

**Inventory Model for Non-Instantaneous Deterioration with Price Dependent Demand,
Linear Holding Cost and Partial Backlogging under Inflationary Environment**

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Abstract

Inventory system is one of the major vexations for an industry. In this research, we suppose an EOQ inventory model for decaying goods having price dependent demand rate. This paper is more optimal as products maintain its value before there is a loss of value for a period of time. This model is studied in the inflationary environment. Here holding cost is taken as variable. Numerical examples and sensitivity analysis are provided to further illustrate the model. The model is optimized and the convexity of the model is shown.

Keywords: EOQ , Inflation, Partial Backlogging, Variable Holding Cost

1. Introduction

In the classical EOQ models, the demand rate is either assumed to be constant or time dependent. But in real scenarios, various factors impact the demand of a product. One of the major factor is price changes. Companies regularly use price change to increase demand. The attractive price of the product motivates customer to buy it and thus increases the overall demand of the product. These price decrease are typically for a relatively short period of time and when the price comes back, naturally the demand decreases. The situation further complicates when the product is deteriorating in nature.

Deterioration leads to decrease in value of the product over a period of time and thus its impact can't be underestimated while deciding on inventory levels. In this paper, we assume that the deterioration is non-instantaneous. Chang and Dye (1999) established an economic order quantity model for deteriorating items with linear demand and partial backlogging. Wu et.al (2006) generalized an non-instantaneous

decaying model with partial shortages. The effect of preservation technology investment on a non-instantaneous deteriorating stock model was considered by Dye (2013). Jaggi (2014) established an optimal replenishment policy for deteriorating items with price dependent demand and time-varying holding cost. Tayal et al. (2014) illustrated an inventory model for deteriorating items with seasonal products and an option of an alternative market. Credit financing in economic ordering policies for non-instantaneous decaying goods with price dependent demand under permissible delay in payments was discussed by Jaggi et al. (2015). Rani et al. (2018) investigated deteriorating inventory optimization for environment savvy customer with learning effect.

Shukla et al. (2013) proposed an EOQ model for deteriorating items with exponential demand rate and shortages. Agarwal and Banarjee (2010), Sarkar and Moon (2014), Khanra *et al.* (2013) and Singh et al. (2015) have studied the inventory model with partially backlogged shortages. Singhal & Singh (2015) introduced a modeling of an inventory system with multi variate demand under volume flexibility and learning. Singh and Rathore (2015) elaborated an Optimal payment policy with preservation technology investment and shortages under trade credit. A fuzzy environment inventory model with partial backlogging under learning effect was developed by Sangal et al. (2016). Agarwal et al. (2016) presented a partial backlogging inventory model for non-instantaneous decaying items under trade credit financing facility. Agarwal et al. (2016) & (2017) studied optimal policy for non-instantaneous decaying stock model with partial shortages.

It is recognized that inflation in world is a monetary situation. Inflation plays very important role in business environment. The concept of inflation is first highlighted by Buzacott (1975). Moon and Lee (2000) established an EOQ model under inflation and discounting with a random product life. Moon et al. (2005) developed a deterministic inventory lot-size models under inflation with for fluctuating demand. Evaluation of a practical inventory control and pricing policy for multi-variable demand under inflation was illustrated by Singh and Jain (2008). Chern et al. (2008) suggested an inventory model with inflation by assuming that the

demand function is fluctuating. Singh et al. (2010) derived an inventory model for deteriorating items with shortages and stock-dependent demand under inflation for two-shops under one management. Singh et al. (2013) generalized an imperfect quality products with learning and inflation under two limited storage capacity. Sarkar et al. (2014) discussed an EMQ model with price and time dependent demand under the effect of reliability and inflation. A model for weibull deteriorate items with dependent demand rate and inflation was considered by Choudhary and Sharma (2015). Green supply chain inventory model for deteriorating items with variable demand under inflation was presented by Rani et al. (2017).

To study this concept in this paper we created a decaying inventory model with price dependent demand rate and time varying holding cost. Shortages are allowed and backlogged is partial under the effect of inflation and time value of money. Solution procedure, numerical example and sensitivity analysis are used for solving this paper by using Mathematica for the feasibility and applicability of our model. Paper is illustrated analytically by minimizing total cost of the system.

2. Assumptions and Nomenclatures

2.1 Assumptions:

The following assumptions are used in the development of the model:

- The demand rate is a function of selling price.
- The inflation is also assumed.
- Linear holding cost is supposed of inventory system.
- The model allows for shortages and the demand is partially backlogged.
- Non-instantaneous deterioration is considered.
- Deterioration rate is constant and there is no replacement or repair of deteriorated units during the period under consideration.

2.2 Notations:

A list of notation is used in this paper summarized below:

- $D(p)$: price dependent demand rate i.e. $D(p) = (a - p) > 0$
- λ : rate of deterioration
- A : ordering cost per order

- C_1 : deterioration cost per unit per unit time
- C_2 : shortage cost per unit per unit time
- π : lost sales cost per unit per unit time
- β : constant function of demand backordered , $0 < \beta < 1$
- r : inflation rate
- $H(t)$: time dependent holding cost $H(t) = (h + \alpha t)$ where $h > 0$ and $\alpha > 0$
- $I_i(t)$: the inventory level at any time t , $i = 1, 2, 3$
- $TC(t_1, T)$: total cost of the model

3. Mathematical formulation of the model

Inventory system goes like this, as the deterioration is non-instantaneous so for the time period $0 \leq t \leq t_d$, no product gets deteriorated. Deterioration of inventory starts after time $t = t_d$. The inventory level reaches its maximum units of item at time $t = 0$.

During the interval $(0, t_d)$ the inventory level is depletes only due to demand rate. In the time duration (t_d, t_1) due to combined impact of demand and deterioration, stock level dropping to zero. During the shortage interval (t_1, T) , the demand is partially backordered.

The differential equation expressing in the interval $(0, t_d)$ governing the system as

$$\frac{dI_1(t)}{dt} = -(a - p) \quad 0 \leq t \leq t_d \quad (3.1)$$

With the boundary condition $I_1(0) = I_{max.}$, we get

$$I_1(t) = I_{max.} - (a - p)t \quad (3.2)$$

The differential equation expressing in the interval (t_d, t_1) governing the system as

$$\frac{dI_2(t)}{dt} + \lambda I_2(t) = -(a - p) \quad t_d \leq t \leq t_1 \quad (3.3)$$

With the boundary condition $I_2(t_1) = 0$, we obtain

$$I_2(t) = \frac{a - p}{\lambda} (e^{\lambda(t_1 - t)} - 1) \quad (3.4)$$

During the shortage interval (t_1, T) , the differential equation governing the system as

$$\frac{dI_3(t)}{dt} = -\beta(a - p), \quad t_1 \leq t \leq T \quad (3.5)$$

With the boundary condition $I_3(t_1) = 0$, we get

$$I_3(t) = \beta(a - p)(t_1 - t) \quad (3.6)$$

Now continuity at $t = t_d$ it follows from equation (3.2) & (3.4) such that

$$I_1(t_d) = I_2(t_d) = I_{max.} - (a - p)t_d = \frac{a - p}{\lambda} (e^{\lambda(t_1 - t_d)} - 1) \quad (3.7)$$

Maximum inventory level per cycle is

$$I_{max.} = (a - p) \left(t_d + \frac{1}{\lambda} (e^{\lambda(t_1 - t_d)} - 1) \right) \quad (3.8)$$

Substituting equation (3.8) into equation (3.2), it gets

$$I_1(t) = (a - p) \left(t_d + \frac{1}{\lambda} (e^{\lambda(t_1 - t_d)} - 1) - t \right) \quad (3.9)$$

Maximum amount of demand backlogged per cycle is

$$S = -I_3(T) = \beta(a - p)(T - t_1) \quad (3.10)$$

The order quantity over the replenishment cycle can be determined as

$$Q = I_{max.} + S \quad (3.11)$$

$$Q = (a - p) \left(t_d + \frac{1}{\lambda} (e^{\lambda(t_1 - t_d)} - 1) + \beta(T - t_1) \right) \quad (3.12)$$

The total inventory cost per cycle consists of the following elements

3.1 Ordering Cost

$$OC = A \quad (3.13)$$

3.2 Present worth Deterioration Cost

The total Deterioration cost for the period t_1 to t_d is

$$DC = C_1 \int_{t_d}^{t_1} \lambda e^{-rt} I_2(t) dt$$

$$DC = C_1 \lambda (a - p) \left[t_1 (t_1 - t_d) - (1 + rt_1) \frac{(t_1^2 - t_d^2)}{2} + r \frac{(t_1^3 - t_d^3)}{3} \right] \quad (3.14)$$

3.3 Present worth Shortage Cost

The total shortage cost for the period t_1 to T is

$$SC = -C_2 \int_{t_1}^T e^{-rt} I_3(t) dt$$

$$SC = -C_2 \beta (a - p) \left[t_1 (T - t_1) - (1 + rt_1) \frac{(T^2 - t_1^2)}{2} + r \frac{(T^3 - t_1^3)}{3} \right] \quad (3.15)$$

3.4 Present worth Holding Cost

The total holding cost for the period 0 to t_1 is

$$\begin{aligned}
 HC &= \left[\int_0^{t_d} (h + \alpha t).e^{-rt} I_1(t) dt + \int_{t_d}^{t_1} (h + \alpha t).e^{-rt} I_2(t) dt \right] \\
 HC &= (a - p) \left[t_1 \left\{ h t_d + (\alpha - rh) \frac{t_d^2}{2} - \alpha r \frac{t_d^3}{3} \right\} - \left\{ h \frac{t_d^2}{2} + (\alpha - rh) \frac{t_d^3}{3} - \alpha r \frac{t_d^4}{4} \right\} \right. \\
 &\quad + t_1 \left\{ h (t_1 - t_d) + (\alpha - rh) \frac{(t_1^2 - t_d^2)}{2} - \alpha r \frac{(t_1^3 - t_d^3)}{3} \right\} - \left\{ \frac{h (t_1^2 - t_d^2)}{2} - \alpha r \right. \\
 &\quad \left. \left. \frac{(t_1^4 - t_d^4)}{4} + (\alpha - rh) \frac{(t_1^3 - t_d^3)}{3} \right\} \right] \quad (3.16)
 \end{aligned}$$

3.5 Present worth Lost Sales Cost

The total Lost Sales cost for the period t_1 to T is

$$\begin{aligned}
 LSC &= \pi \int_{t_1}^T (a - p).e^{-rt}.(1 - \beta) dt \\
 LSC &= \left[\pi(a - p)(1 - \beta) \left\{ (T - t_1) + \frac{r}{2} (t_1^2 - T^2) \right\} \right] \quad (3.17)
 \end{aligned}$$

3.6 The total cost per unit time is

$$TC(t_1, T) = \frac{1}{T} [DC + SC + OC + HC + LSC] \quad (3.18)$$

$$TC(t_1, T) = \frac{1}{T} \left[C_1 \lambda (a - p) \left(t_1 (t_1 - t_d) - (1 + r t_1) \frac{(t_1^2 - t_d^2)}{2} + r \frac{(t_1^3 - t_d^3)}{3} \right) - c_2 \beta (a - p) \right]$$

$$\begin{aligned}
& \left(t_1 (T - t_1) - (1 + rt_1) \frac{(T^2 - t_1^2)}{2} + r \frac{(T^3 - t_1^3)}{3} \right) + A + (a - p) \left\{ t_1 \left\{ ht_d + (\alpha - rh) \frac{t_d^2}{2} \right. \right. \\
& \left. \left. - \alpha r \frac{t_d^3}{3} \right\} - \left\{ h \frac{t_d^2}{2} + (\alpha - rh) \frac{t_d^3}{3} - \alpha r \frac{t_d^4}{4} \right\} + t_1 \left\{ h(t_1 - t_d) + (\alpha - rh) \frac{(t_1^2 - t_d^2)}{2} \right. \right. \\
& \left. \left. - \alpha r \frac{(t_1^3 - t_d^3)}{3} \right\} - \left\{ \frac{h(t_1^2 - t_d^2)}{2} - \alpha r \frac{(t_1^4 - t_d^4)}{4} + (\alpha - rh) \frac{(t_1^3 - t_d^3)}{3} \right\} \right\} + \pi(a - p)(1 - \beta) \\
& \left. \left\{ (T - t_1) + \frac{r}{2}(t_1^2 - T^2) \right\} \right] \tag{3.19}
\end{aligned}$$

Our objective to minimize the total cost function $TC(t_1, T)$. The necessary conditions for minimizing the total cost are

$$\frac{\partial TC(t_1, T)}{\partial t_1} = 0 \text{ \& } \frac{\partial TC(t_1, T)}{\partial T} = 0 \tag{3.20}$$

Provided it satisfies the condition

$$\frac{\partial^2 TC(t_1, T)}{\partial t_1^2} > 0, \frac{\partial^2 TC(t_1, T)}{\partial T^2} > 0 \text{ \& }$$

$$\left[\frac{\partial^2 TC(t_1, T)}{\partial t_1^2} \right] \left[\frac{\partial^2 TC(t_1, T)}{\partial T^2} \right] - \left[\frac{\partial^2 TC(t_1, T)}{\partial T \partial t_1} \right]^2 > 0 \tag{3.21}$$

Using the software Mathematica-5.2, from these equations we can determine the optimum values of t_1 and T and total cost.

4. Numerical Illustration

In this section, for the exemplify of model, we consider following inventory system in which values of different parameters are as follows

$$C_1 = 2, C_2 = 1.4, A = 50, \lambda = 0.01, a = 200, r = 0.05,$$

$$h = 2, \alpha = 0.01, p = 5, t_d = 0.5, \beta = 0.01, \pi = 1$$

Then we get

$$t_1 = 1.50168, T = 5.25835 \text{ and total cost } TC(t_1, T) = \text{Rs.} 454.588.$$

5 .Sensitivity Analysis

In order to study the impact of non-deteriorating period t_d on the policy, we consider the different values of t_d and results are summarized table -1

Table-(1): Effect of non-deteriorating period (t_d) on optimal solution

t_d	t_1	T	DC	Total Cost
0.6	1.50264	5.26067	0.2884	454.52
0.7	1.50360	5.26273	0.2276	454.45
0.8	1.50455	5.26453	0.1743	454.40
0.9	1.50549	5.26607	0.1282	454.35

From table-1, it is observed that as non-instantaneous deteriorating period (t_d) increases, the deterioration cost and total cost $TC(t_1, T)$ will diminish significantly. This shows the importance of life time deterioration on the decision making of optimal policy.

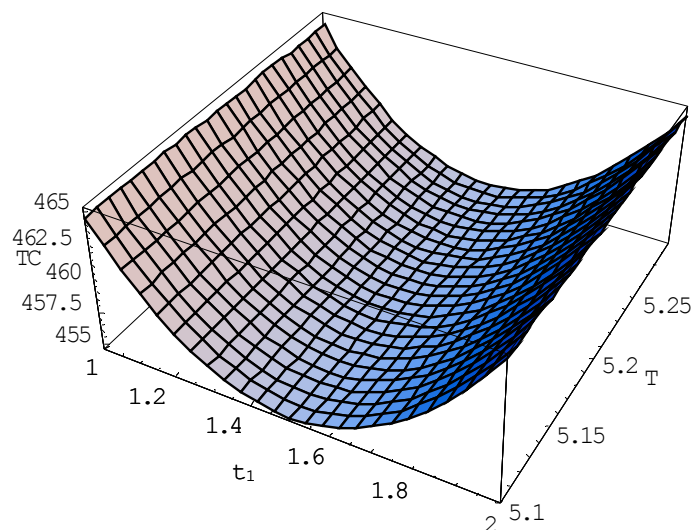


Fig.1: Graph represents the convexity of system w.r.to t_1 &

Table- (2): Sensitivity analysis with respect to diverse parameters

Changing parameter	Change in system	Change in t_1	Change in T	Change in total cost
r	0.06	1.4867	4.7429	440.252
	0.07	1.4726	4.3471	427.389
	0.08	1.4590	4.0304	415.687
	0.09	1.44585	3.7693	404.934
h	2.1	1.43371	5.1262	458.532
	2.2	1.3715	5.0021	462.230
	2.3	1.3145	4.8853	465.706
	2.4	1.2621	4.7751	468.983
α	0.02	1.4964	5.2517	454.789
	0.03	1.4913	5.2452	454.988
	0.04	1.4862	5.2387	455.184
	0.05	1.4812	5.2323	455.380
a	210	1.5017	5.2744	477.414
	220	1.5018	5.2889	500.240
	230	1.5019	5.3021	523.068
	240	1.5020	5.3141	545.897
C_1	3	1.4970	5.2525	454.766
	4	1.4924	5.2467	454.942
	5	1.4878	5.2409	455.117
	6	1.4833	5.2352	455.290
C_2	1.8	1.5080	5.3041	455.430
	2.2	1.5145	5.3508	456.279
	2.6	1.5211	5.3982	457.138
	3.0	1.5278	5.4464	458.006

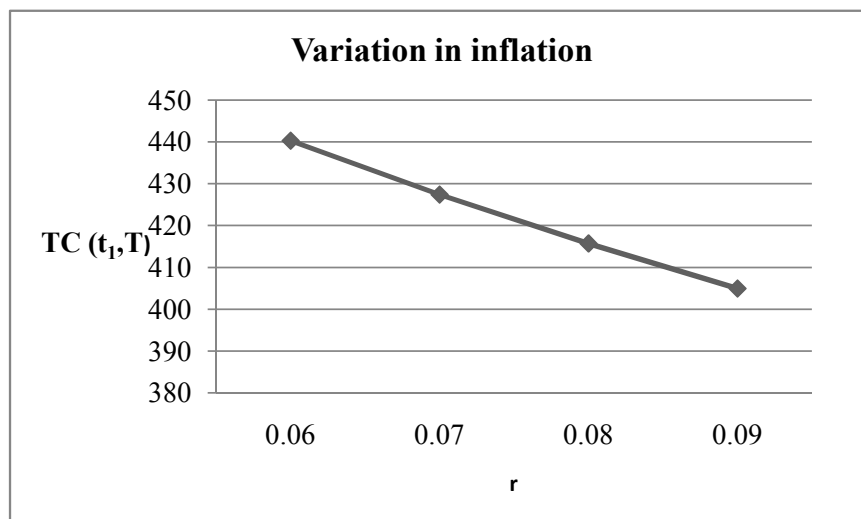


Fig. 2: Sensitivity graph between inflation 'r' and total cost

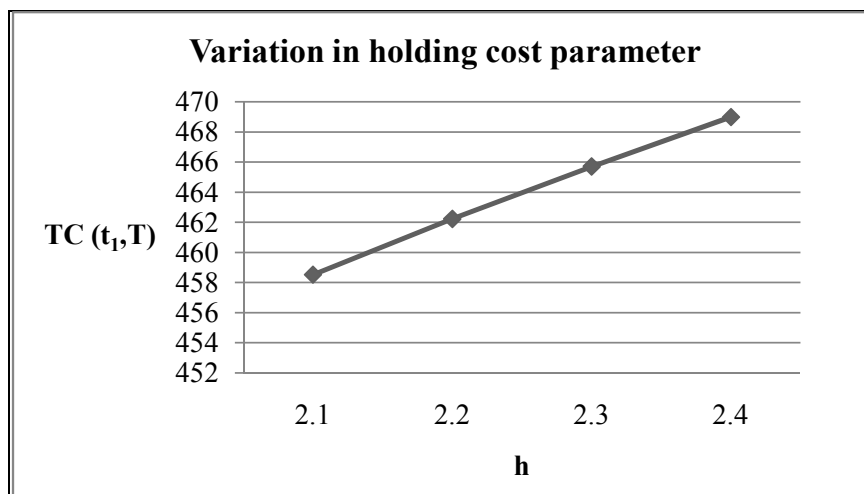


Fig.3: Sensitivity graph between holding cost 'h' and total cost

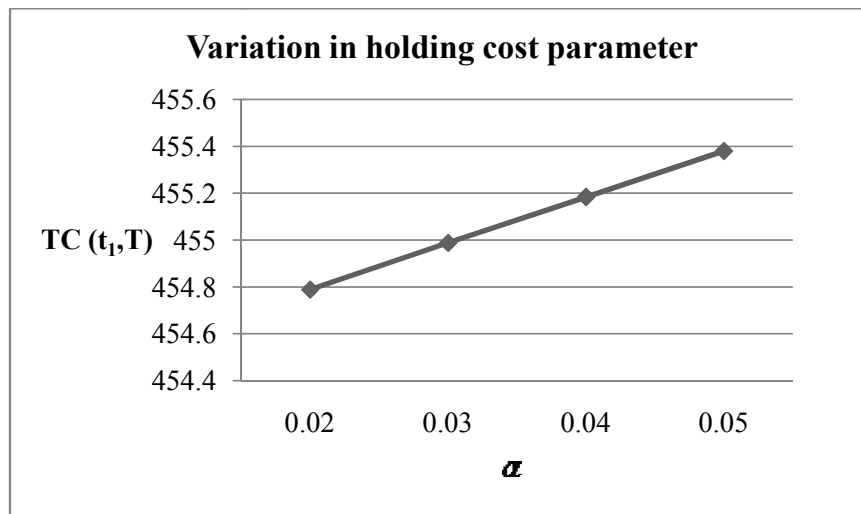


Fig.4: Sensitivity graph between holding cost ' α ' and total cost

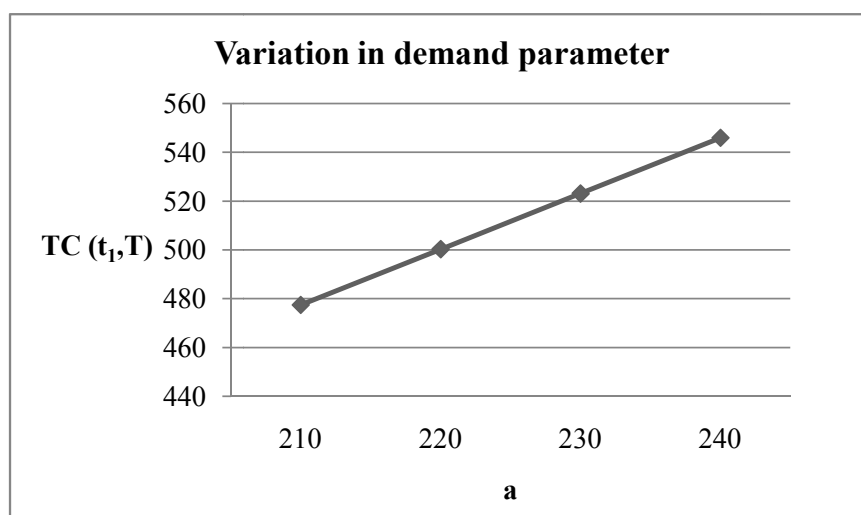


Fig.5: Sensitivity graph between demand parameter ' a ' and total cost

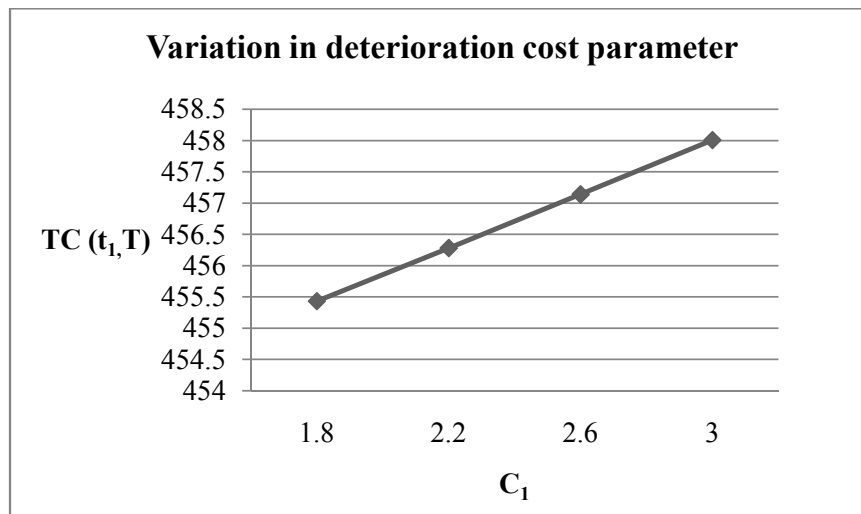


Fig.6: Sensitivity graph between deterioration cost ' C_1 ' and total cost

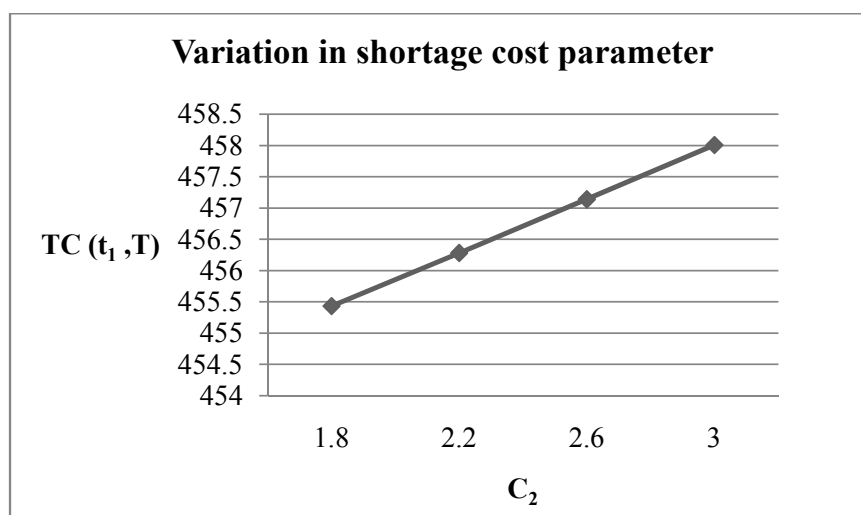


Fig.7: Sensitivity graph between shortage cost ' C_2 ' and total cost

From table-2

- It has observed that both total optimal cost and total cycle time are reducing as inflation rate 'r' (inflation decreases) gradually increases, due to an imbalance between demand and supply of production, distribution cost, raise in taxes etc.

- We investigate its effect on holding cost parameters and the total cost. Figure 3 and 4, It is shown that holding cost increases as the value of holding cost parameters ' h ' & ' α ' increases, consequently the total optimal cost increases and the total cycle length reduces.
- We can see from the table that as the demand parameter ' a ' increases then the total optimal cost increases because stockholders need more commodity to fulfill end customers requirement, but supply is limited by the manufacturer because of that there is a loss in sale, consequently overall optimal cost will be higher.
- It has shown that as deterioration cost parameter ' C_1 ' increases, results enhance in the total optimal cost, but total cycle time reduces because more the destroy or damage of artifacts, results more the optimum cost of the model.
- The total cycle time and total optimal cost increases as there is an increment in shortage cost parameter ' C_2 '. That means there is an excess in demand for commodity but due to lack of availability of artifacts then customers demand will not fulfill their requirements, results higher is the optimal cost.

6. Conclusion and Future Research Direction

In this paper we developed an inventory model when the demand rate is price dependent and inventory is deteriorating in nature. This assumption is more realistic in the market. Companies reduce price for a product quite often and we have seen that it has a high influence on demand and deterioration of products. It is not only effect on fresh fruits, vegetables etc. which is quite visible but also it impacts on various products such as electronic items, perfumes, medicines etc. which gets mostly ignored. We also consider inflation & shortages with partial backlogging and the set up has been chosen to be more realistic & unique in nature.

A numerical study has been shown to elucidate the system. Sensitivity analyses with respect to various parameters are also carried out to obtain a lot of managerial insights. The results of the chapter is not only providing a worthwhile

reference for decision makers in planning and curbing the stock but also offer a useful model for many organizations that use the decision rule to rectify their total cost.

This model is very utile in pragmatically life. There are many bright areas in which direction this model can be prolonged. The proposed model can be elongated such as trade credit, fuzzy optimization technique and learning effect etc.

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