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Capacitated Facility Location Problems, With Partial Assignment and Multiple Time Periods for Effective Management of Municipal Solid Waste

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Abstract

Waste is transferred to transfer stations daily, as many times as required, due to constraints on vehicle capacity. These wastes are segregated and transferred to other facilities like landfill, recycling units, compost plants etc. thrice a week. The model can also be used to find locations of compost plants.The objective of the model is to minimize the total cost of waste management. Mixed integer programming is used to arrive at the optimal solution and binary variables indicate whether solid waste is transported from a transfer station to compost plant. The model is tested using data from various related sources.

AMS Subject Classification: 46N10

Key Words and Phrases: Capacitated facility location models, Multiple time periods, Partial Assignment, Mixed integer programming model.

1 Introduction

Waste management is the set of activities that include (a) collection, transport, treatment and disposal of waste, (b) control, monitoring and regulation of the production, collection, transport, treatment and disposal of waste and (c) prevention of waste production through in-process modifications, reuse and recycling. Most of the existing solid waste treatment system in developing and underdeveloped countries are not very effective.[2]

In this paper, we consider three main concepts, namely:

- Waste generated in wards of a city and usage of transfer stations to minimize vehicle movement.

- Multiple time periods to pick up waste from collection points and removal to transfer stations.
- Opening of facilities like compost plants.

A transfer station generally refers to a place where waste is transferred from smaller collection vehicles to larger, long haul trucks or trains.

2 Literature Review

Marks made the first reference to transfer stations.[8] This model, using mixed integer facility location problem had the objective of finding transfer facilities to serve various demand and disposal facilities so that the total cost of facilities and transshipment is minimized. This model can be considered as the basic model as most of the subsequent models are the modifications of this model. Several modifications to the basic model were given by Marks *et al.*[8] who included transport costs.

The model of Marks and Liebman was modified by Rahman and Kuby[11], who considered a second objective function to minimize the public opposition which is derived empirically from an opinion survey data. A new dimension was thus added by these work location problems to solid waste problem. Shirazi *et al.*[12] applied a novel linear programming model to the city of Tehran to reduce the number of transfer stations and processing units.

Three issues are studied in dynamic location models with a multi-period operating context.This means that the demand varies between different time periods. The main concepts are the best places to locate the available facilities, the best capacity to assign to the generic logistic facility and which periods of time demand a certain amount of production capacity.

Jena *et al.*[6] considered multi-period facility location problem with multiple commodities and multiple capacity levels, which allowed for relocation and temporary closing of parts of the facilities by using particular capacity constraints that involve integer rounding of the allocated demands.

3 Formulation of the problem

We see the formulation and applications of a model based on transfer stations for transport of waste over different time periods.

The model is considered for weekly data, transport from collection point is done daily but only thrice a week from transfer station to landfills and compost plants.

3.1 Assumptions of the Model

1. Waste is collected more than one time from collection points. This could be due to the fact that some residents might not handover waste in the first time period and also to ease burden on roads due to heavy vehicular traffic.
2. Wastes from collection points are moved to the transfer stations.
3. Only waste from residences is considered.

4. Separation of waste is carried out majorly at the source and, where needed, at transfer stations.

5. Costs of transportation depend on distance and waste volume transported.

6. Distances are considered from the center of compost plants/streets/landfill.

7. The compost plant completely composts wet waste and there is no waste needs to be carried to the landfill.

The proposed model in this study has been formulated with the objective of reducing the cost of waste management. While satisfying the demand varying over time and satisfying capacity constraints over time horizon, the fixed and variable costs as well as transportation costs have been considered in this study. By altering the model and its parameters, the model can be used for finding optimum locations for different types of facilities. It has been tested with data from different wards and apartment blocks of Bangalore city.

3.2 Problem Description

Consider a geographical region where households exist. Each household generates waste which has to be cleared over a discrete and finite time horizon.

Each facility has a limit or capacity and every collection point can be served by one or more facilities. Transportation cost of waste is proportional to quantity and distance.

1. Waste is transferred from collection points at various time periods and is taken to transfer station daily.

2. Compostable waste is sent to compost plant thrice a week from transfer stations.

3. Remaining waste is sent to either landfills or to recycling units, depending on their usefulness thrice a week. This reduces transportation cost as well as congestion on roads.

4. Waste is compacted to nearly 18 % of its weight so this point is considered to reduce the weight which exists from transfer station.

5. Capacity and transportation cost remains same in all time periods.

3.3 Indices

- $t = 1, 2, \ldots, n$ denotes set of different time periods at which waste can be picked up

- $T = 1, 2, \ldots, n_1$ denotes the set of transfer stations

- $a = 1, 2, \ldots, n_2$ denotes set of collection points in the administrative area.

- $cp = 1, 2, \ldots, n_3$ denotes set of compost plants which can be set up in the administrative area
- $l = 1, 2, \ldots, n_4$ denotes set of r of landfills (Generally not more than 2)

3.4 Variables (Input parameters) used in the Model

The variables used in the model are as follows:

- W_a = total amount of waste generated at collection point a
- F_T = Fixed cost at transfer station T (this includes labor and storage materials like plastic sheets for covering, waste shovels and other equipment)
- $\dot{F}_{cp}$ = Fixed cost at compost plant per day
- $\ddot{F}_l$ = Fixed cost at landfill
- R_T = Running cost / ton of a transfer station T
- $\dot{R}_{cp}$ = Running cost/ ton of a compost point cp
- $\ddot{R}_l$ = Running cost / ton of a landfill (tons) l
- W_T = capacity of transfer station T (tons)
- $\dot{W}_{cp}$ = capacity of compost plant cp (tons)
- W_l = capacity of landfill l (tons)
- N_{cp}= Number of compost plants constructed
- $(TC)_{aT}$ = Transportation cost/ton from collection point A to transfer station T
- $(\dot{T}C)_{T_{cp}}$ = Transportation cost/ton from transfer station T to compost plant cp
- $(\ddot{T}C)_{Tl}$ = Transportation cost/ton from transfer station T to landfill l

3.5 Decision Variables

The decision variables used in the model are as follows:

- X_{taT} = amount of waste to be moved from collection point a to transfer station T at time period t
- $Y_{T_{cp}}$ =amount (in ton) of solid waste to be removed from transfer station T to compost plant cp
- Z_{T_l} = amount (in ton) of solid waste to be removed from transfer station t to landfill. (A landfill caters to more than one transfer station)

- A_{cp} = A binary variable which can take value of 1 if waste is sent to composting plant at location cp and 0 otherwise

These variables indicate the optimal amount of waste which can be moved at different time periods to transfer stations and also which locations would be feasible for setting up/ reusing a compost plant.

4 Mathematical Formulation of the Proposed Model

The cost of two components are as follows:

1. Fixed and operating cost, calculated according to amount of waste coming in
2. Transportation cost, calculated per distance and amount of waste coming in

In the proposed model, however, the capital costs are ignored as, the cost of setting up transfer stations is quite huge and break-even is achieved after a few years.

The mathematical equation used for the model is given as follows.

$$\begin{aligned} Min\ z \ = \ & \sum_{T=1}^{n} F_T + \sum_{t=1}^{n}\sum_{a=1}^{n_2}\sum_{T=1}^{n_1} R_T X_{taT} + \\ & \sum_{cp=1}^{n_3} \dot{F}_{cp} A_{cp} + \sum_{T=1}^{n}\sum_{cp=1}^{n_3} \dot{R}_{cp} A_{cp} Y_{T_{cp}} + \\ & \sum_{T=1}^{n}\sum_{l=1}^{n_4} \ddot{R}_l Z_{Tl} + \sum_{a=1}^{n_2}\sum_{t=1}^{n_1}\sum_{T=1}^{n} (TC)_{aT} X_{taT} + \\ & \sum_{T=1}^{n}\sum_{cp=1}^{n_3} (\dot{T}C)_{cp} Y_{cp} + \sum_{T=1}^{n}\sum_{l=1}^{n_3} (\ddot{T}C)_{T_l} Z_{tl} \end{aligned}$$

The constraints of the model are such that

$$\sum_{t=1}^{n}\sum_{a=1}^{n_2}\sum_{T=1}^{n_1} X_{taT} \leq W_T \ \forall \ a \tag{1}$$

$$\sum_{t=1}^{n}\sum_{a=1}^{n_2}\sum_{T=1}^{n_1} X_{taT} = \sum_{T=1}^{n_1}\sum_{cp=1}^{n_3} Z_{tcp} + \sum_{t=1}^{n_1}\sum_{l=1}^{n_4} Y_{tl} \tag{2}$$

$$\sum_{t=1}^{n}\sum_{a=1}^{n_2}\sum_{T=1}^{n_1} X_{taT} \leq W_T \ \forall \ T \tag{3}$$

$$\sum_{T=1}^{n_1}\sum_{l=1}^{n_4} Z_{Tl} \leq W_l \ \forall \ l \tag{4}$$

$$\sum_{T=1}^{n_1}\sum_{cp=1}^{n_3} Y_{T_{cp}} \leq W_{cp} \ \forall \ cp \tag{5}$$

and

$$X_{taT} \geq 0, Y \geq 0, Z \geq 0, A_{cp} \in \{0, 1\} \tag{6}$$

The first constraint arises due to the fact that all waste collected at houses should be transported to transfer station in at least one time slot. The second constraint is all waste transported to transfer station should be taken to compost plant and landfill. The third constraint says that the amount of waste sent from all houses should be less than or equal to daily capacity of transfer station. The fourth constraint is regarding the amount of waste sent from all transfer stations to landfill less than or equal to daily capacity of landfill. The next constraint indicates that the amount of waste sent from all transfer stations to compost plant is less than or equal to daily capacity of compost plant. The next constraint is regarding the number of compost units which should not exceed the maximum number of compost units in that area. Non negativity restrictions arise due to the fact that physical quantities of waste have to be transported.

5 Testing of the model using data from various wards of Bangalore city

The city of Bangalore is one of the fastest growing cities in India and faces a huge garbage problem, though large amounts are spent by civic authorities to tackle this problem.

5.1 Transfer stations are introduced along with capacity constraints

We introduce two types of capacity constraints in this mode.

1. Capacity of vehicles
2. Capacity of transfer station

In this case, we consider two costs of transportation - one for transporting from wards to transfer stations and compactors to transport from transfer stations to landfills.

There will be two kinds of running (operating costs) - one at transfer station and one at landfill. For the purposes of calculating distances from transfer station, one in each of the following 6 localities in the ward were considered.

Table 1. Details of collection points

Collection point	Location
A	Basavanagudi
B	RBI extension
C	S.R.Krishnappa Garden
D	Jayanagar 3rd East Block
E	Jayanagar 1st Block
F	Jayanagar 2nd Block

The transfer station is at JP Nagar. These transfer stations being on the border of a minimum of two wards ensures that waste is transported with minimal transportation cost. Distances are calculated from the ward centroids. The optimum allocations from using LINGO17.0 is as follows.

Cost = Rs. 287318.8 per week

Reduction in cost due to application in the model= Rs. 287318.8 per week.

% of reduction in Cost= 9.256323

Thus, transfer stations that compact waste can bring down the cost of transportation significantly. This is due to the fact that

1. Transfer stations compress waste to nearly 18 % of its original weight.
2. Transport from collection points to transfer station is done daily but transport to landfill (longer distance) is done only thrice a week.

5.2 Considering two transfer stations and two compost plants in the model

In this case, we consider two potential locations for compost plants, namely B and C near to transfer stations. These distances have been kept less than 2km from transfer stations, else, if distances are greater, the purpose of model will be lost. Also, in Bangalore city, distances to parks (potential compost centers) is around the same. If we have a constraint on capacity on compost plants, we may require more than one. Then, the dynamic facility model involves finding location of these plants as well as optimum allocation.

Optimum allocation using LINGO is as follows.

Objective function value= Rs. 280362.9 Reduction in cost= Rs. 6955.9 per week % reduction per week= 2.42.

Using this model, we can also determine the amount to be transferred at each time horizon to transfer station.

Table 2: The objective function value and percentage composted

Percentage composted	Objective function value with benefits(Rs)	Amount shipped to landfill(tons)
20	284536.5	9.595872
30	283145.3	8.396388
40	281754.1	7.196904
50	280362.9	5.9974
60	278971.7	4.7979

6 Significance of the model

- This model will be helpful to municipalities and companies involved in waste management for planning for facilities and managing resources.
- The results of this study will add more information to the present literature and pave the way for further research in the area.
- This model can help civic agencies to frame policies and plan waste management programs and help in creating awareness on segregation, composting, etc. By using this model in wards and apartment blocks, the benefits from facilities can be passed on to the citizens.
- There can be quantification of costs through this study and a comparison with other options will give an idea about the optimal situation.

7 Conclusion, Limitations of the Study and Scope of Further Research

The model has to be tested under a large administrative area. Proper segregation and adherence to pick up levels and time windows is crucial for the implementation of the model. A pilot study can be done in a densely populated residential area wherein collection centers and transfer stations are fixed by the administration. It can further be developed exclusively for recycled goods in industrial areas.

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Appendix

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