

AN EPQ INVENTORY MODEL FOR DECAING ITEMS WITH TIME VARYING DEMAND UNDER THE TRADE CREDIT POLICY

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Abstract : This study develops a production inventory model with time dependent demand under the effect of permissible delay. Production rate is demand dependent. The concept of non-instantaneous deterioration is taken into consideration as in some realistic cases deterioration of the product starts after some time. Numerous possible cases on the bases of the positions of trade-credit period and starting point of deterioration have been enlightened. Numerical examples are given to illustrate the proposed model in all cases. The sensitivity analysis is also performed and convexity of the average total cost function for each case is revealed graphically.

1 Introduction

The model incorporating the facility of permissible delay in payment was first discussed by **Haley and Higgins (1973)**[1]. **Goyal (1985)**[2] explored a single item EOQ model under permissible delay in payments. In real life situations, there are products like volatile liquids, medicines, and materials, etc. in which the rate of deterioration is very large. Therefore, the loss due to deterioration should not be ignored. **Aggarwal and Jaggi (1995)**[4] extended **Goyal's (1985)** model to allow for deteriorating items. Shortages are of great importance especially in a model that considers a delay in payment due to the fact that shortages can affect the quantity ordered to benefit from the delay in payment. **Jamal et al. (1997)** [5] generalized the model of **Aggarwal and Jaggi (1995)** to allow for shortages and make it more applicable in the real world. In the above cited references, all the models were developed for a single warehouse and assumed that the available warehouse had unlimited capacity. However, this assumption is debatable in real life situations. A common practical situation is that of a limited storage space, whereas the extra storage capacity

can be acquired in the form of rented warehouse if the retailer's existing storage capacity is insufficient to store the ordered quantity. Motivated by the trade credit policy given by supplier, first time **Shah and Shah (1992)** [3] developed a deterministic inventory model under the permissible delay in payment with two storage facilities. In that model, the items were considered non-deteriorating without shortage and the time horizon was infinite. **Chung and Huang (2004)** [6] extended **Goyal's (1985)** model by considering limited storage capacity of the retailer in which the items were considered non-deteriorating. In that model, shortage was not allowed and the time horizon was infinite with constant demand rate. **Ouyang et al. (2006)** [7] developed an inventory model for deteriorating items with permissible delay in payments. The purpose of that study was to find an optimal replenishment policy for minimizing the total relevant inventory cost. **Chung and Huang (2007)** [8] presented a two-warehouse inventory model for deteriorating items under trade credit financing. In that model, shortages were not allowed and time horizon was infinite. The rate of deterioration in both warehouses was considered the same. **Singh et al. (2008)** [9] presented a two-warehouse inventory model for deteriorating items with constant demand rate where shortages were allowed and partially backlogged. **Singh and Jain (2009)** [10] proposed a deterministic inventory model with time varying deterioration rate and a linear trend in demand over a finite planning horizon. **Geetha and Uthayakumar (2010)** [11] developed an Economic Order Quantity (EOQ) model for deteriorating items with permissible delay in payments and single storage facility. In that model, shortages were partially backlogged. **Ouyang et al. (2015)** [13] proposed an integrated warehouse inventory model with capacity constraint and a permissible delay payment period

that is order-size dependent. **Tiwari et al. (2016)** [14] develops a two warehouse inventory model for non-instantaneous deteriorating items with permissible delay in payments under inflationary conditions. Shortages are allowed and partially backlogged, since customers' willingness to wait decreases over time. **Jaggi et al. (2016)** developed the inventory model with shortages under permissible delay in payments. **Vandana and Sharma (2016)** developed an inventory model for retailer's partial permissible delay-in-payment linked to order quantity with shortage, which is partial backlogged. Here, we consider two different cases, *i.e.* in first, the trade-credit period (M) is greater than or equal to the time interval, that units are depleted to zero due to demand; and later, the trade-credit period is less than the time interval. **Pramanik et al. (2017)** [16] developed an integrated supply chain model under three level trade credit policy with price, credit period and credit amount dependent demand, where a supplier offers a credit period to his/her wholesaler to boost the demand of the item.

In this study, a production inventory model is proposed with trade-credit facility. The demand rate of the item is considered time dependent, and the production rate is assumed variable. The concept of non-instantaneous deterioration is taken into consideration as in some realistic cases deterioration of the product starts after some time. Numerous possible cases on the bases of the positions of trade-credit period and starting point of deterioration have been enlightened. Numerical examples are given to illustrate the proposed model in all cases. The sensitivity analysis is also performed and convexity of the average total cost function for each case is revealed graphically.

2. Notations and Assumptions

2.1. Notations

The following notation is used throughout the model:

- $I(t)$ Inventory level at any time $t, t \geq 0$;
 I_m Maximum inventory level at the time t_1 (units);
 A Setup/ordering cost per cycle;

- $P(t)$ Production rate;
 t_d Time point at which deterioration starts;
 t_1 Time point at which the production stopped;
 t_2 Production cycle time;
 M Delay period provided by the supplier to the manufacturer;
 C_p Cost per item for regular production (\$);
 C_0 Production cost per item for an item produced during overtime (\$);
 C_h Holding cost per unit per unit time (\$);
 C_d Deterioration cost per unit per unit time (\$);
 S Per unit selling price, $S > C_p$ (\$);
 I_e Interest earned per unit per unit time (\$);
 I_c Interest charged per unit per unit time (\$);

2.2. Assumptions

In developing the mathematical model of the inventory system, the following assumptions are made:

1. The inventory system involves only one item.
2. $d(t) = \theta t$ be the deterioration rate of the on-hand inventory at any time t , where $0 < \theta < 1$. The deterioration starts at time $t = t_d$.
3. The deteriorated units are neither repaired nor replaced during the time period t_2 .
4. The demand rate $D(t)$ is time dependent and is defined as $D(t) = \alpha + \beta t$, where parameter $\alpha > 0, \beta \geq 0$.
5. The production rate $P(t)$ is dependent on the demand rate, *i.e.*, $P(t) = KD(t)$ where K is a constant.

3. Model Development

In the model, there are two approaches in the formulation of a production inventory system for decaying item. First, the production stopping time point is lesser than the life time of produced item and second, the production stopping time point is greater than the life time of item. To study these two occurrences, we assumed to case (1) for ($t_d > t_1$) and (2) for ($t_1 > t_d$).

3.1 Suppose ($t_d > t_1$)

In Case 1, the inventory system during a given cycle is depicted in Fig. 1. It is assumed that the initial stock is zero and production starts at $t=0$. During the interval $0 \leq t \leq t_1$, items are produced with a production rate $P(t)$ per unit time and there

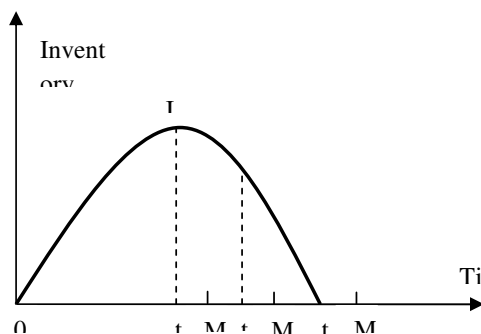


Fig.1: Inventory system when ($t_d > t_1$).

Following the earlier assumptions, the differential equations describing the inventory level $I(t)$ at time t ($0 \leq t \leq t_2$) of this system over the production period are given by

$$\frac{dI_1(t)}{dt} = P(t) - D(t); \quad 0 \leq t \leq t_1 \quad (3.1)$$

$$\frac{dI_2(t)}{dt} = -D(t); \quad t_1 \leq t \leq t_d \quad (3.2)$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -D(t); \quad t_d \leq t \leq t_2 \quad (3.3)$$

with boundary conditions $I_1(0) = 0$ and $I_3(t_2) = 0$.

The solutions of the above mentioned differential equations are given by

$$I_1(t) = (K-1) \left(\alpha t + \frac{\beta t^2}{2} \right) \quad 0 \leq t \leq t_1 \quad (3.4)$$

$$I_2(t) = I_m - \alpha(t-t_1) - \beta \left(\frac{t^2}{2} - \frac{t_1^2}{2} \right)$$

$$t_1 \leq t \leq t_d \quad (3.5)$$

$$I_3(t) = e^{-\frac{\theta t}{2}} \left[\alpha(t_2 - t) + \beta \left(\frac{t_2^2}{2} - \frac{t^2}{2} \right) + \alpha \theta \left(\frac{t_2^3}{6} - \frac{t^3}{6} \right) + \theta \beta \left(\frac{t_2^4}{8} - \frac{t^4}{8} \right) \right]$$

are no deteriorated items during this period. At the time $t = t_1$, stock level reaches to the highest level I_m and then production is stopped. Depletion is taken place by only demand during the period $t_1 \leq t \leq t_d$ and by demand and deterioration both during the period $t_d \leq t \leq t_2$ and at $t = t_2$ the inventory becomes zero.

Now, the setup cost is given by

$$SC_1 = A$$

The holding cost is given by

$$HC_1 = C_h \left[\int_0^{t_1} I_1(t) dt + \int_{t_1}^{t_d} I_2(t) dt + \int_{t_d}^{t_2} I_3(t) dt \right]$$

$$= C_h \left[(K-1) \left(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{6} \right) + I_m(t_d - t_1) - \alpha \left(\frac{t_d^2}{2} + \frac{t_1^2}{2} - t_1 t_d \right) - \frac{\beta}{2} \left(\frac{t_d^3}{3} + \frac{2t_1^3}{3} - t_1^2 t_d \right) + \alpha \left(\frac{t_d^2}{2} + \frac{t_1^2}{2} - t_1 t_d \right) + \frac{\beta}{2} \left(\frac{t_d^3}{3} + \frac{2t_1^3}{3} - t_1^2 t_d \right) + \frac{\alpha \theta}{6} \left(\frac{t_d^4}{4} + \frac{3t_1^4}{4} - t_1^3 t_d \right) + \frac{\beta \theta}{8} \left(\frac{t_d^5}{5} + \frac{4t_1^5}{5} - t_1^4 t_d \right) \right]$$

The deterioration cost by neglecting the higher degree terms of θ is given by

$$DC_1 = C_d \int_{t_d}^{t_2} \theta I_3(t) dt$$

$$= \theta C_d \left[\alpha \left(\frac{t_2^5}{6} + \frac{t_d^5}{3} - \frac{t_1 t_2^2}{2} - \frac{t_1 t_d^2}{2} - \frac{\theta t_2^5}{40} - \frac{\theta t_d^5}{10} + \frac{\theta t_1 t_2^4}{8} \right) + \frac{\beta}{2} \left(\frac{t_2^6}{2} + \frac{t_d^6}{4} - \frac{t_1^2 t_2^2}{2} - \frac{\theta t_2^6}{24} - \frac{\theta t_d^6}{12} + \frac{\theta t_1^2 t_2^4}{8} \right) + \frac{\alpha \theta}{6} \left(\frac{3t_2^6}{10} + \frac{t_d^6}{5} - \frac{t_1^3 t_2^2}{2} - \frac{3\theta t_2^7}{14} - \frac{\theta t_d^7}{14} + \frac{\theta t_1^3 t_2^4}{8} \right) + \frac{\beta \theta}{8} \left(\frac{t_2^8}{3} + \frac{t_d^8}{6} - \frac{t_1^4 t_2^2}{2} - \frac{\theta t_2^8}{16} - \frac{\theta t_d^8}{16} + \frac{\theta t_1^4 t_2^4}{8} \right) \right]$$

The production cost is given by

$$PC_1 = C_p \left(\int_0^{t_1} \alpha dt \right) + C_0 \int_0^{t_1} bK(\alpha + \beta t) dt$$

$$= C_p \alpha t_1 + C_0 bK \left(\alpha t_1 + \frac{\beta t_1^2}{2} \right)$$

Now, permissible delay period M is assumed greater than the production run time t_1 . So, in Case 1 there may arise three cases according to the position of delay period M such as case (i) $t_1 \leq M < t_d < t_2$, case (ii) $t_1 < t_d \leq M < t_2$ and case (iii) $M > t_2$. Now, we will discuss these cases separately one by one.

3.1.1 Special Case (i): When $t_1 \leq M < t_d < t_2$

In this case, interest earned is given by

$$IE_{11} = SI_e \int_0^M (\alpha + \beta t) t dt$$

$$= SI_e \left[\frac{\alpha M^2}{2} + \frac{\beta M^3}{3} \right] \leq t_2 \quad (3.6)$$

Interest charged is given by

$$IC_{11} = C_p I_c \int_M^{t_2} (\alpha + \beta t) dt$$

$$= C_p I_c \left[\frac{\alpha}{2} (t_2^2 - M^2) + \frac{\beta}{3} (t_2^3 - M^3) \right]$$

Now, the average total cost per unit time of inventory system in special case (i) of case 1 is given by

$$ATC_{11}(t_2) = \frac{1}{T} [SC_1 + HC_1 + DC_1 + PC_1 + IC_{11} - IE_{11}] \quad (3.14)$$

3.1.1.1 Solution procedure

The objective of this study is to minimize the average total cost per unit time. The optimal value of t_2 can be determined by solving the equation $dATC_{11}(t_2)/dt_2 = 0$ with the help of software Mathematica 8.0 and the minimum average total cost can be obtained from (3.14).

3.1.2 Special Case (ii): When $t_1 < t_d \leq M < t_2$

In this case, interest earned, interest charged are similar to the case (i).

Now, the average total cost per unit time of inventory system in special case (ii) is given by

$$ATC_{12}(t_2) = \frac{1}{T} [SC_1 + HC_1 + DC_1 + PC_1 + IC_{11} - IE_{11}] \quad (3.15)$$

Solution procedure given in case (i) can be used to find the optimal solution.

3.1.3 Special Case (iii): When $M > t_2$

In this case, interest earned is given by

$$IE_{12} = SI_e \left[\int_0^{t_2} (\alpha + \beta t) dt + (M - t_2) \int_0^{t_2} (\alpha + \beta t) dt \right]$$

$$= SI_e \left[M \left(\alpha t_2 + \frac{\beta t_2^2}{2} \right) - \left(\frac{\alpha t_2^2}{2} + \frac{\beta t_2^3}{6} \right) \right]$$

There is no interest charged for this case, so we have

$$IC_{12} = 0$$

Now, the average total cost per unit time of inventory system in special case (iii) of case 1 is given by

$$ATC_{13}(t_2) = \frac{1}{T} [SC_1 + HC_1 + DC_1 + PC_1 + IC_{12} - IE_{12}] \quad (3.16)$$

4. Numerical Examples

For illustration of all special cases of the two Cases discussed above in developed model, some numerical examples are presented. The figures have been taken randomly from literatures in appropriate units.

Example 1: The input values of the system parameters are taken randomly as follows:

$$A=100, t_d=0.30, \theta=0.06, M=0.25, \alpha=1000, \beta=0.2, C_p=8, C_h=1.8, C_d=6, S=10, I_e=0.08, I_c=0.09$$

Then, the optimal solution is: $t_1^* = 0.253, t_2^* = 0.384, ATC_{11}^*(t_2^*) = 468.868$ and the convexity of the average total cost function is shown graphically in Fig. 3.

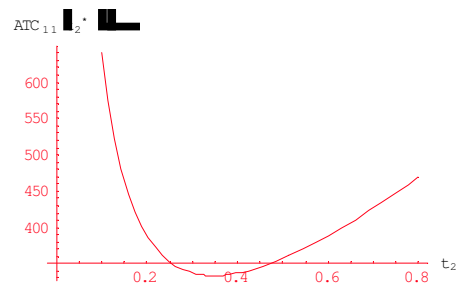


Fig. 2: Convexity of the average total cost function

5. Sensitivity Analysis:

When $(t_1 < t_d)$, to discuss the effect of changes of the model parameters $S, t_d, \alpha, \beta, C_h$ and C_p on the optimal value of the average total cost $ATC_{11}(t_2^*)$, the production stopping time point (t_1^*) , and the cycle time (t_2^*) we have discussed the sensitivity analysis in this section. The different values of these parameter according to $\pm 5\%$ and $\pm 10\%$ change have been taken and the effect on $ATC_{11}(t_2^*), t_1^*$, and t_2^* are presented in the following Table 1. In the similar manner sensitivity analysis can be performed for other cases.

$$(3.18)$$

Table 1: Sensitivity Analysis

Parameters	% Change in Parameter	%Change in the value of		
		t_1^*	t_2^*	$ATC_{11}(t_2^*)$
S	-20%	+0.37	+0.89	-4.81
	-10%	+0.19	+0.58	-2.31
	10%	-0.18	-0.55	+2.20
	20%	-0.38	-0.83	+4.73
t_d	-20%	+4.12	+0.10	-0.46
	-10%	+2.91	+0.08	-0.24
	10%	-3.13	-0.05	+0.27
	20%	-5.24	-0.07	+0.43
α	-20%	+2.62	+0.15	+9.40
	-10%	+1.23	+0.07	+4.69
	10%	-1.53	-0.08	-4.51
	20%	-3.14	-0.16	-9.29
β	-20%	+0.18	-0.08	+0.86
	-10%	+0.11	-0.05	+0.22
	10%	-0.09	+0.05	-0.20
	20%	-0.13	+0.07	-0.83
C_h	-20%	+0.34	+0.52	+1.52
	-10%	+0.23	+0.40	+0.78
	10%	-0.22	-0.41	-0.92
	20%	-0.35	-0.53	-1.58
C_p	-20%	-0.07	-1.34	-4.67
	-10%	-0.04	-0.61	-2.27
	10%	+0.05	+0.72	+2.29
	20%	+0.08	+1.29	+4.71

Some important observations drawn from Table 1 are given as follows:

1. It is observed that from Table 1 as the selling price 'S' increases the average total cost per unit time decreases, because increase in selling price increase the sales revenue and interest earned so the on the whole cost of the system decreases.
2. It is observed from Table 1 that as ' t_d ' increases the average total cost of the system slightly decreases which is evident.
3. It is observed from Table 1 as the demand factors ' α ' and ' β ' increases the average total cost per unit time increases. The reason behind that more demand implies more production so the total average cost increases.
4. As the holding cost parameter ' C_h ' and regular production parameter ' C_p ' increase the average total cost of the system increases which is realistic.

6. Conclusion

In this article, a production inventory model is developed in which production rate is taken as demand dependent. It is assumed that demand rate is a linearly increasing function of time and after a influenced period deterioration occurs with a variable rate. In this model we assumed the concept of permissible delay to make the model more practical. Different cases of trade-credit have been discussed and illustrated with some numerical examples. Sensitivity analysis is also implemented and obtained results shows that the model is quite stable and applicable in real market.

A further research can be done by allowing the model for progressive permissible delay in payment.

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