

A TWO-WAREHOUSE MODEL FOR DETERIORATING ITEMS WITH ADVERTISEMENT DEPENDENT DEMAND AND PRESERVATION TECHNOLOGY INVESTMENT

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Abstract: In present article a two-warehouse inventory model is developed in which the owned warehouse is of limited capacity due to retailer hirers a warehouse with infinite capacity at a high rate of holding cost than the owned warehouse. The demand rate is considered as direct function of advertisement and selling price. The item is deteriorating in nature, so to reduce losses due to deterioration we have considered the concept of preservation technology investment. To make the model more practical we have also focused on trade credit. The model is designed to find the optimal values of total cost, frequency of advertisement and the preservation technology cost. Numerical verification and sensitivity analysis is also carried out with the help of mathematical software MATHEMATICA11.

Keywords: Two-warehouse, Advertisement, Trade Credit, Inflation, Preservation.

1 Introduction

The main task of any organizations is, to gain more profit with high customer satisfaction level. To complete this task successfully, organizations generally try to maintain the relation with their customers. That can only possible by focusing on different parameters. Those are playing a key role in functioning of the system like demand, storage, nature of items, credits for customers etc. So, to boost up the demand, organizations should display their items or in other words one can say that

they should focus on advertisement policies. In this regard, an advertising policy with quantity discounts is developed by **Khouja and Robbins [2003]**.

Chen [2011] has explored a two-level supply chain model focusing on coordination amongst the ordering and advertising policies. A finite horizon inventory model is established by **Palanivel and Uthayakumar [2015]** for deteriorating items with price and advertisement dependent demand rate by considering the influence of inflation. **Goyal et al. [2017]** have established an economic order quantity model with advertisement dependent imprecise demand.

Rabbani et al. [2017] have presented a joint optimal policies for deteriorating items mainly focusing on advertisement and pricing policies. An economic order quantity model for deteriorating items is explored by **Shah and Vaghela [2017]** for deteriorating items in which demand is varying with time and advertisement frequency. An inventory model is described by **Shaikh [2017]** for deteriorating items with advertisement-selling price related demand and different types of trade credit policies.

Another tool for perfect inventory system is storage policies especially in case of deteriorating items. In literature **Jaggi et al. [2017]** has established a model for deteriorating items with two-warehouse inventory system and trade credit. **Palanivel and Uthayakumar [2017]** have explored an inventory model for deteriorating items with partial backlogging and trade credit offer, in case of two-warehouse inventory system. **Shaikh et al. [2017]** have also studied a two-warehouse

inventory model for non-instantaneous deteriorating items.

An optimal ordering policy is developed by **Liao et al. [2018]** for a two-warehouse inventory system with the concept of trade credit, to increase the demand. **Sharmila and Uthayakumar [2018]** have considered the two-warehouse system to store the deteriorating inventory, in a deterministic inventory model with trade credit for a supply chain system.

The preservation technology is also a significant tool to reduce losses, mainly in case of deteriorating items. In inventory research work **Singh and Rathore [2015a]** have considered the concept of preservation technology investment with permissible delay in payments. A two-warehouse inventory model is developed by **Singh and Rathore [2015b]** with preservation technology investment and partial backlogging. **Mishra et al. [2017]** have studied an inventory model for deteriorating items with shortages and preservation technology investment. An optimal replenishment policy is developed by **Pal et al. [2017]** for perishable items with preservation technologies.

In present article to establish a study with concept of advertisement and selling price dependent demand rate, we have developed a two-warehouse inventory model for deteriorating items. The deteriorating rate is put under control by investing in preservation technologies. To attract the more customers we have also considered the concept of trade credit in the inflationary situations. In the next section assumptions and notation are elaborated which are used in the mathematical model formulation.

2 Assumptions and Notation

These are the assumptions and notations which we have incorporated in mathematical model formulation.

2.1 Assumptions

- The advertisement dependent demand rate is as follows:
 $D(A, s, t) = A^a(\alpha_1 - \alpha_2 s + \alpha_3 t)$ where $a, \alpha_1, \alpha_2, \alpha_3 > 0$;
- The shortages are not considered;
- The lead time is zero with instant replenishment;
- The time horizon is taken of infinite length;
- The deteriorating items are the main focus;

- The preservation technologies is inculcating in model formulation;
- The constant rate of inflation is the key for settlement of the model;
- The T_M is the trade credit period which is provided by the supplier for the purchaser. During this time period there is no charge but when T_M get over a high interest rate is charged on unsold amount. So main task for purchaser to set a perfect payment time to reduce the losses due to interest charged.
- The items are stored in two-warehouse system. The Owned Warehouse (OW) has fixed storage capacity of W_2 units on the other hand the Rented Warehouse (RW) has unlimited storage capacity.
- The holding cost (h_2) of RW is larger than the holding cost (h_1) of OW. Purchaser used their stock from RW first and then from OW.
- There is no extra convenes charges between RW and OW as distance between these two is negligible.

2.2 Notation

- C_1 : The order cost per order at $t=0$;
- s : The unit selling price at $t=0$;
- C_s : The purchase cost per unit at $t=0$;
- h_1 : The holding cost per unit per time unit at OW without any interest charges;
- h_2 : The holding cost per unit per time unit at RW without any interest charges;
- A^a : The frequency of advertisement where $a > 0$;
- C_A : The advertisement cost per advertisement;
- θ : The deterioration rate, $0 < \theta < 1$;
- $p(\xi)$: The Preservation technology function with $p(\xi) = \theta(1 - e^{-\mu\xi})$ where $\mu > 0$;
- ξ : The preservation technology cost per unit per time unit;
- τ_θ : The reduced deterioration rate, $\tau_\theta = (\theta - p(\xi))$;
- I_r : The inflation rate, where $0 \leq I_r < 1$;
- I_e : The interest earned per unit per year;
- I_c : The interest paid by purchaser for unsold stock per year;
- t_1 : The time period at which stock level at RW ceases up to zero level;

- T_M : The credit period;
- T : The total cycle length;
- $I(t)$: The instantaneous inventory at any time t ;
- Q : The maximum limit of inventory level during $[0, T]$;
- $TC_1(T, t_1, A, \xi)$: The present worth of total cost per time unit, when $T \leq T_M$ where T, t_1, N, ξ are the decision variables;
- $TC_2(T, t_1, A, \xi)$: The present worth of total cost per time unit, when $T_M < T$ where T, t_1, A, ξ are the decision variables;
- The total cost includes the cost functions which are mentioned below:
 - OC: The order cost;
 - PC: The purchase cost;
 - HC: The present worth of holding cost;
 - IP: The present worth of interest paid;
 - ACA : The present worth of advertisement;
 - IE_1 and IE_2 : The present worth of interest earned from sales revenue while credit period in both the cases when $T \leq T_M$ and $T_M < T$ respectively;

Note: The notations with superscript * represents the optimal value of that particular parameters.

3 Mathematical Model Formulation

At the time $t = 0$ the inventory level is Q as the time precedes the stock level depletes continuously just because of total effect of deterioration and demand. The complete functioning of inventory is elaborated by following differential equations.

$$\frac{dI_1(t)}{dt} + \tau_\theta I_1(t) = -A^a(\alpha_1 - \alpha_2 s + \alpha_3 t); 0 \leq t \leq t_1 \quad (1)$$

And in OW the inventory status is as follows

$$\frac{dI_2(t)}{dt} + \tau_\theta I_2(t) = 0; \quad 0 \leq t \leq t_1 \quad (2)$$

$$\frac{dI_3(t)}{dt} + \tau_\theta I_3(t) = -A^a(\alpha_1 - \alpha_2 s + \alpha_3 t); t_1 \leq t \leq T \quad (3)$$

By applying the limits of stock amount at RW like $I_1(t=0) = W_1, I_1(t=t_1) = 0$ and at OW $I_2(t=t_1) = W_2, I_2(t=T) = 0, I_2(t=t_1) = I_3(t=t_1)$.

Solving (1), (2) and (3) we obtain the instant inventory level that is given by

$$I_1(t) = -A^a e^{-\tau_\theta t} \left[(\alpha_1 - \alpha_2 s)t + \frac{t^2}{2}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta t^3}{3} \right] + W_1 \quad (4)$$

$$I_2(t) = e^{-\tau_\theta t} W_2 \quad (5)$$

$$I_3(t) = -A^a e^{-\tau_\theta t} \left[(\alpha_1 - \alpha_2 s)t + \frac{t^2}{2}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta t^3}{3} \right] + W_2 \quad (6)$$

Where

$$W_1 = A^a e^{-\tau_\theta t_1} \left[(\alpha_1 - \alpha_2 s)t_1 + \frac{t_1^2}{2}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta t_1^3}{3} \right] \quad (7)$$

$$W_2 = \frac{A^a e^{-\tau_\theta t_1}}{(1 - e^{-\tau_\theta t_1})} \left[(\alpha_1 - \alpha_2 s)t_1 + \frac{t_1^2}{2}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta t_1^3}{3} \right] \quad (8)$$

$$Q = W_1 + W_2 \quad (9)$$

The cost functions are as follows:

- The order cost = C_1
- The purchase cost = QC_s
- The advertisement cost = $A C_A$
- The holding cost in the effect of inflation

$$HC = \left[h_2 C_s \int_0^{t_1} I_1(t) e^{-I_r t} dt + h_1 C_s \int_0^{t_1} I_2(t) e^{-I_r t} dt + h_2 C_s \int_{t_1}^T I_3(t) e^{-I_r t} dt \right]$$

$$HC = \left[\begin{aligned} & -A^a h_2 C_s \left[(\alpha_1 - \alpha_2 s) \frac{t_1^2}{2} + \frac{t_1^3}{6}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta t_1^4}{12} \right] \\ & + W_1 t_1 + (\tau_\theta + I_r) A^a \left[(\alpha_1 - \alpha_2 s) \frac{t_1^3}{3} + \frac{t_1^4}{8}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta t_1^5}{15} \right] \\ & + (\tau_\theta + I_r) W_1 \frac{t_1^2}{2} + W_2 h_2 C_s \left[t_1 - (\tau_\theta + I_r) \frac{t_1^2}{2} \right] - A^a h_2 C_s \left[\frac{(\alpha_1 - \alpha_2 s)(T^2 - t_1^2)}{2} + \frac{T^3 - t_1^3}{6}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta (T^4 - t_1^4)}{12} \right] \\ & + (T - t_1) W_2 + (\tau_\theta + I_r) A^a h_2 C_s \left[\frac{(\alpha_1 - \alpha_2 s)(T^3 - t_1^3)}{3} + \frac{T^4 - t_1^4}{8}(\tau_\theta(\alpha_1 - \alpha_2 s) + \alpha_3) + \frac{\alpha_3 \tau_\theta (T^5 - t_1^5)}{15} \right] \\ & + (\tau_\theta + I_r) W_2 h_2 C_s \left(\frac{T^2 - t_1^2}{2} \right) \end{aligned} \right] \quad (10)$$

Now we will calculate the total paid and earned interest by the purchaser. There are two cases according to time period of credit limit as described by: case 1: $T \leq T_M$ case2: $t_1 < T_M \leq T$

Case 1: $T \leq T_M$ In this case as the credit time limit is more than the total cycle period hence there is no paid interest for the purchaser. So the total interest earned is given by as follows. The diagrammatic representation of present case is given in Fig. 1

$$IE_1 = I_e S \left[\int_0^T A^a t (\alpha_1 - \alpha_2 s + \alpha_3 t) e^{-I_r t} dt + T \int_0^{T_M} A^a e^{-I_r t} (\alpha_1 - \alpha_2 s + \alpha_3 t) dt \right]$$

$$IE_1 = I_e S A^a \left[\begin{aligned} & \left[(\alpha_1 - \alpha_2 s) \frac{T^2}{2} + (\alpha_3 - I_r (\alpha_1 - \alpha_2 s)) \frac{T^3}{3} \right] \\ & - \alpha_3 I_r \frac{T^4}{4} \\ & + T \left[(\alpha_1 - \alpha_2 s)(T_M - T) + (\alpha_3 - I_r (\alpha_1 - \alpha_2 s)) \frac{(T_M^2 - T^2)}{2} \right] \\ & - \alpha_3 I_r \frac{(T_M^3 - T^3)}{3} \end{aligned} \right] \quad (11)$$

Hence, here is the total cost function:

$$TC_1(T, t_1, A, \xi) = OC + PC + AC_A + HC - IE_1 \quad (12)$$

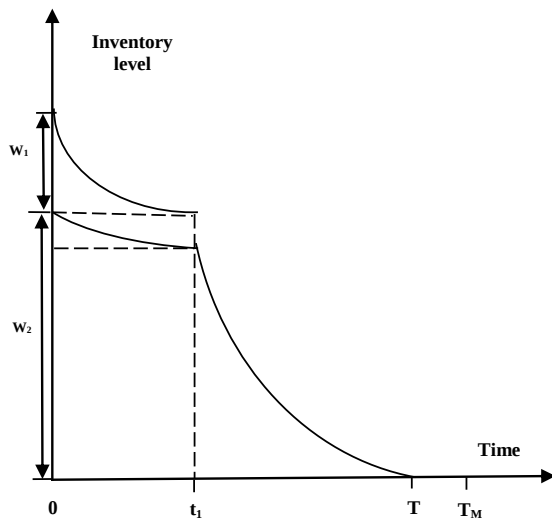


Figure 1. (Case 1: $T \leq T_M$) Two-warehouse inventory system with trade credit.

Case 2: $t_1 < T_M \leq T$ In second case the credit time limit is less than the total cycle length. In such case purchaser have to pay interest for the unsold amount which is charged by the supplier when credit time limit gets over. Hence interest paid by purchaser is given as below. The pictorial representation of this case is given in Fig. 2.

$$IP = I_c C_s \int_0^T I_3(t) e^{-I_r t} dt$$

$$IP = I_c C_s \left[\begin{aligned} & -A^a \left[(\alpha_1 - \alpha_2 s) \frac{(T^2 - T_M^2)}{2} + \frac{(T^3 - T_M^3)}{6} (\tau_\theta (\alpha_1 - \alpha_2 s) + \alpha_3) \right] \\ & + \frac{\alpha_3 \tau_\theta (T^4 - T_M^4)}{12} \end{aligned} \right] \\ + \left[\begin{aligned} & (\alpha_1 - \alpha_2 s) \frac{T^3 - T_M^3}{3} \\ & + \frac{(T^4 - T_M^4)}{8} (\tau_\theta (\alpha_1 - \alpha_2 s) + \alpha_3) \\ & + \frac{\alpha_3 \tau_\theta (T^5 - T_M^5)}{15} \end{aligned} \right] \\ + (\tau_\theta + I_r) W_2 \left(\frac{T^2 - T_M^2}{2} \right) \quad (13)$$

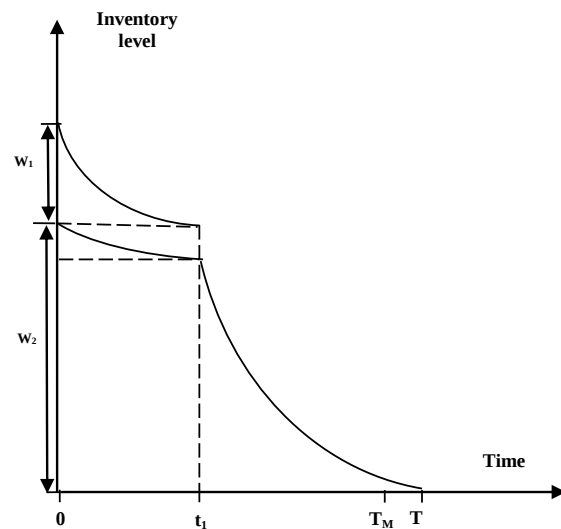


Figure 2. (Case 2: $t_1 < T_M \leq T$) Two-warehouse inventory system with trade credit.

In this case the interest which is earned by the purchaser is as follows:

$$IE_2 = I_e \left[s \int_0^T A^a t (\alpha_1 - \alpha_2 s + \alpha_3 t) e^{-I_r t} dt + (T_M s - C_s T) \int_{T_M}^T A^a e^{-I_r t} (\alpha_1 - \alpha_2 s + \alpha_3 t) dt \right] \\ IE_2 = I_r A^a \left[s \left[\frac{(\alpha_1 - \alpha_2 s) T^2}{2} + (\alpha_3 - I_r (\alpha_1 - \alpha_2 s)) \frac{T^3}{3} \right] - \alpha_3 I_r \frac{T^4}{4} \right] \\ + (T_M s - C_s T) \left[\frac{(\alpha_1 - \alpha_2 s)(T - T_M) + (\alpha_3 - I_r (\alpha_1 - \alpha_2 s)) \frac{(T^2 - T_M^2)}{2}}{2} - \alpha_3 I_r \frac{(T^3 - T_M^3)}{3} \right] \quad (14)$$

Hence the total cost function is as presented below:

$$TC_2(T, t_1, A, \xi) = OC + PC + AC_A + HC + IP - IE_2 \quad (15)$$

The core objective of the system in both the cases is to find the optimum value of the total cost function. For the optimization the necessary and sufficient conditions for $i = 1, 2$ are as follows:

$$\frac{\partial TC_i(T, t_1, A, \xi)}{\partial T} = 0; \frac{\partial TC_i(T, t_1, A, \xi)}{\partial A} = 0; \\ \frac{\partial TC_i(T, t_1, A, \xi)}{\partial \xi} = 0; \frac{\partial TC_i(T, t_1, A, \xi)}{\partial t_1} = 0;$$

Provided $DET(H1) > 0$, $DET(H2) > 0$, $DET(H3)$ where $H1, H2, H3$ are the principal minor of the Hessian matrix. Here is the Hessian Matrix of the total cost function for $i = 1, 2$:

$$\begin{bmatrix} \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial T^2} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial T \partial t_1} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial T \partial A} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial T \partial \xi} \\ \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial t_1 \partial T} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial t_1^2} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial t_1 \partial A} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial t_1 \partial \xi} \\ \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial A \partial T} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial A \partial t_1} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial A^2} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial A \partial \xi} \\ \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial \xi \partial T} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial \xi \partial t_1} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial \xi \partial A} & \frac{\partial^2 TC_i(T, t_1, A, \xi)}{\partial \xi^2} \end{bmatrix}$$

4 Numerical Illustration

We have verified our model by giving this numerical example. The values of different inventory parameters are described as below:

$$C = 151\$, h_1 = 0.16\$, h_2 = 0.10\$, C_s = 21\$, C_3 = 51\$, s = 31\$, I_s = 0.059\$, I_r = 0.089\$, \theta = 0.029, \alpha_1 = 151, \alpha_2 = 3.51, \alpha_3 = 0.21, a = 3.01, T_M = 1.02, \mu = 0.21$$

The optimal values of decision variables are as follows:
 $T^* = 0.94, \xi^* = 12.58, A^* = 0.67, t_1^* = 0.71, TC^* = 69.74$

5 Sensitivity Analysis

We have done a sensitivity analysis to sensitize the study. We have explored the numerical calculations by varying values of some parameters to study the respective change in the optimal values of decision variables. The outputs are mentioned below in table 1.

Table 1: Sensitivity analysis

Para meter	Chan ge in Para meter	Preservati on cost (ξ^*)	Advertisin g frequency (A^*)	Time period (t_1^*)	Total cycle length (T^*)	Total Cost (TC^*)
Θ	0.029	8.249001	0.676101	0.712812	0.940245	65.75
	0.059	9.548010	0.676101	0.712812	0.940245	66.55
	0.079	16.40102	0.676101	0.712812	0.940245	73.37
α_1	141	11.83671	0.782891	0.718812	0.948011	72.71
	146	12.97836	0.756331	0.715812	0.945952	71.18
	156	13.10102	0.657421	0.708812	0.939818	59.89
α_2	3.1	13.61216	0.595981	0.708812	0.938763	66.81
	3.4	13.01671	0.647171	0.711812	0.941122	68.49
	3.7	12.38211	0.698167	0.719812	0.942537	70.55
α_3	0.15	12.51271	0.675781	0.718272	0.941231	69.81
	0.55	13.16571	0.673617	0.708272	0.938918	69.71
	0.65	13.26272	0.670161	0.707272	0.937811	69.62
s	26	11.61721	0.631421	0.707272	0.936782	68.46
	36	13.06182	0.812411	0.718972	0.949721	73.66
	47	13.00561	0.861171	0.728892	0.951221	75.06
μ	0.15	25.78326	0.682628	0.718987	0.941452	69.81
	0.35	8.567134	0.682628	0.718987	0.941452	69.81
	0.45	6.838331	0.682628	0.718987	0.941452	69.81
I_r	0.021	12.64171	0.671671	0.708718	0.939617	69.71
	0.041	12.56171	0.677251	0.717872	0.940918	69.86
	0.051	12.48198	0.679124	0.719811	0.941081	69.99
T_M	1.21	13.99911	0.591711	0.938113	1.121231	65.76
	1.31	14.31526	0.561819	0.967131	1.169891	63.67
	1.41	14.35711	0.541816	0.998712	1.251212	61.81

According to study of data given in above table it is quite clear that this model is very sensitive as by changing the parameters the optimal values are also changed. So, we can say that this model is physically verified model. The convex curve of the total cost function is given as below in Figure 3, Figure 4 and Figure 5.

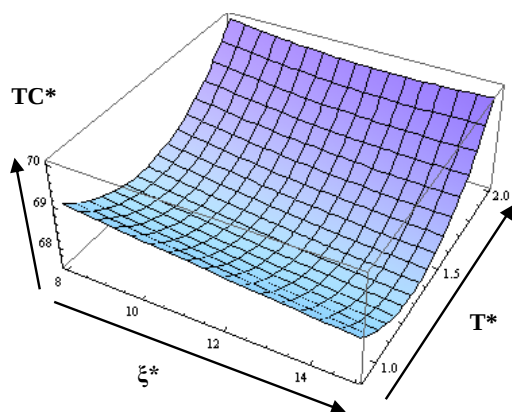


Figure 3. Convex curve of total cost TC^* in regards to preservation cost ξ^* and total cycle length T^*

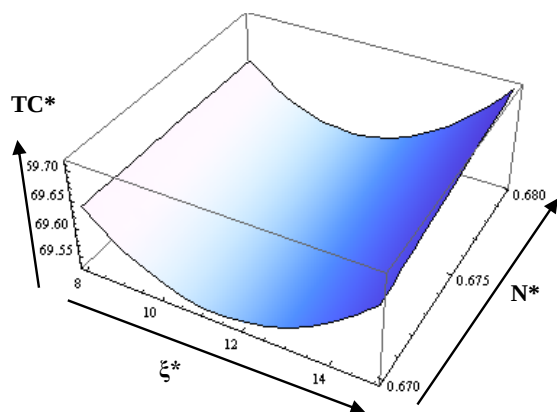


Figure 4. Convex curve of total cost TC^* in regards to preservation cost ξ^* and advertising frequency A^*

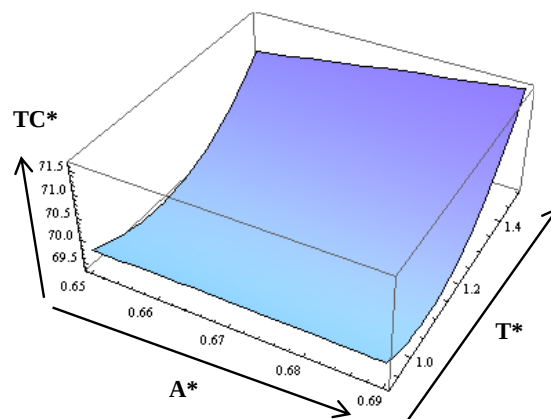


Figure 5. Convexity of total cost TC^* in regards to advertising frequency A^* and total cycle length T^*

Conclusion

In this study, we have explored a model for deteriorating items which are stored at two-warehouse system pertaining preservation technologies. This article also presents a relation between total cost and the advertising frequency by considering the advertising and selling price dependent demand rate. The idea of trade credit and inflation is also implemented in this paper. The study is numerically verified and a sensitivity analysis is also carried out by using the software MATHEMATICA11. This study work further can be extended by incorporating the other constraints of the inventory system.

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