{1}{4} \left( 1 + \frac{x-c}{c-b} \right)^{n+1} & \text{at } a \leq x \leq b \\
\frac{1}{4} \left( 1 + \frac{x-c}{c-b} \right)^{n+1} & \text{at } b \leq x \leq c \\
0 \text{ and } 1 & \text{at } x = b
\end{cases}
\]
Agreement index method:

Figure 2: L-U bud Fuzzy number

\[ i_G(A,H) = \begin{cases} \ \mathcal{I}_1(c-a) \big( \mathcal{I}_2 + a \mathcal{I}_3 + b \mathcal{I}_4 + c \mathcal{I}_5 + \mathcal{I}_6 \big), & \text{where } \mathcal{I}_1 = \frac{2n+1}{(n+1)}, \\
\end{cases} \]

\[ \mathcal{I}_2 = \frac{(\alpha_2 - \alpha_1)(x_1 + x_2)}{2}, \mathcal{I}_3 = \left( \frac{n}{2(2n+1)} \right)^{2n+1} - \alpha_2, \mathcal{I}_6 = -n \frac{(n+1)}{2(n+1)}, \]

\[ \mathcal{I}_4 = -\left( \frac{n}{2(2n+1)} + \frac{n}{2(2n+1)} \right), \mathcal{I}_5 = \left( \alpha_1 - \frac{n}{2(2n+1)} \right).\]

4 Inventory Model in Fuzzy Environment

The proposed inventory model in fuzzy environment is

\[ \text{Min } E(\text{Total Cost}) = \tilde{s} + \tilde{h}_{c1} [ \left( \left( \frac{\alpha_2}{\alpha_1} \right) (t_5 - t_4) \right) \left( \frac{t_5 - t_4}{\theta} \right) + \left( \frac{e^{\theta(t_3-t_4)-1}}{\theta} \right) ] + \tilde{h}_{c2} [ \left( \left( \frac{\alpha_2}{\alpha_1} \right) (t_5 - t_4) \right) \left( \frac{t_5 - t_4}{\theta} \right) + \left( \frac{e^{\theta(t_3-t_4)-1}}{\theta} \right) ] + \tilde{d}_{c1} [ \left( \left( \frac{\alpha_2}{\alpha_1} \right) (t_5 - t_4) \right) \left( \frac{t_5 - t_4}{\theta} \right) ] + \tilde{d}_{c2} [ \left( \left( \frac{\alpha_2}{\alpha_1} \right) (t_5 - t_4) \right) \left( \frac{t_5 - t_4}{\theta} \right) ], \]

where \( \sim \) represents the fuzzification of the parameters. By using the agreement index of L-U bud fuzzy number the total cost is defuzzified.
Min $i_G(E($ Total Cost $), H) = i_G(s_c, H) + i_G(h_{c1}, H)$

$$
= \left[ \left( \frac{a\theta}{\alpha} \right) \left( \frac{t_5-t_2}{\theta} \right) - \left( \frac{a\theta}{\alpha} \right) \left( t_2 - t_1 + \frac{\theta(t_4-t_3)}{\theta} \right) \right] - \left( \frac{a\theta}{\alpha} \right) \theta \left( t_5 - t_3 \right) \right) +
$$

$$
= i_G(h_{c2}, H) \left[ \left( \frac{a\theta}{\alpha} \right) \left( \frac{t_5-t_2}{\theta} \right) + \left( t_2 - t_1 + \frac{\theta(t_4-t_3)}{\theta} \right) \right] -
$$

$$
\left[ \left( \frac{a\theta}{\alpha} \right) \left( \frac{t_5-t_2}{\theta} \right) + \left( t_2 - t_1 + \frac{\theta(t_4-t_3)}{\theta} \right) \right] + i_G(d_{c1}, H)
$$

The optimal value of $t_1, t_2, t_3, t_4, t_5$ and $t_6$ have been obtained from the expected total cost $i_G(E(TC(t_1,t_2,t_3,t_4,t_5,t_6)), H)$ by using MATLAB Software.

5  Numerical Example

The following values of the parameter in proper unit were considered as input for the numerical analysis of the above problem,

$A = 5000, a = 0.7, \theta = 0.6, \mu = 5, \alpha = 5$

$(b_{11}, b_{12}, b_{13}) = (0.16, 0.18, 0.20), (b_{21}, b_{22}, b_{23}) = (0.18, 0.21, 0.24),
(d_{11}, d_{12}, d_{13}) = (0.13, 0.15, 0.17), (d_{21}, d_{22}, d_{23}) = (0.16, 0.19, 0.22).$

Using the analytical expressions, the optimal values of $t_1, t_2, t_3, t_4, t_5, t_6, \text{I}_p, \text{I}_m, \text{I}_a, \text{I}_{p2}$ and the expected total cost are obtained and list out in the following table.

<table>
<thead>
<tr>
<th>Mode</th>
<th>fuzzy</th>
<th>integer</th>
<th>$t_1$/wk</th>
<th>$t_2$/wk</th>
<th>$t_3$/wk</th>
<th>$t_4$/wk</th>
<th>$t_5$/wk</th>
<th>$t_6$/wk</th>
<th>$V_{p1}$/kg</th>
<th>$V_{p2}$/kg</th>
<th>$S_{E TC}$</th>
</tr>
</thead>
</table>

Observation
From the above table, it should be noted that compared to crisp model, the fuzzy model is very effective method in the sense that the expected total cost is obtained in fuzzy model is less than the crisp model.

References

7th edition.


