International Journal of Pure and Applied Mathematics
Volume 118 No. 20 2018, 2585-2593
ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

Some New Results of Triple Sequences Spaces Defined by Double Orlicz Function

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Abstract

In this paper we introduced some new results of triple sequences spaces by using a double Orlicz function and we will examine some properties of these sequence spaces.

Keywords: Triple sequence, p-convergent, double Orlicz function.

1 . Introduction

A triple sequence (real or complex) can be defined as a function $T: N \times N \times N \to R(C)$ where N, R and C denote the sets of natural numbers, real numbers and complex numbers respectively [5] [6] .The Orlicz function has been founded by Prof. Wlayshaw Roman Orlicz from Poland and carried his name , so he was constructed the Orlicz space [2]. Some new results of triple sequences spaces would studied by double Orlicz function using a function Ψ where

$\Psi = (\Psi_1(k), \Psi_2(t))$.

Let $(k,t) = (k_{h,d,b},\ t_{h,d,b})$ be a triple infinite array of elements $(k_{h,d,b},\ t_{h,d,b})$, where $k = (k_{h,d,b})$ be an infinite array of elements $k_{h,d,b}$ and $t = (t_{h,d,b})$ be an infinite array of elements $t_{h,d,b}$. Let μ^3 denote the set of all triple sequences of real or complex numbers. Lindenstrauss and Tzafriri [3] used the idea of Orlicz function to construct Orlicz sequence space T_Ψ ,for that idea, we will construct a triple sequence space as follows :

$$T_\Psi^3 = \left\{ (\mathrm{k},\mathrm{t}) \in \mu 3 \ : \sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \ \left\{ \Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right), \Psi_2\left(\frac{|t_{h,d,b}|}{\rho}\right)\right\} < \infty \text{, for some} \rho > 0 \right\} ,$$

this means $T_\Psi^3 = (3T_{\Psi_1}, 3T_{\Psi_2})$, where

$$3T_{\Psi_1} = \left\{ \mathrm{k} = (\mathrm{k}_{\mathrm{h,d,b}}) \in 3\mu \ : \sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} \Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right) < \infty \text{, for some } \rho > 0 \right\}$$

and

$3T_{\Psi_2}=\left\{t=(t_{h,d,b})\in\ 3\mu\ :\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty}\Psi_2\left(\frac{|t_{h,d,b}|}{\rho}\right)\ <\infty\text{, for some }\rho>0\right\}.$

The space T_{Ψ}^{3} with the norm

$\|(k,t)\|_{\Psi}=\inf\left\{\rho>0:\ \sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty}sup\ \left\{\Psi_1\left(\frac{|k_{h,d,b}|}{\rho}\right),\Psi_2\left(\frac{|t_{h,d,b}|}{\rho}\right)\right\}\leq 1\right\},$

where

$\|(k)\|_{\Psi_1}=\inf\left\{\rho>0:\ \sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty}\Psi_1\left(\frac{|k_{h,d,b}|}{\rho}\right)\ \leq 1\right\},$

$\|(t)\|_{\Psi_2}=\inf\left\{\rho>0:\ \sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty}\Psi_2\left(\frac{|t_{h,d,b}|}{\rho}\right)\ \leq 1\right\},$

becomes a Banach space which is called a double Orlicz of triple sequence spaces.

Also in this topic ,we introduce and examine some properties of triple sequence spaces defined by a double Orlicz function Ψ , which generalize the well- known a double Orlicz of triple sequence space T_{Ψ}^{3} and strongly summable triple sequence spaces

$[C,1,1,1,G]^{3}=[\,[C,1,1,1,G\,],[C,1,1,1,G]\,],$

$[C,1,1,1,G]_{0}^{3}=[\,[\,C,1,1,1,G]_0,[C,1,1,1,G]_0\,]$, and

$[C,1,1,1,G]_{\infty}^{3}=[\,[C,1,1,1,G]_{\infty},[\,C,1,1,1,G]_{\infty}],$

and $G=(G_{h,d,b})$ be any factorable triple sequences of strictly positive real numbers.

2 . Definition and Preliminaries

Let us define the N- function $\Psi(k,t)$ in the term of a triple sequence spaces.

Definition.2.1.

Let $=(k_{h,d,b})$, $t=(t_{h,d,b})$ be a triple sequences. A triple sequences $(k,t)=(k_{h,d,b}\ ,\ t_{h,d,b})$ be a bounded , if there exists a positive number J such that $|(k_{h,d,b},t_{h,d,b})|<J$ for all h,d,b.

By means of Pringsheim definition [1] , we will give the following definition:

Definition. 2.2.

A triple sequence $(k,t)=(k_{h,d,b}\ ,\ t_{h,d,b})$ has limit (γ_1,γ_2) in Pringsheim's sense denoted by $P-\ lim(k,t)=(\gamma_1,\gamma_2)$ where $(P-\ lim\ k=\gamma_1\ ,P-\ limt=\gamma_2\ )$ provided that given $\epsilon>0$ there exists $N\in\mathbb{N}$ such that

$|(k_{h,d,b} - \gamma_1 , t_{h,d,b} - \gamma_2)| < \epsilon$ whenever $k , t > N$ .We shall describe such an k,t more briefly as ‘’P −convergent ‘’.

Definition.2.3.[7].[8].

A double Orlicz function is a function $\Psi: [0,\infty) \times [0,\infty) \to [0,\infty) \times [0,\infty)$such that

$\Psi(k,t) = (\Psi_1(k), \Psi_2(t))$,

$\Psi_1 : [0,\infty) \to [0,\infty)$ and $\Psi_2: [0,\infty) \to [0,\infty)$ such that Ψ_1, Ψ_2 are Orlicz function which is continuous , non – decreasing, even , convex , and satisfies the following conditions :

1) $\Psi_1(0) = 0$, $\Psi_2(0) = 0 \Longrightarrow \Psi(0,0) = (\Psi_1(0) , \Psi_2(0)) = (0,0)$
2) $\Psi_1(k) > 0$, $\Psi_2(0) > 0 \Longrightarrow \Psi(k,t) = (\Psi_1(k), \Psi_2(t)) > (0,0)$

for $k > 0$, $t > 0$ we mean by $\Psi(k,t) > (0,0)$ that $\Psi_1(k) > 0$, $\Psi_2(t) > 0$

3) $\Psi_1(k) \to \infty$, $\Psi_2(t) \to \infty$ as $k, t \to \infty$ then ,

$\Psi(k,t) = (\Psi_1(k), \Psi_2(t))) \to (\infty,\infty)$ as $(k,t) \to (\infty,\infty)$

we mean by $\Psi(k,t) \to (\infty,\infty)$ that $\Psi_1(k) \to \infty$, $\Psi_2(t) \to \infty$.

Definition.2.4.

Let Ψ be a double Orlicz function and $G = (G_{h,d,b})$ be any factorable triple sequences of strictly positive real numbers .

We will define the following triple sequence spaces as follows:

$$T_\Psi^3(G) = \left\{ (\mathrm{k},\mathrm{t}) \in \mu^3 : \sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{ \left(\Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}}, \left(\Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}}|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}} \right\} < \infty , \text{ for some } \rho > 0 \right\},$$

this means, that $T_\Psi^3(G) = (3T_{\Psi_1}(G) , 3T_{\Psi_2}(G))$

$\mu^3(\Psi, G) =$

$$\left\{ (\mathrm{k},\mathrm{t}) \in \mu^3 : \mathrm{P} - \lim_{\mathrm{n,r,s}} \frac{1}{\mathrm{nrs}} \sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup \left\{ \left(\Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}} - \ell_1|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}}, \left(\Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}} - \ell_2|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}} \right\} \text{ for some } \rho > 0 \right\}$$

i.e. $\mu^3(\Psi, G) = (3\mu(\Psi_1, G), 3\mu(\Psi_2, G))$

$\mu_0^3(\Psi, G)$=

$$\left\{(\mathrm{k},\mathrm{t}) \in \mu^3 : \mathrm{P}-\lim_{\mathrm{n,r,s}} \frac{1}{\mathrm{nrs}} \sum_{h=1}^{n} \sum_{d=1}^{r} \sum_{b=1}^{s} sup \left\{ \left(\Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}}, \left(\Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}}|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}} \right\} = 0, \text{ for some } \rho > 0\right\}$$

i.e. $\mu_0^3(\Psi, G) = (3\mu_0(\Psi_1, G), 3\mu_0(\Psi_2, G))$

$\mu_\infty^3(\Psi, G)$=

$$\left\{(\mathrm{k},\mathrm{t}) \in \mu^3 : sup_{n,r,s} \frac{1}{nrs} \sum_{h=1}^{n} \sum_{d=1}^{r} \sum_{b=1}^{s} sup \left\{ \left(\Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}}, \left(\Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}}|}{\rho}\right)\right)^{\mathrm{G}_{\mathrm{h,d,b}}} \right\} < \infty, \text{ for some } \rho > 0\right\}$$

i.e. $\mu_\infty^3(\Psi, G) = (3\mu_\infty(\Psi_1, G), 3\mu_\infty(\Psi_2, G))$

For a double Orlicz function $\Psi(k,t) = (\Psi_1(k), \Psi_2(t)), B = [C,1,1,1]$ and $G_{h,d,b}$=1 , we now introduce the following triple sequence spaces :

$\mu^3(\Psi)$ =

$$\left\{(\mathrm{k},\mathrm{t}) \in \mu^3 : \mathrm{P}-\lim_{\mathrm{n,r,s}} \frac{1}{\mathrm{nrs}} \sum_{h=1}^{n} \sum_{d=1}^{r} \sum_{b=1}^{s} sup \left\{ \Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}} - \ell_1|}{\rho}\right), \Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}} - \ell_2|}{\rho}\right) \right\} = 0, \text{ for some } \rho > 0\right\}$$

i.e. $\mu^3(\Psi) = (3\mu(\Psi_1), 3\mu(\Psi_2))$

$\mu_0^3(\Psi)$=

$$\left\{(\mathrm{k},\mathrm{t}) \in \mu^3 : \mathrm{P}-\lim_{\mathrm{n,r,s}} \frac{1}{\mathrm{nrs}} \sum_{h=1}^{n} \sum_{d=1}^{r} \sum_{b=1}^{s} sup \left\{ \Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right), \Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}}|}{\rho}\right) \right\} = 0, \text{ for some } \rho > 0\right\}$$

i.e. $\mu_0^3(\Psi) = (3\mu_0(\Psi_1), 3\mu_0(\Psi_2))$

$\mu_\infty^3(\Psi)$=

$$\left\{(\mathrm{k},\mathrm{t}) \in \mu^3 : sup_{n,r,s} \frac{1}{nrs} \sum_{h=1}^{n} \sum_{d=1}^{r} \sum_{b=1}^{s} sup \left\{ \Psi_1\left(\frac{|\mathrm{k}_{\mathrm{h,d,b}}|}{\rho}\right), \Psi_2\left(\frac{|\mathrm{t}_{\mathrm{h,d,b}}|}{\rho}\right) \right\} < \infty, \text{ for some } \rho > 0\right\}$$

i.e. $\mu_\infty^3(\Psi) = (3\mu_\infty(\Psi_1), 3\mu_\infty(\Psi_2))$

when $\Psi_1(k) = k$ and $\Psi_2(t) = t$,hence $\Psi(k,t) = (\Psi_1(k), \Psi_2(t)) = (k,t)$, if we let $G_{h,d,b}=1$ for all h, d and b , then $T^3_{\Psi}(G)$, $\mu^3(\Psi, G)$,$\mu^3_0(\Psi, G)$ and $\mu^3_\infty(\Psi, G)$ reduce to T^3_{Ψ}, $\mu^3(\Psi)$, $\mu^3_0(\Psi)$ and $\mu^3_\infty(\Psi)$ respectively .

Definition.2.5.[8].

A double Orlicz function $\Psi(k,t) = (\Psi_1(k), \Psi_2(t))$ is said to satisfy Δ_2-condition for all values of k,t , if there exists aconstant $x > 0$ such that $\Psi_1(2k) \leq x\,\Psi_1(k)$ and $\Psi_2(2t) \leq x\,\Psi_2(t)$ for all $x \geq 0, t \geq 0$, then $\Psi(2k,2t) = (\Psi_1(2k), \Psi_2(2t)) \leq (x\,\Psi_1(k), x\,\Psi_2(t)) = x(\Psi_1(k), \Psi_2(t)) = x\,\Psi(k,t)$, for all $x \geq 0, t \geq 0$.

Lemma.2.1.[8].

Let $\Psi(k,t) = (\Psi_1(k), \Psi_2(t))$ be adouble Orlicz function which satisfies Δ_2-condition ,and let $0 < \delta < 1$. Then for each $k \geq \delta$,$t \geq \delta$, we have $\Psi_1(k) < xk\frac{1}{\delta}\Psi_1(2)$, $\Psi_2(k) < xt\frac{1}{\delta}\Psi_2(2)$,for some constant $x > 0$, $\Psi(k,t) = (\Psi_1(k), \Psi_2(t)) < (x\frac{1}{\delta}k\Psi_1(2), x\frac{1}{\delta}t\Psi_2(2)) = x\frac{1}{\delta}(k\Psi_1(2), t\Psi_2(2)) = x\frac{1}{\delta}(k,t)(\Psi_1(2), \Psi_2(2)) = x\frac{1}{\delta}(k,t)\Psi(2,2)$, for some constant x.

3.Main result

Theorem.3.1.

Let $H = sup_{h,d,b} G_{h,d,b}$, then $T^3_{\Psi}(G)$ is a linear set over the set of complex numbers$\mathbb{C}^2$.

Proof. Let $k = (k_{h,d,b})$ and $a = (a_{h,d,b}) \in 3T_{\Psi_1}(G)$, $t = (t_{h,d,b})$, $j = (j_{h,d,b}) \in 3T_{\Psi_2}(G)$

and consequently $(k,t) = (k_{h,d,b}, t_{h,d,b})$,$(a,j) = (a_{h,d,b}, j_{h,d,b})$ be elements of $T^3_{\Psi}(G)$ and let (α,α) , $(\beta, \beta) \in \mathbb{C}^2$ are complex numbers . In prove the result , we must to find some $\rho_3>0$ such that

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|\alpha k_{h,d,b}+\beta a_{h,d,b}|}{\rho_3}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|\alpha t_{h,d,b}+\beta j_{h,d,b}|}{\rho_3}\right)\right)^{G_{h,d,b}}\right\} < \infty .$$

Since (k,t) , (a,j) are in $T^3_{\Psi}(G)$,) , $\Psi(k,t) = (\Psi_1(k), \Psi_2(t)$ be a double Orlicz function, then there exist some positive $\rho_1$and ρ_2 such that

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|k_{h,d,b}|}{\rho_1}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|t_{h,d,b}|}{\rho_1}\right)\right)^{G_{h,d,b}}\right\} < \infty$$

and

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|a_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|j_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}\right\} < \infty$$

In similar case of Parashar and Choudhary [4] ,we set ρ_3=max $\{3|\alpha|\rho_1, 3|\beta|\rho_2\}$. Since Ψ_1, Ψ_2 *and* Ψ are non – decreasing and convex ,then

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|\alpha k_{h,d,b}+\beta a_{h,d,b}|}{\rho_3}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|\alpha t_{h,d,b}+\beta j_{h,d,b}|}{\rho_3}\right)\right)^{G_{h,d,b}}\right\} \leq$$

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|\alpha k_{h,d,b}|}{\rho_3}+\frac{|\beta a_{h,d,b}|}{\rho_3}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|\alpha t_{h,d,b}|}{\rho_3}+\frac{|\beta j_{h,d,b}|}{\rho_3}\right)\right)^{G_{h,d,b}}\right\} \leq$$

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} \frac{1}{2^{G_{h,d,b}}} sup \left\{\left(\Psi_1\left(\frac{|k_{h,d,b}|}{\rho_1}\right)+\left(\frac{|a_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|t_{h,d,b}|}{\rho_1}\right)+\left(\frac{|j_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}\right\} <$$

$$\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|k_{h,d,b}|}{\rho_1}\right)+\left(\frac{|a_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|t_{h,d,b}|}{\rho_1}\right)+\left(\frac{|j_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}\right\} \leq$$

$$Y\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|k_{h,d,b}|}{\rho_1}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|t_{h,d,b}|}{\rho_1}\right)\right)^{G_{h,d,b}}\right\} +$$

$$Y\sum_{h=1}^{\infty}\sum_{d=1}^{\infty}\sum_{b=1}^{\infty} sup \left\{\left(\Psi_1\left(\frac{|a_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}, \left(\Psi_2\left(\frac{|j_{h,d,b}|}{\rho_2}\right)\right)^{G_{h,d,b}}\right\} \leq \infty,$$

where $Y = max\{1, 2^{H-1}\}$ .Thus T_Ψ^3(G) is a linear space.■

Theorem.3.2.

For any double Orlicz function Ψ which satisfies Δ_2-condition the following are satisfied ,

1) $[C,1,1,1]^3 \subset \mu^3(\Psi)$.
2) $[C,1,1,1]_0^3 \subset \mu_0^3(\Psi)$, *and*
3) $[C,1,1,1]_\infty^3 \subset \mu_\infty^3(\Psi)$.

Proof. Let $k = (k_{h,d,b})$, $t = (t_{h,d,b}) \in$ [C,1,1,1],hence (k,t) =$(k_{h,d,b}, t_{h,d,b}) \in$ [C,1,1,1]^3 , thus

$$E_{n,r,s} = \frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s}\left|(k_{h,d,b-\ell_1}, t_{h,d,b-\ell_2})\right| \to 0 \; as \; n,r,s \to \infty$$

Let $y_{h,d,b} = \left|(k_{h,d,b-\ell_1}, t_{h,d,b-\ell_2})\right|$.

Let $\epsilon > 0$ and choose δ with $0 < \delta < 1$ such that $\Psi_1(z_1) < \epsilon$ for $0 \leq z_1 \leq \delta$, and $\Psi_2(z_2) < \epsilon$ for $0 \leq z_2 \leq \delta$,hence ,

$\Psi(z_1, z_2) = (\Psi_1(z_1), \Psi_2(z_2)) < (\epsilon, \epsilon)$for $(0,0) \leq (z_1, z_2) \leq (\delta, \delta)$.

Thus , by the lemma.2.1.[8], we get the following

$\frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup\{\Psi_1(|k_{h,d,b}-\ell_1|),\Psi_2(|t_{h,d,b}-\ell_2|)\}=$

$\frac{1}{nrs}\sum_{h=1\&y_{h,d,b}\leq\delta}^{n}\sum_{d=1\&y_{h,d,b}\leq\delta}^{r}\sum_{b=1\&y_{h,d,b}\leq\delta}^{s} sup\{\Psi_1(|k_{h,d,b}-\ell_1|),\Psi_2(|t_{h,d,b}-\ell_2|)\}+$

$\frac{1}{nrs}\sum_{h=1\&y_{h,d,b}>\delta}^{n}\sum_{d=1\&y_{h,d,b}>\delta}^{r}\sum_{b=1\&y_{h,d,b}>\delta}^{s} sup\{\Psi_1(|k_{h,d,b}-\ell_1|),\Psi_2(|t_{h,d,b}-\ell_2|)\} < \frac{1}{nrs}\epsilon\,(nrs) + \frac{1}{nrs}\,x\,\frac{1}{\delta}\,(\Psi_1(2)\,,\Psi_1(2))(nrs)\,E_{n,r,s}$.

Therefore ,as n, r *and* s go to infinity , in Pringsheim 's sense it follows that $(k_{h,d,b}, t_{h,d,b}) \in \mu^3(\Psi)$.

Part 2 and 3 follow similar arguments as part 1and therefore it's clear .

This completes the proof.

Theorem.3.3.

1)Let $0 < \inf G_{h,d,b} \leq G_{h,d,b} \leq 1$. Then $\mu^3(\Psi, G) \subseteq \mu^3(\Psi)$.

2)Let $1 \leq G_{h,d,b} \leq \sup G_{h,d,b} < \infty$. Then $\mu^3(\Psi) \subseteq \mu^3(\Psi, G)$.

Proof .1) Let k=$(k_{h,d,b}) \in 3\mu(\Psi_1$,G) and t =$(t_{h,d,b}) \in 3\mu(\Psi_2$,G),and consequently (k,t)=($k_{h,d,b}, t_{h,d,b}$)$\in \mu^3(\Psi, G)$, since $0 < \inf G_{h,d,b} \leq 1$,we get the following:

$\frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup\left\{\Psi_1\left(\frac{|k_{h,d,b}-\ell_1|}{\rho}\right),\Psi_2\left(\frac{|t_{h,d,b}-\ell_2|}{\rho}\right)\right\} \leq$

$\frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup\left\{\Psi_1\left(\frac{|k_{h,d,b}-\ell_1|}{\rho}\right)^{G_{h,d,b}},\Psi_2\left(\frac{|t_{h,d,b}-\ell_2|}{\rho}\right)^{G_{h,d,b}}\right\}$.

Thus $(k,t) \in \mu^3(\Psi)$.

2) Let $G_{h,d,b} \geq 1$, for each h,d and b, and let $\sup G_{h,d,b} < \infty$.

Let k=$(k_{h,d,b}) \in 3\mu(\Psi_1)$ and t=$(t_{h,d,b}) \in 3\mu(\Psi_2)$, hence (k,t)= ($k_{h,d,b}, t_{h,d,b}$)$\in \mu^3(\Psi)$.

Then, for each $0 < \epsilon < 1$, we have $(0,0) < (\epsilon,\epsilon) < (1,1)$, there exists appositive integer N such that

$\frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup\left\{\Psi_1\left(\frac{|k_{h,d,b}-\ell_1|}{\rho}\right),\Psi_2\left(\frac{|t_{h,d,b}-\ell_2|}{\rho}\right)\right\} \leq \epsilon < 1$, for all $n, r, s \geq N$.

This implies that

$\frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup\left\{\Psi_1\left(\frac{|k_{h,d,b}-\ell_1|}{\rho}\right)^{G_{h,d,b}},\Psi_2\left(\frac{|t_{h,d,b}-\ell_2|}{\rho}\right)^{G_{h,d,b}}\right\} \leq$

$\frac{1}{nrs}\sum_{h=1}^{n}\sum_{d=1}^{r}\sum_{b=1}^{s} sup\left\{\Psi_1\left(\frac{|k_{h,d,b}-\ell_1|}{\rho}\right),\Psi_2\left(\frac{|t_{h,d,b}-\ell_2|}{\rho}\right)\right\}$.

Therefore , $(k,t)\in\mu^3(\Psi,G)$. This completes the proof.■

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