Abstract: Fractal dimension (FD) is most popular and significant characteristic of fractal theory and has been broadly used in image processing in terms of image analysis, texture segmentation, shape classification and identifying the image features such as roughness and smoothness of an digital image. Since noisy images can lead to estimation of inaccurate fractal dimension due to addition of noise factor. In this work, the pursuance of noise and inaccuracy results may come for the presence of noise on estimation of fractal dimension is studied by using different material images. For this purpose, we used improved differential box counting method with three sample gray scale images are used, In addition we used three kinds of noise technique like Gaussian noise, salt and pepper noise and speckle noise for generating noised image. Finally, we evaluate FD from noise image as well as original image and consequent error is address by means of both RMSE and PSNR. Our experimental work shows that the three chosen noise methods have an indicative reflex on FD estimation. Finally we evaluate the average percentage error and address as an error percentage for genuine estimation of fractal dimension of noise image.

Keywords: Fractal dimension, Error analysis, Salt and pepper noise, Gaussian noise, Speckle noise, RMSE, PNSR, improved differential box counting.

1. Introduction

The perception of fractal geometry was initially setup by mandelbrot [1] to explain self-similar sets called fractals. Fractal geometry comes into play where conventional euclidean geometry fails to express the usual or spited set of natural features as well as complex objects [2]. It yields a numerical representation for various complex real world natural objects. One of the major properties of fractal theory called Self-similarity and it also useful for estimation of fractal dimension (FD). In this regard, many methods have been developed but most have their practical and theoretical limitations. Fractal dimension was broadly used in various applications like segmentation, analysis of texture, image analysis and classification [3], [4], [5], [6], [7]. Pentland [2] provides the concept of smoothness vs. roughness of image surface with FD as 2 in case of smooth image surface and FD as 3 for maximum rough surface image called salt & pepper surface. Many researchers contributed their effort on estimating fractal dimension in the field of fractal geometry. Thus, several concepts have been proposed in this regard, however gangepain and carmes [8] presented the famous reticular cell-counting method which enhanced upon by Voss [9] by leading probability theory. Afterward, Keller et al. [10, 11] gave additional enhancement by a way of linear interpolation. Voss retrograde and divided these concepts into 3 key group called, box-counting, variance and spectral methods [12]. Out of these three methods box counting method is most popular because its easiness and easy implementation [13]. In this regard many efficient box counting methods were proposed to estimate FD [8], [14], [15], [16], [17], [18].

This article is prepared as follows. In section 2, we illustrate about principle idea of fractal theory and improved differential box counting method for estimating FD. Section 3 discusses about different noise models. Section 4 describes about material used. Section 5 representing proposed method. Section 6 represents experimental setup. Section 7 represents concluding notes.

2. Estimation Of Fractal Dimension

The surface roughness estimation or FD is an essential characteristic of fractal theory for the reason that it has grab information regarding their geometric arrangement. Fractal dimensions of the entire images generally
spreading of pixels. The basic idea for evaluation of FD of an entire image is depend upon the principle of self-similarity. From the property of self-similarity we can say that fractal is normally an irregular or inexact geometric shape that can be broken down in smaller pieces; each is related to the original.

Fractal dimension D of a set X is defined by equation (1).

\[
D = \log(N) / \log(1/r)
\]  

(1)

Where \( N \) is the entire number of dissimilar copies related to X and X is scaled down by a fraction of \( 1/r \). Fractal dimension are usually evaluate the surface roughness of images and accordingly it revels the variation among different grey levels that are found on the image, from the above equation (1) the value of \( N \) is the total no of boxes and has to be evaluated by means of technique called box-counting and the fractal dimension is evaluated accordingly. In this regard several box counting measures are developed. However, we have considering the improved version of differential box counting method [18] since it removes some demerits of differential box counting method [14] that is

- Over counting the amount of boxes covering the image intensity surface.
- Under counting the amount of boxes may occur at the boundary of the neighboring box blocks.

A. Improved Differential Box Counting Method

Liu.et.al [18] was presented improve differential box-counting method for evaluating FD of gray scale image. In this regard author taking into consideration of difference of boxes where the maximum and minimum intensity value falls, they took an image of dimension \( M \times M \) in a 3D plane, where \( (x, y) \) plane representing arrangement of pixel in an image plane, and the third axis \( (z) \) representing gray level. Then the entire no of pixels has been scaled down into block of size of \( l \times l \) where \( l \) are lies among \( M/2 \) and 1. Subsequently they evaluate the reduction factor \( r = 1/M \). For each and every scaled down block, there is a stake of boxes of size \( s \times s \times s' \), where \( s' \) indicates height of each box, another assumption they took \( G/s' = M/s \), where G represents gray-level. Suppose the minimum and maximum gray-levels are \( I_{min} \) and \( I_{max} \) correspondingly in the \((i,j)\)th block. Then the total amount of boxes needed to wrap the block in \( z \) direction is \( n_{old} \) and after shifting the \( \delta \) positions from \( n_{old} \), \( n_{new} \) is calculated. Maximum of \( n_{old} \) and \( n_{new} \) is taken as nr. \( n_r = \max(n_{old}, n_{new}) \). \( n_{old} \) or \( n_{new} \) is calculated as follows:

\[
\begin{cases} 
\text{ceil}(\frac{l_{max}-l_{min}+1}{s'})I_{max} - I_{min} & \text{if } n_r \neq n_{old} \\
1 & \text{otherwise} 
\end{cases}
\]

\( n_r = \sum n_r(i,j) \)

3. Noise Model

Noise represents surplus information which deteriorates image quality. During any processing phase of digital image such as image acquisition, image transmission etc the noise was present in an images. Many aspects are accountable that leads to noise in an image. During image acquisition by camera there are many factors are like sensor temperature, illumination level, dust particle are responsible for creating noise in the image. Subsequently variety of noise models available so far, despite in this research we have considered only three models, that is salt and pepper, gaussian and speckle noise. In this case we form a noisy image as below equation.

\[ n(a,b) = o(a,b) + n(a,b) \]

where \((a,b)\) original image intensity is value and \( n(a,b) \) is the noise present in an image and \( o(a,b) \) is the resultant image.

Salt and pepper Noise

These types of noise are otherwise called Impulse noise or bipolar noise [19, 20], mainly this types of noised are produced due to rapid and sudden disturbance in image signal. These effects can haphazardly scattered in terms of white and black dot pixel above the image. Due to this nature the original intensity values are restored by infected intensity values either by maximum or minimum intensity value for gray scale image i.e., 255 or 0 respectively. The salt & pepper noise is usually arising by means of malfunctioning camera’s sensor, or malfunction of memory cell or by error due to synchronization in the image digitizing.
Gaussian Noise

Gaussian noise also called amplifier noise as it arises in amplifiers or detectors. Random fluctuations in the signal can cause Gaussian noise to the images. It is a kind of additive noise caused by addition of random values to the pixel values. It follows normal distribution. Its probability density function \([19-21]\) is given below.

\[
p(z) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)
\]

Speckle Noise

These types of noise are multiplicative in nature, Hence this can be viewed as arbitrary no multiplication with intensity value of an image. This can be represented as follows

\[
N = \text{Img} + \text{ns*Img}
\]

where, \(\text{Img}\) is the original image, \(\text{ns}\) is the multiplicative uniform noise added in terms two factor called mean and variance and \(N\) is the resultant speckle noised image. This noise is common in coherent light imaging systems like radars & lasers. Its probability density function is given below.

\[
p(z) = \begin{cases} 
\frac{1}{b-a} & \text{if } a \leq z \leq b \\
0 & \text{otherwise}
\end{cases}
\]

4. Material Used

In this section, we are considering three standard researches gray scale images of size 128×128, specifically Lena, froot and baboon respectively, which are represented in Figure. 1. Each images having gray values range of 0 to 255 with 8-bit pixel depth. In order to obtain noise image, we are added noise to each images with suitable extent. Noise specifications are varies based on the noise model. The respected parameters or specifications for salt and pepper, gaussian and speckle noise are decided as 0.005, 0.001 and 0.005 respectively. These values of specification or noise density for individual noise are capricious and taken after accurate selection. Hence, we are considering these three above mention value and enforced on original taken images and subsequent interpretation are taken care so that the consequence of noise is clearly observable in the images. The noisy resultant images are obtained from figure. 1 using three noise techniques is represented in Figure. 2 - 4 respectively.

5. Proposed Method

In this research we have estimated fractal dimension of 3 different types of gray scale images of size 128x128 and checked the noise effect on FD of above mentioned images are tested, for this purpose we examined with various types of noise factors are discussed in section III. The FD of original images and noisy images has been estimated by using Improved Differential Box Counting method and compared. For further validation we evaluate percentage of error and RMSE to set a average percentage error value for estimation of accurate surface roughness of digital images.

A.Methodology

To ascertain the correctness of the proposed method three standard gray scale images were chosen. In addition of noise, we have chosen three noise technique called salt and pepper, Gaussian and speckle noise with different density of 0.5, 0.1, and 0.1 percentages. The algorithm description of the proposed method for estimation of fractal dimension of digital images using improved differential box counting method are discussed in section 2. Our procedure mechanism comprises of mainly three steps. First steps included the estimation of surface roughness (FD) of input data set images and second steps included noise model, each image added with noise and estimate surface roughness of noised images. Finally last step included the analysis of error percentage in terms of RMSE. The root mean square error (RMSE) is estimated in terms original data set images with noised images and estimate PNSR among noised and original images to show the accuracy.

6. Result And Discussion

Proposed method is implemented on matlab12 in windows 8 64 bit operating system, Intel (R) i7 - 4770 CPU @ 3.40 GHz. As fractal dimension basically used to estimate surface roughness, hence in this experiment we first estimate fractal dimension of three data set image namely lena, baboon and froot image are represented on figure 1 and there corresponding FD
values are reported on Table 1. In addition to generate noised image, we are choosing three noise models described in section 3. After generate of noise model we estimate fractal dimension of noised image by improved box counting mechanism and result were represented on Table 1. From this both tables we have seen that fractal dimension is increased in case of noised images. In case of lena the FD value will increase from 2.4355 to 2.4611 for salt and peeper and 2.4355 to 2.4451 for Gaussian and 2.4355 to 2.4528 for speckle. Similarly in other cases the FD value also increased. Therefore we are evaluating what amount of percentage error is increased in case of noised image; Table 2 represents percentage error and Table 3 and Table 4 representing RMSE and PNSR. RMSE and PNSR are inversely proportional to each other; Less RMSE value indicates the result will be closer to the original. In this case we are considering PNSR to check the image correctness between original and noised image. From the Figure 5; it shows that when RMSE value decreased mean while PNSR will increased; From this above experimental work we are finding the average percentage error on each noised image less than equal to 1, Hence from this experiment we conclude that if we set the percentage error as 1, then we can estimate accurate fractal dimension of noised image.

<table>
<thead>
<tr>
<th>Image Name</th>
<th>FD of Original Image</th>
<th>Salt &amp; Pepper</th>
<th>Gaussian</th>
<th>Speckle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>2.3765</td>
<td>2.4085</td>
<td>2.4014</td>
<td>2.4085</td>
</tr>
<tr>
<td>Frost</td>
<td>2.4611</td>
<td>2.4922</td>
<td>2.4790</td>
<td>2.4801</td>
</tr>
<tr>
<td>Baboon</td>
<td>2.4355</td>
<td>2.4611</td>
<td>2.4481</td>
<td>2.4528</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image Name</th>
<th>Percentage of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>0.01</td>
</tr>
<tr>
<td>Frost</td>
<td>0.025</td>
</tr>
<tr>
<td>Baboon</td>
<td>0.015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image Name</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>6.6271</td>
</tr>
<tr>
<td>Frost</td>
<td>6.5947</td>
</tr>
<tr>
<td>Baboon</td>
<td>6.9471</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image Name</th>
<th>PNSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>31.7983</td>
</tr>
<tr>
<td>Frost</td>
<td>31.7808</td>
</tr>
<tr>
<td>Baboon</td>
<td>32.4642</td>
</tr>
</tbody>
</table>

Figure.5. FD variation of original vs noised images

Figure.7. RMSE vs PSNR of Noised images
7. Conclusion

In this research, we are focusing with the noise effect in estimation of FD of gray scale images, since in addition of noise to images become rougher, as a result the resultant fractal dimensions increase because of intensity values present within a box are deviate with some extends and the difference among the maximum and minimum intensity point inside the box is changes with double standard deviation. Due to this noise factor fractal dimension could be increased, so it necessary to analysis what percentages of error are increased in case of noise image. Once the percentage of error identified then the noise image could be used directly for estimating the actual FD.

References
