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Even-Odd Cordial Labeling Graphs

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Abstract

Let G be a graph with $n \geq 2$ vertices. A bijection vertex labeling $f: V(G) \rightarrow \{1,2,3\ldots,n\}$ induces an edge labeling $f^*: E(G) \rightarrow \{0,1\}$ defined by f^*

$$f^*(uv) = \begin{cases} 1, & \text{if both f(u) and f(v) are even or odd} \\ 0, & \text{otherwise.} \end{cases}$$

For each uv $\in E(G)$, is called an even-odd labeling. A graph G is called an even-odd cordial if $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. In this paper we study some properties of even-odd labeling and we investigate even-odd cordial labeling of star $k_{1,n}$, path P_n, the corona product of P_n and K_1, Shell S_n and DS (P_n).

1. Introduction

We begin with a graph G = (V(G), E(G)) with p vertices and q edges. For standard terminology and notations related to graph theory we refer to Gross and Yellen [7]. We will provide brief summary of definitions and other information which are prerequisites for the present investigations.

Definition 1.1

The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (edge labeling).

A brief account of various graph labeling problems and latest bibliographic references can be found in Gallian [5].

Definition 1.2

A mapping f : V(G)→{0,1} is called a binary vertex labeling of G and f (v) is called the label of the vertex v of G under f.

For each edge e = uv, the induced edge labeling f^* : E(G) → {0,1} is given by

$$f^*(e) = |f(u) - f(v)|$$

$$\left.\begin{array}{l}\text{Let } v_f(i) = \text{number of vertices of } G \text{ having label i under f} \\ e_{f^*}(i) = \text{number of edges of } G \text{ having label i under } f^*\end{array}\right\} \text{where i = 0 or 1}$$

Definition 1.3

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. A graph G is cordial if it admits cordial labeling .

The concept of cordial labeling was introduced by Cahit [4] as a weaker version of graceful [9] and harmonious [6] labeling. The partial motivation to define cordial labeling was an attempt to settle tree conjectures for graceful and harmonious labeling. It turns out to be relatively easy to prove that every tree admits a cordial labeling. In the same paper Cahit [4] has investigated some classes of cordial graphs and a necessary condition for an Eulerian graph to be cordial graph. This concept is explored by many researchers like Andar et al. [1, 2], Vaidya and Dani [11, 12]. Middle graphs of path, crown, $k_{1,n}$, tadpole T(n, l+1) admit cordial labeling as proved by Vaidya and Vihol [13] while Vaidya and Shah [14, 15] have discussed cordial labeling of some bistar related graphs and cordial labeling of some snake related graphs. Motivated through the concept of cordial labeling Babujee and Shobana [3] introduced the concepts of cordial languages and cordial numbers. Lawrence and Koilraj [8] have discussed cordial labeling for the splitting graph of some standard graphs. Kalidass and shanthi [19] have studied discussion on total signed product cordial labeling for some standard graphs.

Definition 1.4

For a graph G the splitting graph $S^{'}(G)$ of a graph G is obtained by adding a new vertex $v^{'}$ corresponding to each vertex v of G such that N(v) = N($v^{'}$).

Definition 1.5

Let G = (V(G), E(G)) be a graph with V = $S_1 \cup S_2 \cup S_3 \cup \ldots S_t \cup T$ where each S_i is a set of all vertices of the same degree with at least two elements and T = V – $\bigcup_{i=1}^{t} S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices w_1, w_2, w_3,... , w_t and joining to each vertex of, S_i for 1 ≤ i ≤ t.

Definition 1.6

A shell S_n is the graph obtained by taking n-3 concurrent chords in cycle, C_n. The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called fan and is denoted by f_{n-1}.

i.e. $S_n = f_{n-1} = P_{n-1} + K_1$.

Definition 1.7

Let G be a graph with n ≥ 2 vertices. A bijection vertex labeling f : V(G)→{1,2,3...,n} induces an edge labeling f^* : E(G)→ {0,1} defined by f^*

$$f^*(uv) = \begin{cases} 1, \text{ if both f(u) and f(v) are even or odd} \\ 0, \text{ otherwise.} \end{cases}$$

For each uv ∈ E(G), is called an even-odd labeling

Definition 1.8

Let G be a graph with n ≥ 2 vertices and let f be an even-odd labeling. An even-odd labeling f of G is even-odd cordial labeling if $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. If G admits an even-odd cordial labeling, then G is called an even-odd cordial.

2. Main Results

Theorem 2.1

If G is an even-odd cordial graph of even size, then $G - e$ is also an even-odd cordial graph.

Proof

Consider an even-odd cordial graph G with p vertices and q edges

Let q be an even size of the even-odd cordial graph G.

Then it follows that $e_{f^*}(0) = e_{f^*}(1) = \frac{q}{2}$

Let ‘e’ be any edge in G which is labeled either ‘0’ or ‘1’

In $G-e$, we have either

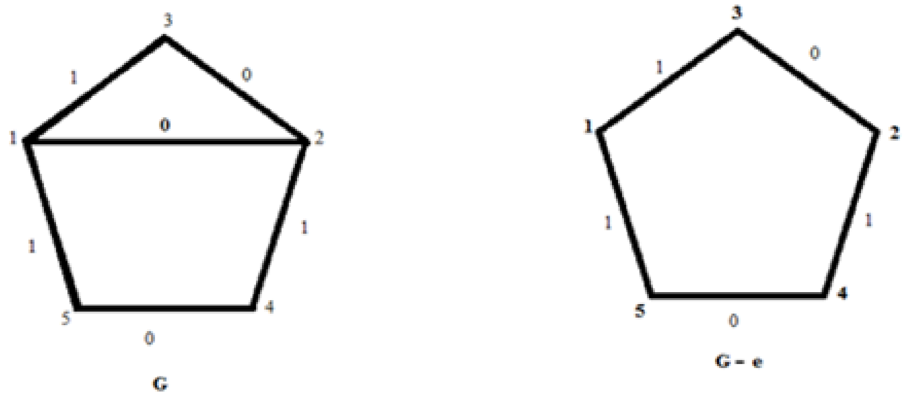
$e_{f^*}(0) = e_{f^*}(1) + 1$ (or) $e_{f^*}(1) = e_{f^*}(0) + 1$

In either case we have $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

$\therefore$ $G-e$ is an even-odd cordial.

Example 1

Even-odd cordial labeling for an even size of G and $G-e$



Theorem 2.2

If G is an even-odd cordial graph of even size, then $G+e$ is also an even-odd cordial graph.

Proof

Consider an even-odd cordial graph G with p vertices and q edges

Let q be an even size of an even-odd cordial graph G, $e_{f^*}(0) = e_{f^*}(1) = \frac{q}{2}$

Let u and v be any two non-adjacent vertices of G

Let $G^{'}$ = G + uv

Suppose f(u) and f(v) are both even or odd, then

$$e_{f^*}(1) = \frac{q}{2} + 1 \qquad e_{f^*}(0) = \frac{q}{2}$$

Suppose f(u) and f(v) are either even or odd, then

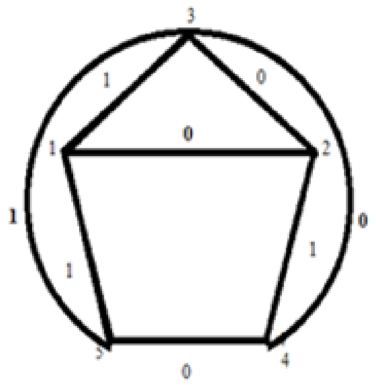
$$e_{f^*}(1) = \frac{q}{2} \qquad e_{f^*}(0) = \frac{q}{2} + 1$$

Hence, $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

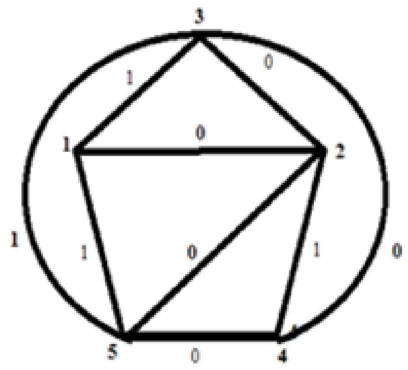
∴ G' is an even-odd cordial.

Example 2

Even-odd cordial labeling for an even size of G and $G + e$



G

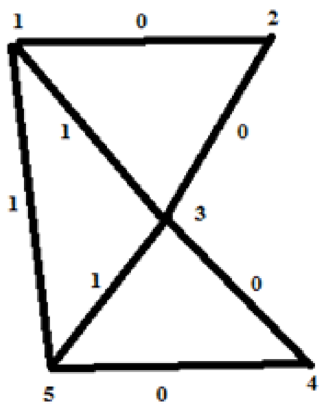


G + e

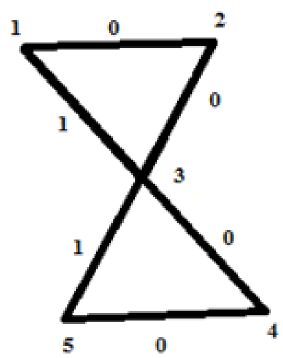
Remark

If G is an even-odd cordial of odd size, then $G - e$ and $G + e$ may not be an even-odd cordial.

Example 3



G



G-e

Here G is an even-odd cordial but $G - e$ is not an even-odd cordial

Theorem 2.3

A star graph $k_{1,n}$ is an even-odd cordial.

Proof

Let v be the apex vertex of $k_{1,n}$ and let $v_1, v_{2,} v_{3 \ldots\ldots} v_n$ be the remaining vertices of $k_{1,n}$

Define f : V(G)→{1,2,3…,n+1}

by f(v) = 1 and f(v_i) = i+1

Case (i)

Suppose n is even

$$e_{f^*}(1) = e_{f^*}(0) = \frac{n}{2}$$
$$\therefore\ e_{f^*}(1) - e_{f^*}(0) = 0$$

Hence, $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

Case (ii)

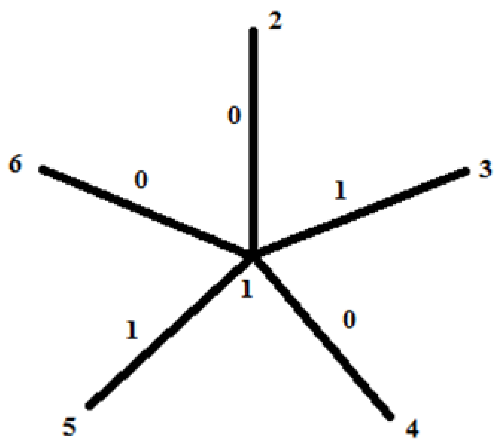
Suppose n is odd

$$e_{f^*}(0) = \frac{n+1}{2} \qquad e_{f^*}(1) = \frac{n-1}{2}$$
$$\therefore |e_{f^*}(0) - e_{f^*}(1)| \leq 1$$

Hence a star graph $k_{1,n}$ is an even-odd cordial.

Example 4

$k_{1,5}$ and its even-odd cordial labeling



Theorem 2.4

Path P_n(n≥2) is an even-odd cordial.

Proof

Let P_n be the path with n vertices and n-1edges

Define f:V(P_n)→{1,2,3,4,..,n}

By

$$f(v_i) = \begin{cases} i, & \text{if } i \equiv 0 (\text{mod } 4) \quad \text{or} \quad i \equiv 1 (\text{mod } 4), \\ i+1, & \text{if } i \equiv 2 (\text{mod } 4), \\ i-1, & \text{if } i \equiv 3 (\text{mod } 4). \end{cases}$$

Case(i)

Suppose n is even

$$e_{f^*}(0) = \frac{n}{2} - 1 \qquad e_{f^*}(1) = \frac{n}{2}$$

Case (ii)

Suppose n is odd

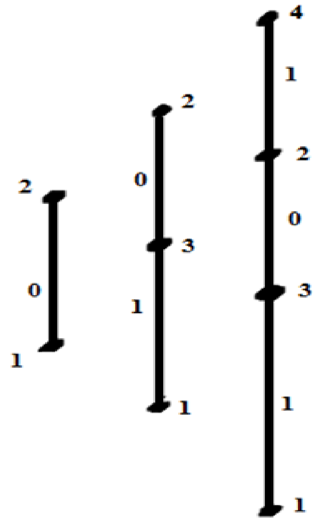
$$e_{f^*}(1) = e_{f^*}(0) = \frac{n-1}{2}$$

Hence, $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

∴ Path P_n(n≥2) is an even-odd cordial.

Example 5

Path P_2, P_3, P_4 and their even-odd cordial labeling



Theorem 2.5

$P_n \odot K_1$(n≥2) is an even-odd cordial.

Proof

Let P_n be a path with n vertices V(P_n) = $\{u_1, u_2, u_3, \ldots, u_n\}$

Also, let v_i be the pendant vertices which is adjacent to u_i

Let the vertex labeling of P_n as 1,2,4,…,2(n-1) and let the vertex labeling of apex of K_1 as

3, 5, 7,…, 2n-1

By the above labeling pattern, we have

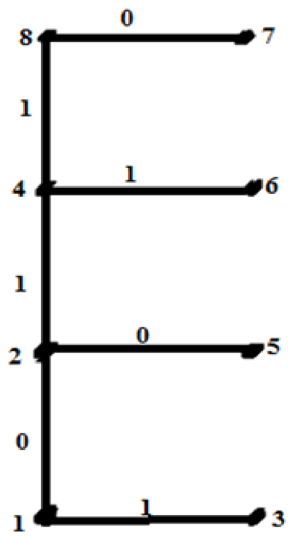
$$e_{f^*}(1) = \mathrm{n} \quad e_{f^*}(0) = \mathrm{n} - 1$$

Hence, $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

∴ $P_n \odot K_1$ (n≥2) is an even-odd cordial.

Example 6

$P_4 \odot K_1$ and its even-odd cordial labelling.



Theorem 2.6

Shell S_n(n≥2) is an even-odd cordial.

Proof

Let S_n *be the Shell with n vertices and 2n-3 edges*

Define mapping f: V(S_n) → {1,2,3,...,n} as given below.

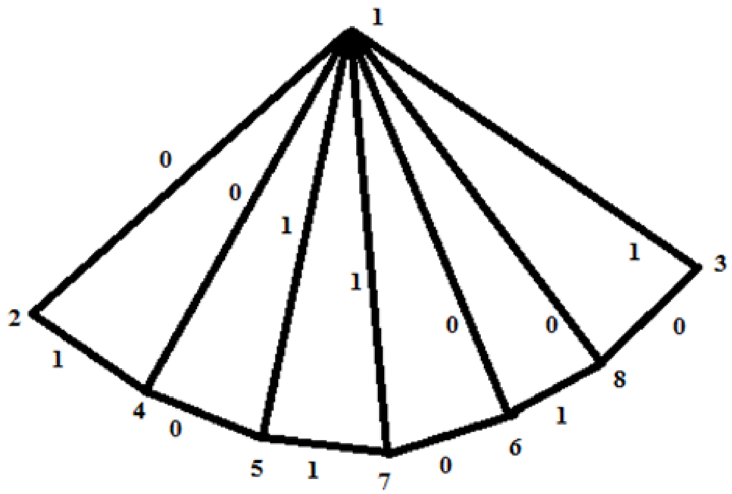
$$f(v_i) = \begin{cases} i, & \text{if } i \equiv 2 \pmod 4, \\ i+1, & \text{if } i \equiv 3,4 \pmod 4, \\ i+2, & \text{if } i \equiv 1 \pmod 4. \end{cases}$$

From the above labeling pattern, we have $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

∴ Shell S_n(n≥2) is an even-odd cordial.

Example 7

Shell S_8 and its even-odd cordial labeling



Theorem 2.7

The Graph DS (P_n) (n≥2) is an even-odd cordial graph.

Proof:

Let DS (P_n) be the degree splitting graph of P_n with n+2 vertices and 2n-1 edges.

Define mapping f: V(P_n) → {1,2,3,4,...,n} as given below.

Case(i)

Suppose n is even

$$f(w_1) = n+2$$
$$f(w_2) = n+1$$

Case(ii)

Suppose n is odd

$$f(w_1) = n+1$$
$$f(w_2) = n+2$$
$$f(v_1) = 1 ,\ f(v_2) = 2 ,\ f(v_n) = 3$$

$$f(v_i) = \begin{cases} i, & \text{if } i \equiv 0 \pmod 4, \\ i+1, & \text{if } i \equiv 1,2 \pmod 4, \\ i+2, & \text{if } i \equiv 3 \pmod 4. \end{cases} \qquad \text{except } i \neq 2$$

By the above labeling pattern, we have

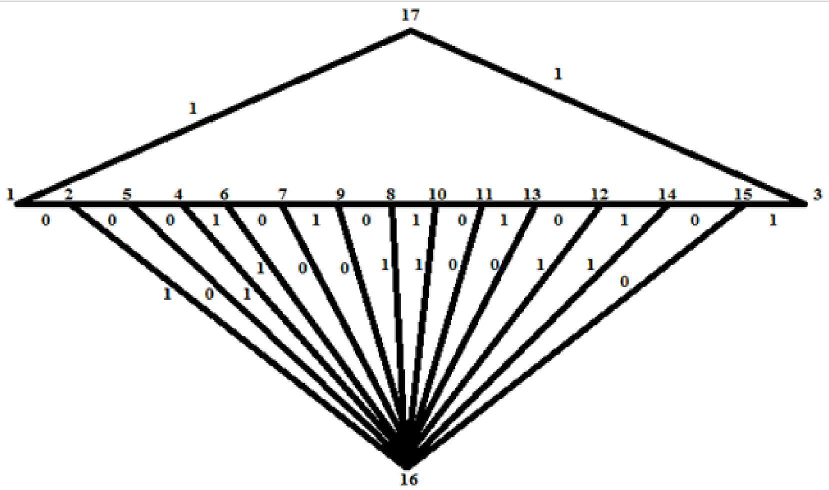
$e_{f^*}(1) = n \qquad e_{f^*}(0) = n - 1$

Hence, $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$

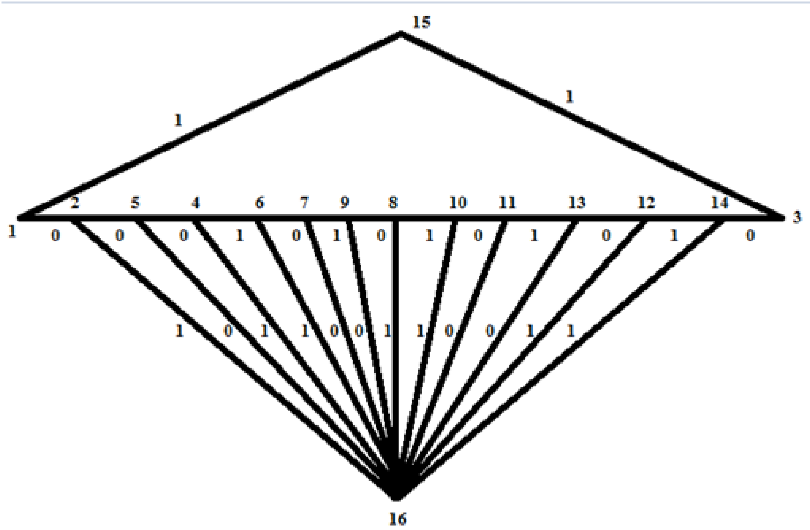
∴ DS (P_n) (n≥2) is an even-odd cordial.

Example 8

DS (P_{15}) and its even-odd cordial labelling



DS (P_{14}) and its even-odd cordial labeling



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