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An Adoptable Technique to Solve Fuzzy Assignment Problem

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Abstract

Assignment problem is an outstanding subject and is used all the time in solving Problems of engineering and management science. In this paper C_{ij} denotes the cost for assigning jobs $i = 1, 2, ..., m$ to the person $j = 1, 2, ..., n$. In this research paper C_{ij} has been considered to be trapezoidal fuzzy number which are more realistic and general in nature. A Numerical example is taken to illustrate the solution procedure.
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1 Introduction

Assignment Problem is a special type of linear programming problem(LPP) in which our objective is to assign a number of origins to the equivalent number of destinations at any minimum cost (or maximum profit). In a assignment problem, jobs n are to be performed by persons n contingent upon their efficiency to do the job. Every one of the calculations created to locate the ideal arrangement of transportation problems are material to Assignment Problem. In any case, because of its high decadence nature, Kuhn H.W, presented an uncommonly composed calculation purported Hungarian strategy for taking care of Assignment problem in crisp environment.

As of late, Fuzzy Assignment Problems have become much thought. Lin and Wen tackled the assignment problem with fuzzy interval numbers expenses by a labeling algorithm. A method is to solve fuzzy Assignment Problem, with fuzzy cost C_{ij}. Since the objectives are to maximize the total profit or to minimize the total cost, Subject to some crisp constraints, the objective function is considered also as a fuzzy number.

Branch and bound algorithm outline worldview which is by and large utilized for solving combinatorial optimization problems. These problem regularly exponential as far as times unpredictability and may require investigating all conceivable change in most pessimistic scenario. Branch and bound technique is to solve these problems relatively quickly.

Ranking function to solve fuzzy assignment problem using Branch and Bound Technique with Trapezoidal fuzzy numbers. Fuzzy sets was presented by Zadeh in 1965. Give us another scientific apparatus to manage uncertainty of information.

2 Preliminaries

Definition 1. (Fuzzy set)

Let X be a non-empty set. A fuzzy set A in X is represented by its membership function $A : X \to [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

Definition 2. (α - Cut)

The α - Cut of a α- level set of a fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α.
(i.e.,) $\tilde{A}_\alpha = \{ \ x|\mu \ \tilde{A} \ (x) \geq \alpha \ \}$.

Definition 3. (Strong α - Cut)

The Strong α - Cut of a α- level set of a fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α.
(i.e.,) $\tilde{A}^+_\alpha = \{ \ x|\mu \ \tilde{A} \ (x) \geq \alpha \ \}$.

Definition 4. (Fuzzy number)

A fuzzy set A on $\mathbb{R}$ must possess at least the following three properties to qualify as a fuzzy number,
(i) A must be a normal fuzzy set,
(ii) $^\alpha\tilde{A}$ must be closed interval for every $\alpha \in [0, 1]$
(iii) The support of A, must be bounded.

Definition 5. (Trapezoidal fuzzy number)

The fuzzy number A is defined to be a trapezoidal fuzzy number if its membership functions $\mu\tilde{A} \ : \ R \to [0, 1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if x \in [a_1, a_2], \\ 1 & if x \in [a_2, a_3], \\ \frac{a_4-x}{a_4-a_3} & if x \in [a_3, a_4], \\ 0 & otherwise. \end{cases}$$

where $a_1 \leq a_2 \leq a_3 \leq a_4$.

3 Robust's Ranking Technique

Robust's ranking technique which satisfy compensation, linearity additively properties and provides results which are consist human intuition. If $\tilde{a}$ is a fuzzy number then the Robust Ranking is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_L^\alpha, a_R^\alpha)\mathrm{d}\alpha.$$

Where (a_L^α, a_R^α) is the α-level cut of the fuzzy number $\tilde{a}$.

The Robust's ranking for triangular fuzzy number is :

$$R(\tilde{a}) = \int_0^1 0.5[(b-a)\alpha + a, c-(c-b)\alpha]\mathrm{d}\alpha.$$

The Robust's ranking for trapezoidal fuzzy number is :

$$R(\tilde{a}) = \int_0^1 0.5[(b-a)\alpha + a, d-(d-c)\alpha]\mathrm{d}\alpha.$$

This method is for ranking the objective values. The Robust ranking index $R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

4 Algorithm for Assignment Problem using Branch and Bound Method

In this section we give Branch and Bound method for solving fuzzy assignment Problem to obtain the fuzzy total optimal cost with fuzzy members.

Step 1

The objective is to assigning all the jobs to find minimize the total cost of the available persons. One person to one job when the time or costs $\tilde{a}_{ij}$ are fuzzy numbers.

Step 2

The total cost becomes a fuzzy number. Robusts ranking technique, which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition.

Step 3

Hence, we cannot be minimized directly to solve the problem. Hence we defuzzify the fuzzy costs coeffient into crisp one by the fuzzy number ranking method.

Step 4

By using Branch and bound algorithm we find the minimum cost value.

Step 5

Futher Branching the corresponding results in convensional assignment problem in the LPP form. By using this procedure we find the fuzzy optimal cost.

5 Numerical Example

5.1 Robust's Ranking Function for Traphezoidal Fuzzy Number

Assignment:1

Give us a chance to consider a Fuzzy Assignment problem with rows representing to a person $A_1,\ A_2,\ A_3,\ A_4$ and columns represents to the jobs $B_1,\ B_2,\ B_3,\ B_4$. The cost matrix $\tilde{C_{ij}}$ is given then whose elements are trapezoidal fuzzy numbers. In this problem we have to discover the assignment of person to jobs that will minimize the aggregate fuzzy cost. Find the optimal assignment so that the total cost of job assignment becomes minimum.

	B_1	B_2	B_3	B_4
A_1	$(2,3,5,6)$	$(6,4,2,0)$	$(7,8,10,11)$	$(3,5,5,7)$
A_2	$(7,8,10,11)$	$(4,9,6,5)$	$(6,9,4,1)$	$(5,8,9,10)$
A_3	$(15,10,5,4)$	$(5,8,11,12)$	$(7,6,6,5)$	$(6,9,4,1)$
A_3	$(1,2,4,5)$	$(1,4,9,16)$	$(8,5,1,6)$	$(6,8,10,12)$

Solution:

The formulation of fuzzy assignment problem is

Min { R(2,3,5,6)x_{11} + R(6,4,2,0)x_{12} + R(7,8,10,11) x_{13}+ R(3,5,5,7)x_{14} + R(7,8,10,11) x_{21}+ R(4,9,6,5)x_{22} + R(6,9,4,1)x_{23} + R(5,8,9,10)x_{24}+ R(15,10,5,4)x_{31} + R(5,8,11,12) x_{32}+ R(7,6,6,5) x_{33} + R(6,9,4,1)x_{34} + R(1,2,4,5)x_{41} + R(1,4,9,16)x_{42} + R(8,5,1,6) x_{43}+ R(6,8,10,12)x_{44} },

Subject to

$x_{11}+x_{12}+x_{13}+x_{14}$ =1, $x_{11}+x_{21}+x_{31}+x_{41}$ =1,
$x_{21}+x_{22}+x_{23}+x_{24}$ =1, $x_{12}+x_{22}+x_{32}+x_{42}$ =1,
$x_{31}+x_{32}+x_{33}+x_{34}$ =1, $x_{13}+x_{23}+x_{33}+x_{43}$ =1,
$x_{31}+x_{32}+x_{33}+x_{44}$ =1, $x_{13}+x_{23}+x_{33}+x_{44}$ =1,

where $x_{ij} \in [0,1]$,

$$For\ which\ (a_\alpha^L, a_\alpha^U) = \int_0^1 0.5[(b-a)\alpha + a, d-(d-c)\alpha]\mathrm{d}\alpha.$$

Now we calculate $R(2,3,5,6)$ by proposed method.

$$\begin{aligned} R(C_{11}) &= \int_0^1 0.5(\alpha+2+6-\alpha)\mathrm{d}\alpha, \\ &= 4. \end{aligned}$$

Similarly,

$R(C_{12})=3.\ \ R(C_{13})=9.\ \ R(C_{14})=5.\ \ R(C_{21})=9.\ \ R(C_{22})=6.$
$R(C_{23})=5.\ \ R(C_{24})=8.\ \ R(C_{31})=8.\ \ R(C_{32})=9.\ \ R(C_{33})=6.$
$R(C_{34})=5.\ \ R(C_{41})=3.\ \ R(C_{42})=15.\ R(C_{43})=5.\ \ R(C_{44})=9.$

We replace these values for their corresponding to given problem then the results in a typical assignment problem in the LPP form. by solving it, we get the solution a Crisp form of the problem:

	B_1	B_2	B_3	B_4
A_1	4	3	9	5
A_2	9	6	5	8
A_3	8	9	6	5
A_4	3	15	5	9

Initially, no job is assigned for any operator, so the assignment (σ) at the root (level 0) of the branching tree is a null set and the corresponding lower bound is also 0 for each.

Branching guideline : 1 The four different sub problem under the root nodes are shown as in Figure: 2.1. lower bound the solution problems shown on its right hand side.

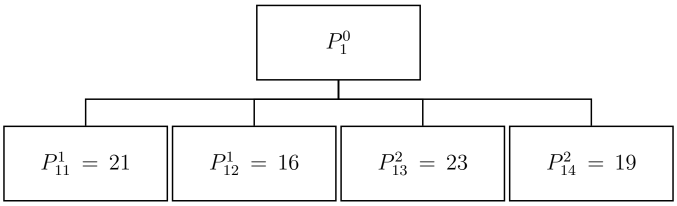

Figure: 2.1

$$V_\sigma = \sum_{i,j\in A} C_{ij} + \sum_{i\in X}\left(\sum_{j\in Y} \min C_{ij}\right),$$

Where $\sigma = \{(11)\}$, $A = \{(11)\}$, $X = \{2,3,4\}$, $Y = \{2,3,4\}$. Then

$$V_{11} = C_{11} + \sum_{i\in 2,3,4}\left(\sum_{j\in 2,3,4} \min C_{ij}\right),$$

$$\begin{aligned} P_{11}^1 &= 4 + (6+6+5), & P_{12}^1 &= 3 + (5+5+3), \\ &= 21. & &= 16. \\ P_{13}^1 &= 9 + (6+5+3), & P_{12}^1 &= 5 + (5+6+3), \\ &= 23. & &= 19. \end{aligned}$$

Branching guideline : 2 Further branching is done from the terminal node which has the least lower bound at this stage, the nodes P_{11}^1, P_{12}^1, P_{13}^1, P_{14}^1 are the terminal nodes. The node P_{12}^1 has the least lower bound. Hence, further branching from this node is shown as in Figure: 2.2.

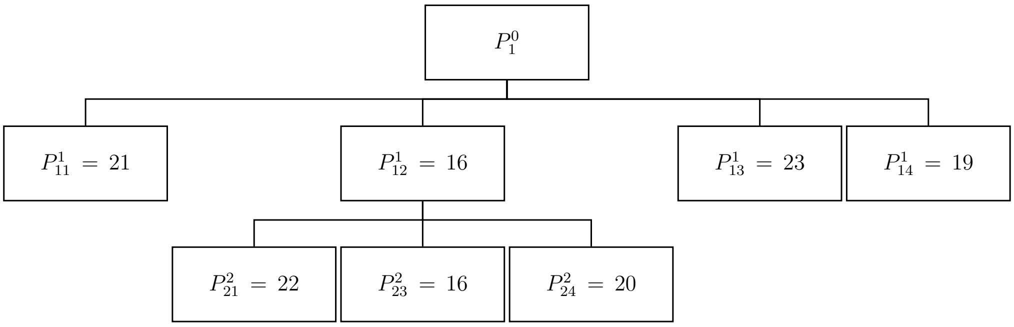

Figure: 2.2

$$V_{21} = C_{11} + C_{22} + \sum_{i \in 3,4} \left(\sum_{j \in 3,4} minC_{ij} \right),$$

$P_{22}^1 = 3 + 9 + 5 + 5,$ $=22.$ $\quad P_{23}^1 = 3 + 5 + (5 + 3),$ $= 16.$ $\quad P_{23}^1 = 3 + 8 + 6 + 3,$ $= 20.$

Branching guideline : 3 At this stage the nodes P_{12}^1, P_{21}^2, P_{23}^2, P_{24}^2, are the terminal node. The node P_{23}^2 has the least lower bound. Hence, further branching from this node is shown as in Figure: 2.3.

$$V_{31} = C_{12} + C_{23} + C_{31} + \sum_{i \in 4} \left(\sum_{j \in 4} minC_{ij} \right),$$

$$P_{31}^3 = 3 + 5 + 8 + 9, \quad P_{34}^3 = 3 + 5 + 5 + 3,$$
$$= 25. \qquad = 16.$$

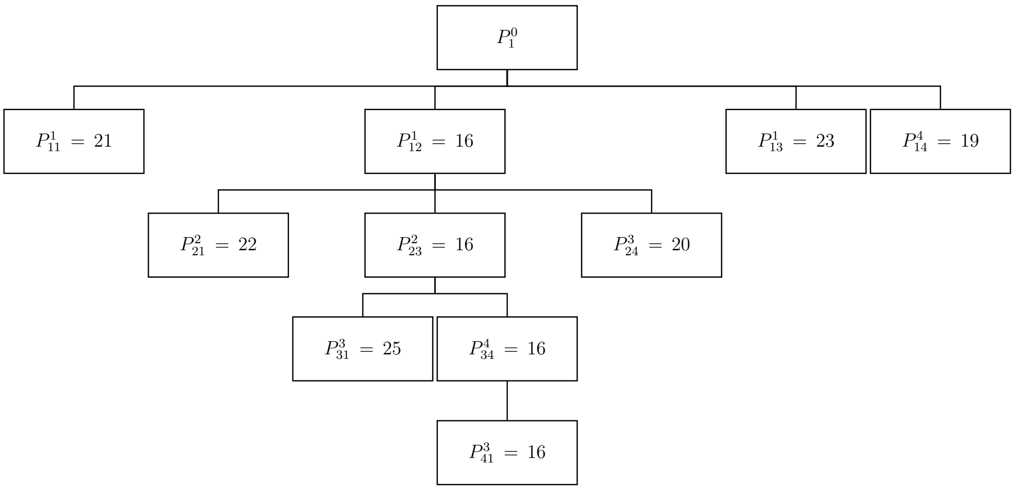

Figure: 2.3

The optimal assignment path is (1,2), (2,3), (3,4), (4,1).
The calculation of fuzzy optimal cost is
(6, 4, 2, 0) + (6, 9, 4, 1) + (6, 9, 4, 1) + (1, 2, 4, 5)= (19, 24, 14, 7).
Also, 3 + 5 + 5 +3 = **16**.

6 Conclusion

In this paper, the assignment cost has been considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. By using Robust ranking indices the fuzzy assignment problem has been converted into a crisp assignment problem and then Branch and bound method has been applied to find an optimal solution. Numerical example has been shown that the total cost obtained is optimal. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function.

References

[1] **Bai. X. J, Liu. Y. K and Shen S. Y,** Fuzzy generalized assignment problem with credibility constraints, Proceedings of the Eighth International Conference on Machine Learning and Cybernetics, Baoding, (2009), 657-662.

[2] **Chang. W. K,** Ranking of fuzzy utilities with triangular membership function. In International Conference on Plicy Analysis and Informations Systems Tamkang University, (1981), 163-171.

[3] **Chen. S. H,** Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, (1985), 113-129.

[4] **Dubois. D, and Fortemps. P,** Computing improved optimal solutions to max-min flexible constraint satisfaction problems, Operations Research, (1999), 95-126.

[5] **Fortemps, P, and Roubens, M.,** Ranking and defuzzification methods based area compensation, fuzzy sets and systems, (1996), 319-330.

[6] **Geetha. S, and Nair. K. P. K,** A variation of the assignment problem, European Journal of Operation Research, (1993), 422-426.

[7] **Kalaiarasi. K, Sindhu. S and Arunadevi. M,** Optimization of fuzzy assignment model with triangular fuzzy numbers, International Journal of Innovative Science, Engineering and Technology, (2014), 2348-7968.

[8] **Kuhn. H. W,** The Hungarian method for assignment problem, Naval Research Logistics Quarterly, (1955), 83-97.

[9] **Marius Posta., et.al,** ”An Exact Method with variable fixing for solving the generalized assignment problem, CIRRELT, (2011).

[10] **Nagarajan. R and solairaju. A,** A computing improved fuzzy optimal Hungarian. Assignment problem with fuzzy cost under Robust ranking technique, internal journal of computer application, (2010), 6-13.

[11] **S. Narayanamorthy, S. Kalyani,** A Modified Concept of the Optimal Solution of the Transportation Problem in Fuzzy Environment, *IJPEM*, vol 9, PP: 1575-1580, (2014).

[12] **S. Narayanamorthy, S Maheswari,** Finding Fuzzy Critical Path by Metric Distance Ranking Method Using Fuzzy Numbers, (2014).

[13] **Ventepaka Yadaiah, Haragopal. V. V,** "A new approach of solving single objective unbalanced assignment problem", American Journal of Operation Research, (2016), 81-89.

[14] **Votaw. D.F and Orden. A,** The personal assignment problem, Symposium of linear in qualities and programming, Project SCOOP 10, US Air Force Washington, (1952), 155-163.

[15] **Wang. X,** Fuzzy optimal assignment problem, Fuzzy Mathematics, (1987), 101-108.

[16] **Yager. R. R,** A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, (1981), 143-161.

[17] **Zadeh, L. A,** Fuzzy sets, Information and Control, (1965), 338-353.