SLIDING MODE CONTROL FOR A SECOND ORDER UNSTABLE SYSTEM

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Abstract:
Linear Systems on using PD or PID controller might become unstable due to increase in dead time or due to parametric uncertainties. This paper presents the control strategy when there is a high delay time and the system is unstable. The oscillatory response of the system is eliminated and its robust property is maintained. A second order system with two unstable poles has been discussed for their robustness properties. The comparative performances under various disturbances is obtained by simulation.

Keywords: High delay time, Sliding Mode Controller, Oscillatory response, chattering

Introduction:
In Industrial Control Systems, PID controller is a commonly used controller. It continuously calculates the error value e(t) and applies a correction based on the proportional, integral, derivative terms. While PID controllers are applicable to many control problems, and often perform satisfactorily without any improvements or only coarse tuning, they can perform poorly in some applications, and do not in general provides Optimal control [1]. PID controllers, when used alone, can give poor performance when the PID loop gains must be reduced so that the control system does not overshoot, oscillate or hunt about the control set point value. They also have difficulties in the presence of non-linearity’s, may trade-off regulation versus response time, do not react to changing process behavior (say, the process changes after it has warmed up), and have lag in responding to large disturbances.

In order to overcome those hunting problems in systems, a special control strategy called Sliding Mode Control or SMC, which is a nonlinear control method that alters dynamics of a non-linear systems by application of discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behavior. One application of sliding mode controller is the control of electric drives operated by switching power converters. But in this paper, the implementation of the sliding mode controller is used for disturbance rejection due to parametric uncertainties and unmodelled dynamics.

Control Strategy:
With the transfer function of a general second order system with two unstable poles, the dead time in it can be approximated by either Taylor's series or by Pade approximation series method[2].

\[
\frac{X(s)}{U(s)} = \frac{Ke^{-Ts}}{\alpha s + 1}
\]

\[
\frac{X(s)}{U(s)} = \frac{K}{(\alpha s + 1)(\beta s + 1)}
\]

The State space equation for the above transfer function is found and the state variables obtained from the equation are used for the control laws of SMC.

The control strategies in SMC involves two phases. Reaching phase and Sliding phase. The reaching phase is a discontinuous control input in
which the system states are driven from initial value to the set point, where it has to be slided.\[3\] The Sliding phase induced by continuous control input in which the system states are in a sliding motion.

In SMC, there are five main control laws that determines the robustness property of any systems.\[4\]

**Constant Rate reaching law:**

\[
\dot{s} = -Q \text{sgn}(S) \quad Q > d_{\text{max}}
\]

Where \(d_{\text{max}} = \text{magnitude of external bounded disturbance.}\)

The reaching rate is directly dependent on the magnitude of the gain. But the main problem with the large gain is that, the chattering magnitude is directly proportional to the gain of the discontinuous term.

To improve the system performance during the sliding phase, the system should reach the equilibrium point at a faster rate. The different sliding modes that are used for finite time convergence at a faster rate are Terminal sliding mode (TSM) \[6\] and Fast terminal sliding mode (FTSM) \[7\]-\[9\] control.

In TSM, sliding surface is defined as

\[
S = x_2 + \beta x_1^{q/p} \quad \beta > 0
\]

Where p and q are odd integers, \(q > p\)

If \(p=q\), the terminal sliding mode surface becomes a linear sliding surface.

In TSM, a nonlinear sliding surface is proposed. The equilibrium is a terminal attractor i.e the states can be reached in finite time and stable.

In FTSM, generally sliding surface is given by,

\[
S = x_2 + \alpha x_1 + \beta x_1^{q/p}
\]

When \(x_1\) is far away from origin, dominates, so convergence is faster. For values of close to origin \(\beta x_1^{q/p}\) is the dominating term. The convergence is enhanced in the fast terminal sliding mode by introducing a linear term in the terminal sliding surface.

The net law reduces this chattering magnitude considerably.

**Constant Plus Proportional rate reaching law:**

\[
\dot{s} = -KS - Q \text{sgn}(S) \quad K,Q > d_{\text{max}}
\]

Where \(Q = \text{internal disturbance in the system.}\)

**Super twisting law:**\[5\]

It is a second order controlling law which has a continuous control input.

\[
\dot{s} = -K |S|^{1/2} \text{sgn}(S) + Z
\]

\[
\dot{Z} = -K_2 sgn(S) + d \quad K_2 > d_{\text{max}}
\]

modes that are used for finite time convergence at a faster rate are Terminal sliding mode (TSM) \[6\] and Fast terminal sliding mode (FTSM)\[7\]-\[9\] control.

**SIMULATION RESULTS:**

The transfer function of the second order unstable system with two unstable poles which is taken for study is\[10\]

\[
p(s) = \frac{2}{(3s-1)(s-1)} e^{-0.3s}
\]

The system with dead time is approximated by Taylor’s series as

\[
G(s) = \frac{2}{(3s-1)(s-1)(0.3s+1)}
\]

State space representation of the system is

\[
\begin{bmatrix}
A = & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
-1.1111 & 4.1111 & -2 & 0 \\
\end{bmatrix}

B = \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}

C = \begin{bmatrix}
0 & 0 & 2.2222 \\
\end{bmatrix}

D = 0
\]
Simulation Output:

Fig 1. Discontinuous control input of second order sliding mode control

Fig 2. Response of constant rate reaching law

Fig 3. Response for Constant Proportional Rate Reaching Law

Fig 4. Response for Terminal Sliding Mode Control

Fig 5. Plot Response for Super Twisting Control

Fig 6. Response plot for Terminal Sliding Mode Control

Fig 7. Plot response for Fast Terminal Sliding Mode
Comparative performance metrics:

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>high frequency disturbance</th>
<th>Low frequency disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant rate reaching law</td>
<td>1.623</td>
<td>8.71</td>
</tr>
<tr>
<td>Constant plus proportional rate reaching law</td>
<td>66.76</td>
<td>70.99</td>
</tr>
<tr>
<td>Super twisting sliding mode</td>
<td>817.42</td>
<td>133.02</td>
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<tr>
<td>Terminal sliding mode</td>
<td>72.73</td>
<td>71.03</td>
</tr>
<tr>
<td>Fast Terminal sliding mode</td>
<td>504.33</td>
<td>453.92</td>
</tr>
</tbody>
</table>

Tab 1. Performance of the controller with high frequency and Low Frequency disturbances

CONCLUSION

Sliding mode controller for a higher order unstable system has been presented [3], [11]. The sliding phase of the system is achieved without any alternative disturbances and external bounded disturbances. The control parametrizes gives the output results at various frequency values. [12] A general second order system with two highly unstable poles has been controlled which accepts the controller equation.

REFERENCES
