

# NORMAL SPACE IN SOFT IDEAL TOPOLOGY

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**Abstract:** In this paper the concept of soft  $\beta$  – I – normal space is introduced and the properties of soft  $\beta$  – I – normal space is studied. Also the concept of soft  $\beta$  – I – regular space is introduced and studied their properties. 2010 AMS Classification: 54A05, 54A10

**Keywords:** Soft set, soft topological space, soft  $\beta$  open sets, soft  $\beta$  -  $\tilde{I}$ - normal space, soft  $\beta$  –  $\tilde{I}$  – regular space.

## 1. INTRODUCTION

The theory of soft set was applied successfully by Molodtsov [7] in several directions as a general tool in dealing with the vagueness. The representation of operation on soft sets [5], application and the properties of the theory of soft sets have been increased [1], [4], [6], and [9]. The soft topological spaces were introduced by Shabir and Naz[8] over the initial universe with fixed set of the parameters. The concept of open sets, closed sets, closure and interior in soft set theory were investigated by them. The concept of  $\beta$  – open soft sets were introduced by Kandil et al.[3]. He also introduced the definition of semi-regular and semi-normal spaces via ideals using soft sets. A. C. Guler and G. Kale [2] have discussed about regularity and normality on soft ideal topological spaces. In this paper we define the soft  $\beta$  – ideal normal space and soft  $\beta$  – ideal regular space study some of their properties.

## 2. PRELIMINARIES

**Definition 2.1**[8] Let  $\tilde{\tau}$  be the collection of soft sets over  $X$ , then  $\tilde{\tau}$  is said to be soft topology on  $X$  if, (1)  $\tilde{X}$  and  $\tilde{\Phi} \in \tilde{\tau}$ , where  $\tilde{X}(e) = X$  and  $\tilde{\Phi}(e) = \Phi$ , for all  $e \in E$ . (2) Union of any number of soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ . (3) Intersection of any two soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ . The triplet  $(X, \tilde{\tau}, E)$  is called soft topological space over  $X$ . The members in  $\tilde{\tau}$  are said to be soft open sets in  $X$ .

**Definition 2.2** [8] A soft subset  $F_A$  of a soft topological space  $X$  is said to be soft closed set if  $X - F_A$  is soft open.

**Definition 2.3** [2] Let  $(X, \tilde{\tau}, E)$  be soft topological space and  $(F, A) \in SS(X)_E$ . The soft closure  $(F, A)$ , denoted by  $cl(F, A)$ ;

$cl(F, A) = \tilde{\cap} \{ (H, C) : (H, C), \text{ is closed soft set and } (F, A) \subseteq (H, C) \}.$

where  $(G, B)$  is the largest open set over  $U$ .

**Definition 2.4** [2] Let  $(X, \tilde{\tau}, E)$  be a soft topological space and  $(F, A) \in SS(X)_E$ . The soft interior of  $(G, B)$ , denoted by  $int(G, B)$ ;

$\text{int}(G, B) = \bigcup \{(H, C) : (H, C), \text{ is an open soft set and } (H, C) \subseteq (G, B)\}.$   
 where  $(G, B)$  is the largest open set over  $X$ .

**Lemma 2.1** Let  $(Y, \tilde{\tau}_Y, E)$  be a soft subspace of a soft topological space  $(X, \tilde{\tau}, E)$  and  $(F, E) \in SS(X)_E$ , then,

- (1) If  $(F, E)$  is a soft open set in  $Y$  iff  $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$  for some  $(G, E) \in \tilde{\tau}$ .
- (2) If  $(F, E)$  is a soft closed set in  $Y$  iff  $(F, E) = \tilde{Y} \tilde{\cap} (H, E)$  is  $\tilde{\tau}$  – soft closed set in  $X$ .

**Proof:**

(1) Let  $(X, \tilde{\tau}, E)$  be a soft topological space, let  $(F, E) \in SS(X)_E$  and  $Y$  be a non-empty subset of  $X$ , then by the definition of soft subset  $(F, E)$  over the set  $Y$ ,  $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$  for some  $(G, E) \in \tilde{\tau}$ . Converse is obvious as  $(F, E)$  is a soft subset of  $(Y, \tilde{\tau}_Y, E)$ , therefore  $(F, E)$  is a soft open set in  $Y$ .

(2) If  $(F, E)$  is soft closed in  $Y$ , then  $(F, E) = \tilde{Y} - (H, E)$  for some  $(G, E) \in \tilde{\tau}_Y$ . Let  $(G, E) = \tilde{Y} \tilde{\cap} (H, E)$ , for some  $(H, E) \in \tilde{\tau}$ .

$$(F, E) = \tilde{Y} - (G, E) = \tilde{Y} - (\tilde{Y} \tilde{\cap} (H, E)) = \tilde{Y} - (H, E) = \tilde{Y} \tilde{\cap} (X - (H, E)) = \tilde{Y} \tilde{\cap} (\overline{H, E}).$$

$(F, E) = \tilde{Y} \tilde{\cap} (\overline{H, E})$ , where  $(\overline{H, E})$  is soft closed in  $X$  as  $(H, E) \in \tilde{\tau}$ .

Conversely, assume that  $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$ , for soft closed set  $(G, E)$  in  $X$  implies that

$(\overline{G, E}) \in \tilde{\tau}$ . If  $(G, E) = X - (H, E)$ , where  $(H, E) \in \tilde{\tau}$ , then  $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$

$$= \tilde{Y} \tilde{\cap} (X - (H, E)) = \tilde{Y} - (H, E) = \tilde{Y} - (\tilde{Y} \tilde{\cap} (H, E)). \text{ Thus } (F, E) = \tilde{Y} \tilde{\cap} (H, E). \text{ Since}$$

$(H, E) \in \tilde{\tau}$ , so  $(\tilde{Y} \tilde{\cap} (H, E)) \in \tilde{\tau}_Y$  and hence  $(F, E)$  is a soft closed set in  $Y$ .

### Definition 2.5

Let  $SS(X)_A$  and  $SS(Y)_B$  be the family of soft sets,  $u : X \rightarrow Y$  and  $p : A \rightarrow B$  be mappings.

Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a mapping, then

(i) If  $(F, A) \in SS(X)_A$ , then the image of  $(F, A)$  under  $f_{pu}$ , written as

$f_{pu}(F, A) = (f_{pu}(F), p(A))$  is a soft set in  $SS(Y)_B$  such that

$$f_{pu}(F)(b) = \begin{cases} \bigcup_{x \in p^{-1}(b) \cap A} u(F(x)), & p^{-1}(b) \cap A \neq \Phi \\ \Phi, & \text{otherwise} \end{cases}, \text{ for all } b \in B$$

(ii) If  $(G, B) \in SS(Y)_B$ , then the image of  $(G, B)$  under  $f_{pu}$ , written as

$f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$  is a soft set in  $SS(X)_A$  such that

$$f_{pu}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B \\ \Phi, & \text{otherwise} \end{cases}, \text{ for all } a \in A.$$

The soft function  $f_{pu}$  is called surjective if  $p$  and  $u$  are surjective, also is said to be injective if  $p$  and  $u$  are injective. The soft function  $f_{pu}$  is called bijective if  $p$  and  $u$  are both surjective and injective.

**Definition 2.6** [3] Let  $(X, \tilde{\tau}_1, A)$  and  $(Y, \tilde{\tau}_2, B)$  be the soft topological spaces and  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be a function, then the function  $f_{pu}$  is called

- 1) Soft continuous if  $f_{pu}^{-1}[G, B] \in \tilde{\tau}_1$ , for all  $(G, B) \in \tilde{\tau}_2$ .
- 2) Soft open mapping if  $f_{pu}(G, A) \in \tilde{\tau}_2$ ,  $(G, A) \in \tilde{\tau}_1$ .
- 3) Soft homeomorphism if its bijective, soft continuous and soft open mapping.

**Lemma 2.2** Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$  be an injective soft function, then  $f_{pu}[F_A - G_A] = f_{pu}(F_A) - f_{pu}(G_A)$ .

**Proof:**

$$\text{Let } y \in f_{pu}[F_A - G_A](b) = \begin{cases} \bigcup_{x \in p^{-1}(b) \cap A} u([F_A - G_A](x)), & p^{-1}(b) \cap A \neq \Phi \\ \Phi, & \text{otherwise} \end{cases}$$

for all  $b \in B$ . Since  $f_{pu}$  injective, then  $u$  is injective and  $y \in u[F(a) - G(a)]$ . It follows that,

$y \in u[F(a)]$  and  $y \notin u[G(a)]$ . Hence  $y \in f_{pu}[F_A(b)]$  and  $y \notin f_{pu}[G_A(b)]$ .  
 Therefore,  $y \in f_{pu}[F_A(b)] - f_{pu}[G_A(b)] = f_{pu}[F_A - G_A](b)$ . So,  $y \in f_{pu}(F_A) - f_{pu}(G_A)$ .  
 Thus  $f_{pu}[F_A - G_A] = f_{pu}(F_A) - f_{pu}(G_A)$ .

**Definition 2.7** [3] Let  $(X, \tau, E)$  be soft topological space, soft set  $(F, E)$  is said to be soft  $\beta$  open set if  $(F, E) \subseteq cl(int(cl(F, E)))$ .

**Definition 2.8** [3] Let  $(X, \tau, E)$  be soft topological space, soft set  $(\overline{F}, E)$  is said to be soft  $\beta$  closed set if  $(F, E)$  is soft  $\beta$  open.

**Definition 2.9** [3] Let non empty collection of soft sets be  $\tilde{I}$  over the universe  $X$  with the fixed set of parameters  $E$ , then  $\tilde{I} \subseteq SS(X)_E$  is called soft Ideal on  $X$  with a fixed set  $E$  if,

- (1)  $(F, E) \in \tilde{I}$  and  $(G, E) \in \tilde{I} \Rightarrow (F, E) \tilde{\cup} (G, E) \in \tilde{I}$ .
- (2)  $(F, E) \in \tilde{I}$  and  $(G, E) \subseteq (F, E) \Rightarrow (G, E) \in \tilde{I}$ .  $\tilde{I}$  is closed under finite unions and soft subsets.

**Definition 2.10** Let  $(X, \tau, E)$  be the soft topological space over  $X$  and let  $(H, E)$  be a soft closed set in  $X$  and  $x \in X$  such that  $x \notin (H, E)$ , if  $\exists$  open soft sets  $(F_1, E)$  and  $(F_2, E)$  such that  $x \in (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \tilde{\cap} (F_2, E) = \Phi$ , then  $(X, \tau, E)$  is called asoft regular space.

**Lemma 2.3** Let  $(X, \tau_1, A, \tilde{I})$  be soft topological space with soft ideal,  $(Y, \tau_2, B)$  be soft topological space and  $f_{pu} : (X, \tau_1, A, \tilde{I}) \rightarrow (Y, \tau_2, B)$  be soft function, then  $f_{pu}(\tilde{I}) = \{f_{pu}((F, A)) : (F, A) \in \tilde{I}\}$  is called soft ideal on  $Y$ .

**Proof:**

Let  $f_{pu}(F, A) \in f_{pu}(\tilde{I})$  and  $f_{pu}(G, A) \subseteq f_{pu}(F, A)$ , then  $(F, A) \in \tilde{I}$  and  $(G, A) \subseteq (F, A)$ . Since  $\tilde{I}$  is soft ideal then  $(G, A) \in \tilde{I}$ . Thus  $f_{pu}(G, A) \in f_{pu}(\tilde{I})$ . Let  $f_{pu}(F, A) \in f_{pu}(\tilde{I})$  and  $f_{pu}(G, A) \in f_{pu}(\tilde{I})$ , then  $(F, A) \in \tilde{I}$  and  $(G, A) \in \tilde{I}$ . Since  $\tilde{I}$  is soft ideal, then  $(F, A) \tilde{\cup} (G, A) \in \tilde{I}$ . Thus  $f_{pu}((F, A) \tilde{\cup} (G, A)) = f_{pu}(F, A) \tilde{\cup} f_{pu}(G, A) \in f_{pu}(\tilde{I})$ . Therefore  $f_{pu}(\tilde{I})$  is a soft ideal on  $Y$ .

**Definition 2.11** Let  $(X, \tau, E)$  be soft topological space over  $X$ ;  $(H, E)$  &  $(K, E)$  be two disjoint soft closed sets over  $X$ , if there exist soft open sets  $(F, E)$  &  $(G, E)$  such that  $(H, E) \subseteq (F, E)$ ,  $(K, E) \subseteq (G, E)$  and  $(F, E) \tilde{\cap} (G, E) = \Phi$ , then is called soft normal space.

**Example 2.1** Let  $X = \{h_1, h_2\}$ ,  $E = \{e_1, e_2\}$ ,  $\tau = \{X, \Phi, (F_1, E), (F_2, E)\}$  where  $(F_1, E), (F_2, E)$  the soft sets over  $X$  defined as follows:

$F_1(e_1) = \{h_2\}$ ;  $F_1(e_2) = \{h_1, h_2\}$ ;  $F_2(e_1) = \{h_2\}$ ;  $F_2(e_2) = \{h_1\}$ , then  $(X, \tau, E)$  is not asoft normal space.

**Definition 2.12** Let  $(X, \tau, E)$  be the soft topological space over  $X$ ;  $(F, E)$  &  $(G, E)$  be soft closed sets over  $X$  such that  $(F, E) \tilde{\cap} (G, E) = \Phi$ . If there exist soft  $\beta$  open sets  $(F_1, E)$  &  $(F_2, E)$  such that  $(F, E) \subseteq (F_1, E)$ ,  $(G, E) \subseteq (F_2, E)$  and  $(F_1, E) \tilde{\cap} (F_2, E) = \Phi$ , then is called soft  $\beta$  normal space.

**Theorem 2.4:** Let  $(X, \tau, E)$  be soft ideal topological space then the following are equivalent:

- (1)  $(X, \tau, E)$  is a soft  $\beta$  normal space
- (2) For all soft  $\beta$  – closed set  $(H, E)$  and for each neighbourhood  $(U, E)$  of  $(F, E)$  there exist soft  $\beta$  – open neighbourhood  $(V, E)$  of  $(F, E)$  such that  $S\beta cl(V, E) \subseteq (U, E)$ .

- (3) For each pair of the soft  $\beta$ -closed sets  $(H, E)$  and  $(K, E)$  there exists a soft  $\beta$ -open neighbourhood  $(U, E)$  of  $(F, E)$  such that  $S\beta cl(U, E) \tilde{\cap} (G, E) = \tilde{\Phi}$ .
- (4) For each pair of soft  $\beta$ -closed sets  $(H, E)$  and  $(K, E)$  there exists a soft  $\beta$ -open neighbourhood  $(U, E)$  of  $(H, E)$  and  $(V, E)$  of  $(K, E)$  such that  $(U, E) \tilde{\cap} (V, E) = \tilde{\Phi}$

**Proof:**

(1)  $\Rightarrow$  (2) Let  $(X, \tilde{\tau}, E)$  is a soft  $\beta$  normal space. Let  $(H, E)$  be the soft  $\beta$ -closed sets and  $(U, E)$  be any open neighbourhood of  $(H, E)$ . Let the soft  $\beta$ -closed sets be  $(H, E)$  and  $X - (U, E)$  and  $(H, E) \subseteq (U, E)$  implies that  $(H, E) \tilde{\cap} (X - (U, E)) = \tilde{\Phi}$ . Since  $X$  is soft  $\beta$  normal space there exist an soft  $\beta$ -open neighbourhood of  $(V, E)$  of  $(H, E)$  and  $(W, E)$  of  $X - (U, E)$  such that  $(V, E) \tilde{\cap} (W, E) = \tilde{\Phi}$ . Now  $(V, E) \tilde{\cap} (W, E) = \tilde{\Phi} \Rightarrow (V, E) \subseteq X - (W, E)$ . Since  $X - (W, E)$  is soft  $\beta$ -closed,  $S\beta cl(V, E) \subseteq S\beta cl(X - (W, E)) = X - (W, E)$ . Therefore  $S\beta cl(V, E) \tilde{\cap} (W, E) = \tilde{\Phi}$  and  $S\beta cl(V, E) \tilde{\cap} X - (U, E) \subseteq S\beta cl(V, E) \tilde{\cap} (W, E) = \tilde{\Phi}$ , thus  $S\beta cl(V, E) \subseteq (U, E)$ .

(2)  $\Rightarrow$  (3) Let  $(H, E)$  and  $(K, E)$  be disjoint soft  $\beta$ -closed sets. Since  $(H, E) \tilde{\cap} (K, E) = \tilde{\Phi}$  and  $X - (K, E)$  soft  $\beta$ -open, we have  $(F, E) \subseteq X - (K, E)$ . Hence  $X - (K, E)$  is a soft  $\beta$ -open neighbourhood of the  $(U, E)$  of  $(H, E)$  such that  $S\beta cl(U, E) \subseteq X - (K, E)$ . Therefore  $S\beta cl(U, E) \tilde{\cap} (K, E) = \tilde{\Phi}$ .

(3)  $\Rightarrow$  (4) Let  $(H, E)$  and  $(K, E)$  be the disjoint soft  $\beta$ -closed sets. By (3) there exist an soft  $\beta$ -open neighbourhood of  $(U, E)$  of  $(H, E)$  such that  $S\beta cl(U, E) \tilde{\cap} (K, E) = \tilde{\Phi}$ . Now  $(K, E)$  and  $S\beta cl(U, E) \tilde{\cap} (K, E) = \tilde{\Phi}$ . Now  $(K, E)$  and  $S\beta cl(U, E)$  are disjoint soft  $\beta$ -closed sets. Hence by (3) there exist a soft  $\beta$ -open neighbourhood of  $(V, E)$  of  $(K, E)$  such that  $S\beta cl(V, E) \tilde{\cap} (K, E) = \tilde{\Phi}$ .

(4)  $\Rightarrow$  (1) Let  $(H, E)$  and  $(K, E)$  be disjoint soft  $\beta$ -closed sets. Then by (4) there exist soft  $\beta$ -open neighbourhood of  $(U, E)$  of  $(H, E)$  and  $(V, E)$  of  $(K, E)$  such that  $S\beta cl(V, E) \tilde{\cap} S\beta cl(U, E) = \tilde{\Phi}$ . Since  $\tilde{\Phi} \subseteq (U, E) \tilde{\cap} (V, E) = \tilde{\Phi}$ . Hence  $(X, \tilde{\tau}, E)$  is a soft  $\beta$  normal space.

**3. SOFT B – I – NORMAL SPACE VIA SOFT IDEALS**

**Definition 3.1** Let  $(X, \tilde{\tau}, E, \tilde{I})$  be soft ideal topological space and  $(H, E), (K, E)$  be disjoint  $\beta$  closed sets over  $X$ , then  $(U, \tilde{\tau}, E, \tilde{I})$  is called a soft  $\beta$ - $\tilde{I}$ -normal space if there exist soft open sets  $(F, E)$  and  $(G, E)$  such that  $(H, E) - (G, E) \in \tilde{I}$  and  $(K, E) - (F, E) \in \tilde{I}$ .

**Example 3.1** Let  $X = \{a, b, c\}$ ,  $E = \{e_1, e_2\}$ ,  $\tilde{\tau} = \{X, \tilde{\Phi}, (H_1, E), (H_2, E), (H_3, E)\}$  where  $(H_1, E), (H_2, E), (H_3, E)$  the soft sets over  $X$  defined as follows:

$$(H_1, E) = \{(e_1, \{b\}), (e_2, \{a\})\}; (H_2, E) = \{(e_1, \{a, b\}), (e_2, \{b, c\})\};$$

$$(H_3, E) = \{(e_1, \{b, c\}), (e_2, \{a, b\})\} \text{ and } \tilde{I} = \{\tilde{\Phi}, (I_1, E), (I_2, E), (I_3, E)\}$$

$$(I_1, E) = \{(e_1, \{\Phi\}), (e_2, \{a\})\}; (I_2, E) = \{(e_1, \{a\}), (e_2, \{c\})\};$$

$$(I_3, E) = \{(e_1, \{\Phi\}), (e_2, \{c\})\}, \text{ then } (X, \tilde{\tau}, E, \tilde{I}) \text{ is a soft } \beta - \tilde{I} - \text{normal space.}$$

**Proposition 3.1** (1) Every soft  $\beta$  normal space is a soft  $\beta - \tilde{I}$ -normal.

(2) If  $\tilde{I} = \{\tilde{\Phi}\}$ , then  $(X, \tilde{\tau}, E, \tilde{I})$  is soft  $\beta$  normal space iff it is soft  $\beta - \tilde{I}$ -normal space.

**Theorem 3.1** The soft topological space  $(X, \tilde{\tau}_2, E, \tilde{I})$  is soft  $\beta - \tilde{I}$ -normal space iff  $(X, \tilde{\tau}_1, E, \tilde{I})$  is a soft  $\beta - \tilde{I}$ -normal space.

**Proof.**

Let  $(X, \tilde{\tau}_2, E, \tilde{I})$  be soft  $\beta$  ideal normal space and soft  $\beta$  closed sets be  $(H, E)$  and  $(K, E)$  in  $\tilde{\tau}_1$  as  $\tilde{\tau}_2 \subseteq \tilde{\tau}_1$ .

$\Rightarrow$   $(H, E)$  and  $(K, E)$  are disjoint  $\tilde{\tau}_2$ -soft  $\beta$  closed sets. Since  $(X, \tilde{\tau}_2, E, \tilde{I})$  is soft  $\beta - \tilde{I}$ -normal space, there exist disjoint  $\tilde{\tau}_2$ -soft  $\beta$  open sets,  $(F, E)$  and  $(G, E)$  such that

$(K, E) - (F, E) \in \tilde{I}$  and  $(H, E) - (G, E) \in \tilde{I}$ . Since  $(F, E)$  and  $(G, E)$  are  $\tilde{\tau}_2$ -soft  $\beta$  open sets, then,  $(F, E) = (A, E) - (I_1, E)$  and  $(G, E) = (B, E) - (I_2, E)$ , where  $(A, E), (B, E) \in \tilde{\tau}_1$  and  $(I_1, E), (I_2, E) \in \tilde{I}$ . Thus  $(K, E) - (F, E) = (K, E) - ((A, E) - (I_1, E)) \in \tilde{I}$  and  $(H, E) - (G, E) = (H, E) - ((B, E) - (I_2, E)) \in \tilde{I}$ . From definition of soft ideal  $(K, E) - (A, E) \in \tilde{I}$  and  $(H, E) - (B, E) \in \tilde{I}$ , therefore  $(X, \tilde{\tau}_1, E, \tilde{I})$  is soft  $\beta - \tilde{I}$  - normal space. Conversely, let  $(X, \tilde{\tau}_1, E, \tilde{I})$  soft  $\beta$  ideal normal space and let  $\tilde{\tau}_2$  - soft  $\beta$  closed sets be  $(H, E)$  and  $(K, E)$ . Since  $(\overline{H}, \overline{E})$  and  $(\overline{K}, \overline{E})$  are  $\tilde{\tau}_2$ - $\beta$  open soft sets, then  $(\overline{H}, \overline{E}) = (G, E) - (I_1, E)$  and  $(\overline{K}, \overline{E}) = (F, E) - (I_2, E)$ , where  $(G, E), (F, E) \in \tilde{\tau}_1$  and  $(I_1, E), (I_2, E) \in \tilde{I} \Rightarrow (\overline{G}, \overline{E})$  and  $(\overline{F}, \overline{E})$  are disjoint  $\tilde{\tau}_1$  - soft  $\beta$  closed sets. Since  $(X, \tilde{\tau}_1, E, \tilde{I})$  is soft  $\beta - \tilde{I}$  - normal space, there exist disjoint  $\tilde{\tau}_1$  - soft  $\beta$  open sets  $(A, E)$  and  $(B, E)$  such that  $(\overline{F}, \overline{E}) - (A, E) \in \tilde{I}$  and  $(\overline{G}, \overline{E}) - (B, E) \in \tilde{I}$ . Implies  $(\overline{F}, \overline{E}) - (A, E) = [(K, E) - (I_1, E)] - (A, E) \in \tilde{I}$  and  $(\overline{G}, \overline{E}) - (B, E) = [(H, E) - (I_2, E)] - (B, E) \in \tilde{I}$ . So  $(K, E) - (A, E) \in \tilde{I}$  and  $(H, E) - (B, E) \in \tilde{I}$ . Therefore  $(X, \tilde{\tau}_2, E, \tilde{I})$  is soft  $\beta - \tilde{I}$  - normal space.

**Theorem 3.2** A soft  $\beta$  closed subspace  $(Y, \tilde{\tau}_Y, E, \tilde{I}_Y)$  of a soft  $\beta - \tilde{I}$  - normal space  $(X, \tilde{\tau}, E, \tilde{I})$  is soft  $\beta - \tilde{I}_Y$  - normal.

**Proof.**

Let  $(H, E), (K, E)$  be two disjoint soft  $\beta$  - closed sets over  $Y$  as  $\tilde{Y}$  is a soft closed set over  $X$ .

By hypothesis, there exist disjoint soft  $\beta$  - open sets  $(F, E), (G, E)$  over  $X$  such that

$(H, E) - (G, E) \in \tilde{I}$  and  $(K, E) - (F, E) \in \tilde{I}$ . It implies that  $\tilde{Y} \tilde{\cap} [(H, E) - (G, E)] \in \tilde{I}_Y$  and

$\tilde{Y} \tilde{\cap} [(K, E) - (F, E)] \in \tilde{I}_Y$ . Hence  $(H, E) - [\tilde{Y} \tilde{\cap} (G, E)] \in \tilde{I}_Y$  and  $(K, E) - [\tilde{Y} \tilde{\cap} (F, E)] \in \tilde{I}_Y$ , where  $\tilde{Y} \tilde{\cap} (G, E)$  and  $\tilde{Y} \tilde{\cap} (F, E)$  are disjoint soft  $\beta$  - open sets over  $Y$ . Therefore  $(X, \tilde{\tau}, E, \tilde{I})$  is soft  $\beta - \tilde{I}_Y$  - normal space.

**Theorem 3.3** Let  $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$ , be a soft homeomorphism function. If  $(X, \tilde{\tau}_1, \tilde{I}, A)$  is a soft  $\beta - \tilde{I}$  - normal space, then  $(Y, \tilde{\tau}_1, \tilde{I}, B)$  is a soft  $\beta - f_{pu}(\tilde{I})$  - normal space.

**Proof.**

Let the disjoint sets in  $Y$  be  $(F, B)$  and  $(G, B)$ . Given,  $f_{pu}$  is a soft homeomorphism, let  $(U, A)$  and  $(V, A)$  as disjoint soft closed sets in  $X$  defined as  $(U, A) = f_{pu}^{-1}(F, B)$  and  $(V, A) = f_{pu}^{-1}(G, B)$  Such that,  $(U, A) \tilde{\cap} (V, A) = f_{pu}^{-1}[(F, B) \tilde{\cap} (G, B)] = f_{pu}^{-1}(\tilde{\Phi}_B) = \tilde{\Phi}_A$ . By hypothesis, there exist open soft sets  $(K, A)$  and  $(H, A)$  in  $X$  Such that  $(U, A) - (K, A) \in \tilde{I}$  and  $(V, A) - (H, A) \in \tilde{I}$ . From lemma 2.2 and 2.3, it follows that,  $f_{pu}[(U, A) - (K, A)] = f_{pu}(U, A) - f_{pu}(K, A) \in f_{pu}(\tilde{I})$  and  $f_{pu}[(V, A) - (H, A)] = f_{pu}(V, A) - f_{pu}(H, A) \in f_{pu}(\tilde{I})$ . Since  $f_{pu}$  is a soft homeomorphism. Then  $f_{pu}(K, A) = (K, B)$  and  $f_{pu}(H, A) = (H, B)$  are disjoint open soft sets in  $Y$ , thus  $(Y, \tilde{\tau}_2, \tilde{I}, B)$  is a soft  $\beta - f_{pu}(\tilde{I})$  - normal space.

**Definition 3.2** Let  $(X, \tilde{\tau}, E, \tilde{I})$  be soft topological space with soft ideal and  $(H, E)$  be soft  $\beta$  closed set over  $X$  such that  $x \notin (H, E)$ , for  $x \in X$ , then  $(X, \tilde{\tau}, E, \tilde{I})$  is called soft  $\beta - \tilde{I}$  - regular space if there exist disjoint, if there exists soft open sets  $(F, E)$  and  $(G, E)$  such that  $x \notin (F, E)$  and  $(H, E) - (G, E) \in \tilde{I}$ .

**Example 3.2** Let  $X = \{h_1, h_2, h_3\}$ ,  $E = \{e_1, e_2\}$ ,  $\tilde{\tau} = \{X, \tilde{\Phi}, (F_1, E), (F_2, E), (F_3, E)\}$  where  $(F_1, E), (F_2, E), (F_3, E)$  the soft sets over  $X$  defined as follows:  
 $F_1(e_1) = \{h_3\}; F_1(e_2) = \{h_1\}; F_2(e_1) = \{h_2\}; F_2(e_2) = \{h_2\}; F_3(e_1) = \{h_1\}; F_3(e_2) = \{h_2, h_3\}$  and  $\tilde{I} = \{\tilde{\Phi}, (I_1, E), (I_2, E), (I_3, E)\}$  where,  $(I_1, E) = \{h_3\}; (I_2, E) = \{\Phi\}; (I_3, E) = \{h_1\}$ , then  $(X, \tilde{\tau}, E, \tilde{I})$  is a soft  $\beta - \tilde{I}$ -normal space.

**Theorem 3.4:** Let  $(X, \tilde{\tau}, E, \tilde{I})$  be soft ideal topological space then the following are equivalent:

(1)  $(X, \tilde{\tau}, E, \tilde{I})$  is a soft  $\beta - \tilde{I}$  - regular space

- (2) For each  $x \in X$  and soft  $\beta$  – open set  $(F, E)$  containing  $x$  there is a soft  $\beta$  – open set  $(G, E)$  containing  $x$ , such that  $(H, E) - (G, E) \in \tilde{I}$  and  $S\beta cl(G, E) - (F, E) \in \tilde{I}$ .
- (3) ) For each  $x \in X$  and soft  $\beta$  – closed set  $(H, E)$  containing  $x$  there is soft  $\beta$  – open set  $(G, E)$  containing  $x$  such that  $S\beta cl(G, E) \tilde{\cap} (H, E) \in \tilde{I}$ .

**Proof.**

- (1)  $\Rightarrow$  (2) Let  $x \in X$  and soft  $\beta$  – open set  $(F, E)$  containing  $x$ , then there exist disjoint and soft  $\beta$  – open set  $(G, E)$  and  $(D, E)$  such that  $x \in (G, E)$  and  $(X - (F, E)) - (D, E) \in \tilde{I}$  implies if  $(X - (F, E)) - (D, E) = (I, E) \in \tilde{I}$ , then  $(X - (F, E)) \tilde{\cap} (D, E) \tilde{\cap} (I, E)$ . The disjoint and soft  $\beta$  – open set  $(G, E)$  and  $(D, E)$  implies that  $(G, E) \tilde{\cap} X - (D, E)$  and  $S\beta cl(G, E) \tilde{\cap} X - (D, E)$ ,  
 $S\beta cl(G, E) - (F, E) \tilde{\cap} (X - (D, E)) \tilde{\cap} ((D, E) \tilde{\cap} I) = (X - (D, E)) \tilde{\cap} (I, E) \tilde{\cap} (I, E) \in \tilde{I}$ .
- (2)  $\Rightarrow$  (3) Let  $(H, E)$  soft  $\beta$  – closed in  $X$  such that  $x \notin (H, E)$ . Then there exist an soft  $\beta$  – open set  $(G, E)$  containing  $x$  such that  $S\beta cl(G, E) - X - (H, E) \in \tilde{I}$ , implies that  $S\beta cl(G, E) \tilde{\cap} (H, E) \in \tilde{I}$ . If  $S\beta cl(G, E) \tilde{\cap} (H, E) = (I, E) \in \tilde{I}$ .
- (3)  $\Rightarrow$  (1) Let  $(H, E)$  soft  $\beta$  – be closed in  $X$  such that  $x \notin (H, E)$ . Then there exist an soft  $\beta$  – open set  $(G, E)$  containing  $x$  such that  $S\beta cl(G, E) \tilde{\cap} (H, E) = (I, E) \in \tilde{I}$ , then  $(H, E) - (X - S\beta cl(G, E)) = (I, E) \in \tilde{I}$ . Therefore  $(X, \tau, E, \tilde{I})$  is a soft  $\beta$  –  $\tilde{I}$  – regular space.

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