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Study onConnected Network throughMiddle Graph Approach

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Abstract

In this paper we created middle graph approach for the connected network topology. And this study leads to new approach in different way from all the other generation of connected network topology after studying of connected domination set. Also we obtained new results for induced graph of some special type of graphs like C_n+ C_n and special graph like C_n-C_n. We observed diameter, domination number, planarity and number of independent dominant sets for theCycles and middle graph of the cycles for the above mentioned network graphs.

Key Words: Cycle, diameter, domination set, independence domination set,middle graph,planar graph.

1. Introduction

The study of dominating sets in graphs was began by Ore and Berge. The domination number, independent dominance number are introduced by Cockayne and Hedetniemi. AsHedetniemi&Laskar (1990) note the domination problem was studied from the 1950'sonwards, but the rate of research on domination significantly increased in the mid-1970's, in 1972, Richard Karb proved the set cover problem to be NP-complete. Dominating sets are of practical interest in several areas like networks, computer science engineering.In wireless networking dominating sets are used to find efficient routs with ad-hoc mobile networks (MANET). They have also been used in document summarization and in designing secure systems for electrical grids. One of the reasons for the recent revival of interest in graph theory among students of electrical engineering is the application of graph theory to the analysis and design of electrical networks An electrical network is a collection of interconnected electrical elements such as resisters, capacitors, inductors, diodes, transistors, vacuum tubes, etc.,

In this paper, all the graph considered here are simple, finite, nontrivial, undirected and connected. Wireless networks consist of nodes which can communicate with each other over the wireless link. The planarity and related other related concepts are useful in many practical situation , for instance the design of printed circuit board the electrical engineer must know if he can make the required connections.

2. Prelimineries

Definitions

Cycle: A cycleis a walk with different vertices except for initial and end vertex.

Middle graph: The Middle graph of a graph denoted by M (G) is a graph whose vertex set is V (G)∪E (G), and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is an edge incident with it.

Domination set: A Set D of a graph G = (V, E) is called a dominating set if every vertex in V-D is adjacent to some vertex in D.The domination number γ (G) of G is the minimum cardinality taken over all dominating set of G**.**

Independent Domination set:A domination set S of vertices of graph G is an independent dominating set of G if any two vertices in S are notadjacent.

Planar Graph: Agraph G is said to be planar if there exits some geometric representation of G which can be drawn on a plane such that no two of its edges are intersect.

Diameter: The diameter of a connected graph is defined as the largest distance between two vertices in the graph. $\lceil x \rceil$ Denotes the smallest integer not less than x ,$\lfloor x \rfloor$ Denotes the greatest integer not greater than x

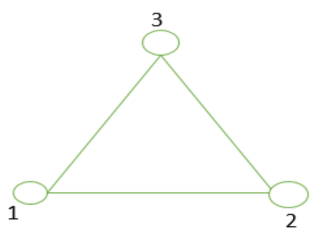
Split domination set:ADomination set D of a graph G = (V, E) is a split domination set if the induced sub graph V-D is disconnected. The split domination number γ_s (G) of G is minimum cardinality of a split domination set.

Strong Split domination set: A Domination set D of a graph G = (V, E) is a strong split domination set if the induced sub graph V-D is totally disconnected with atleast two vertices.

The strong split domination number γ_{ss} (G) of G is minimum cardinality of a strong split domination set.

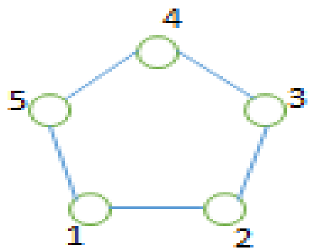
Study on Cycle Cn (n ≥ 3)

Consider Cycle - C_3



a) Diameter is 1.
b) Domination number is 1.
c) Independent domination sets are {{1}, {2}, {3}}.
d) Number of Independent domination set is 3.

Consider Cycle - C_5



a) Diameter is 2.
b) Domination number is 2.
c) Independent domination sets are {{1,3},{1,4},{2,4},{2,5},{3,5}}.
d) Number of Independent domination set is 5.

Results Observed

1. Diameter of $C_n = \begin{Bmatrix} \frac{n}{2} if n is even \\ \left\lfloor \frac{n}{2} \right\rfloor if n is odd \end{Bmatrix}$.
2. Independent Domination number of

$$C_n=\begin{cases} \frac{n}{3} \, if n is multiple of \ 3 \\ \left\lceil \frac{n}{3} \right\rceil otherwise \end{cases}.$$

3. Number of independent domination sets for

$$C_n=\begin{cases} 3 & if n is multiple of \ 3 \\ 2 & if n = 4 \\ & n otherwise \end{cases}.$$

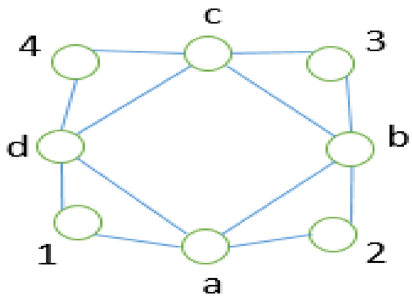
4. Covering number of Cycle $(\mathbf{C_n})=\begin{cases} \frac{n}{2} if n is even \\ \left\lceil \frac{n}{2} \right\rceil if n is odd \end{cases}$.

5. All independent domination sets of $\mathbf{C_n}(n > 3)$ are split dominationset.

6. All independent domination sets of $\mathbf{C_{3i+1}}$ $(i \geq 1)$ are strong splitdomination set.

Study on Middle Graph of Cycle (M(Cn))

Consider M (C4)



a. Diameter is 3.

b. Domination number is 2

c. Independent domination sets are {{a,c},{b,d}}.

d. Number of Independent domination set is 2.

Results Observed

1. Diameter of M $(C_n)=\begin{cases} \frac{n}{2}+1 & if n is even \\ \left\lceil \frac{n}{2} \right\rceil if n is odd \end{cases}$

2. Independent Domination number of $M(C_n) = \begin{cases} \frac{n}{2} if n is even \\ \left\lceil \frac{n}{2} \right\rceil if n is odd \end{cases}$.

3. Number of independent domination set of

$$M(C_n) = \begin{cases} 2 & if n is even \\ & n if n is odd \end{cases}$$

4. Domination number of $M(C_n)$ =

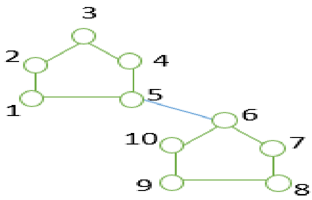
$$\begin{cases} \text{Diameter of M } (C_n) - 1 if n is even \\ \text{Diameter of M } (C_n) \ if n is odd \end{cases}.$$

5. The elements in Independent domination sets ofM (C_{2n}) are the edgeswhichare distance of two in C_{2n}.for all n>1.

6. Let D be an independent domination set of $M(C_n)$. The induced graph < V-D > of

$$M(C_n) = \begin{Bmatrix} \frac{n}{2} components of\, p_3 if\, n is\, even \\ \frac{n-3}{2} components of\, P_3\, and P_4 \quad if\, n is\, odd \end{Bmatrix}.$$

Consider C5 – C5as follows



a) Diameter is 5.
b) Domination number is 4.
c) Independent domination sets are as follows, {{1,4,6, 8},{1,4,6,9}, {1,3,6,8} ,{1,3,6,9} , {1,4,7,9} , {1,4,7,10},{2,5,7,9},{2,5 ,7,10}, {2,5,8,10}, {3,5,7,9}, {3,5,7,10}, {3,5,8,10}, {2,4,6,9}, {1,3,7,9},{1,3,7,10}}.
d) Number of Independent domination set is 15.

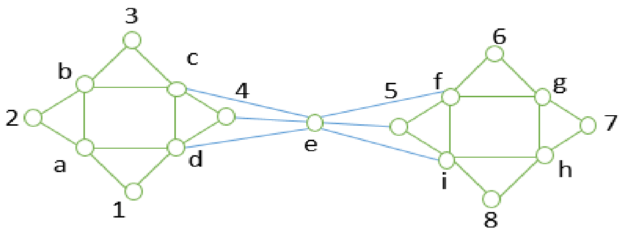
Results Observed

1) Diameter of C_n - C_n= $\begin{Bmatrix} n+1 & if\, n is\, even \\ & n if\, n is\, odd \end{Bmatrix}$.

2)Domination number of C_n - C_n = $\begin{Bmatrix} \frac{2n}{3} if\, n = 3i\ (i \geq 1) \\ \left\lceil \frac{2n}{3} \right\rceil otherwise \end{Bmatrix}$.

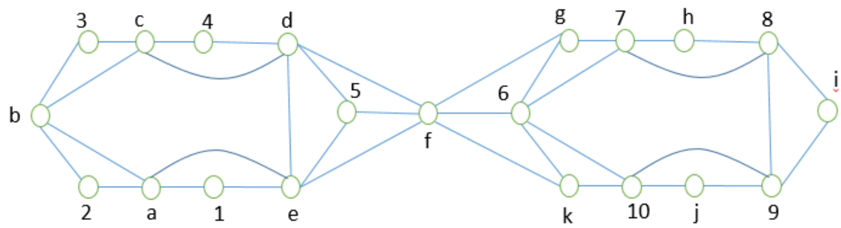
Study on Middle Graph of Cn – Cn(M (Cn – Cn))

Consider M(C4 – C4)



a) Diameter is 6.
b) Domination number is 4.
c) Independent domination sets are {{b,d,f,h},{b,d,I,g},{a,c,f,h},{a,c,I,g}}.
d) Number of Independent domination set is 4.

Consider M($C_5 - C_5$).



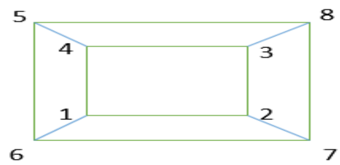
a) Diameter is 6.
b) Domination number is5.

Results Observed

1) Diameter of M (C_n-C_n)= $\begin{Bmatrix} n+1 & ifnisodd \\ n+2 & ifniseven \end{Bmatrix}$.

2) Domination number ofM (C_n -C_n)= n for all n ≥ 3.

Consider C4 + C4



a. Diameter is 3.
b. Domination number is 2.
c. Independent domination sets are {{1,8},{2,5},(3,6},{4,8}}.
d. Number of Independent domination set is 4.

Results Observed

1) Diameter of C_n+C_n= $\begin{Bmatrix} \frac{n}{2}+1 & ifniseven \\ & \left\lceil\frac{n}{2}\right\rceil ifnisodd \end{Bmatrix}$.

2) Domination number of

$$C_n + C_n = \begin{Bmatrix} \left\lceil\frac{n}{2}\right\rceil ifn = 2i+1\ (i \geq 1) \\ \frac{n}{2} ifn = 4i\ (i \geq 1) \\ \frac{n}{2}+1 \qquad otherwise \end{Bmatrix}.$$

3)Let D be an independent domination set ofC_n + C_n(n ≠ 4i + 1. i ≥1),

The induced graph < V-D > =

$$\begin{cases} P_{\frac{3n-1}{2}} if n = 4i - 1\ (i \geq 1) \\ C_{\frac{3n}{2}} if n = 4i\ (i \geq 1) \\ P_{\frac{3n-4}{2}} with one isolatted vertex if n = 4i + 2 (i \geq 1) \end{cases}.$$

4)Let D be an Independent domination set of $C_{4i+1} + C_{4i+1} (i \geq 1)$,

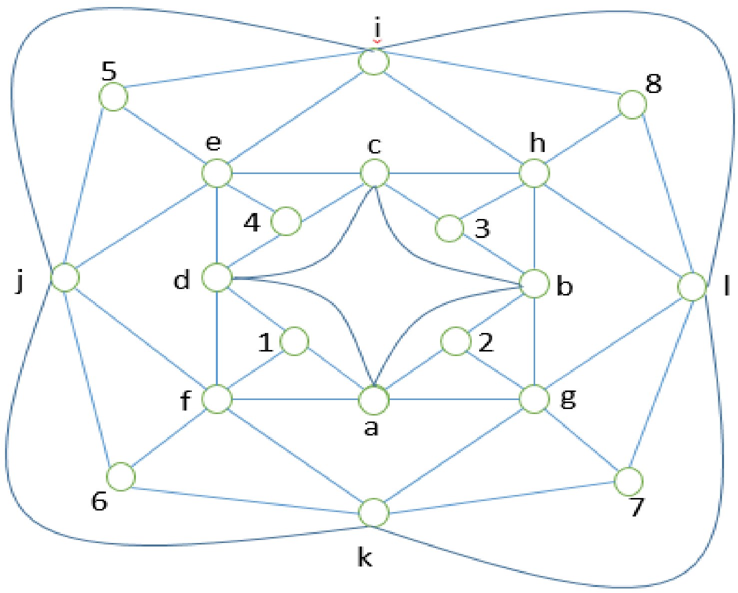
The induced graph < V-D >= $P_{\frac{3n-7}{2}}$ with two isolated vertex.

5) TheDomination number and Independent domination number are not same in $C_{4n+1} + C_{4n+1} (n \geq 1)$.

6) Domination number of

$$C_n + C_n = \begin{cases} (\text{Diameter of } (C_n + C_n)) - 1 & if n = 4i\ (i \geq 1) \\ \text{Diameter of } (C_n + C_n) & if n is otherwise \end{cases}.$$

ConsiderM (C4 + C4)



a. Diameter is 4.
b. Domination number is 4.
c. Middle graph of $C_4 + C_4$ is planar graph.

Results Observed

1) Diameter of M $(C_n + C_n) = \begin{cases} \frac{n}{2} + 2 & if n is even \\ \frac{n}{2} + 1 & if n is odd \end{cases}$.

2) Domination number of M $(C_n + C_n)$= n, for all n.

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