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Skin Friction Analysis of Parabolic Started Infinite Vertical Plate with Variable Temperature and Variable Mass Diffusion in the Presence of Magnetic Field

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Abstract

The present paper is on study of the influence of flow past an impulsively started infinite vertical plate with variable temperature and variable mass diffusion in the presence of transverse applied magnetic field. The basic governing equations are reduced to non-dimensional equations valid with the imposed initial and boundary conditions. The exact solutions are obtained by using Laplace transform technique. A magnetic field of uniform strength is applied normal to the direction to the flow. The numerical computations are carried out for the values of the physical parameters such as velocity, temperature, concentration, skin friction, Sherwood number and Nusselt number are discussed and presented graphically.

Key Words: Parabolic, variable temperature, MHD, vertical plate, variable mass diffusion, magnetic field.

1. Introduction

The experimental and theoretical studies of magneto-hydrodynamics flows are paramount from a technological point of view, since they have many applications, as in magneto-hydrodynamics electrically conducting fluids etc., The influence of a magnetic field on viscous incompressible flow of electrically conducting fluid is of accent in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, magnetic materials processing, glass manufacturing control processes and the paper industry in different geophysical cases etc., In many process industries, the cooling of threads or sheets of a bit polymer materials is of importance in the production line. Magneto convection plays an imperative role in various industrial applications including magnetic control of molten iron flow in the steel industry and liquid metal cooling in nuclear reactors.

The heat Transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in non-conventional energy sources such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal diffusion is of considerable interest in nature and in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fibre and granular insulation, electronic system cooling, cool combustors, oil extraction, thermal energy storage and flow through filtering devices, porous material regenerative heat exchanger. Many transport processes exist in nature and industrial application in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of species. In the last few decades several efforts have been made to solve the problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering.

MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. [10]. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate were also studied by Soundalgekar et al. [10]. The dimensionless governing equations were solved using Laplace transform technique. In all above studies the stationary vertical plate considered. Raptis and Perdikis [8] studied the effects free convection flow past a moving vertical plate. The governing equations were solved analytically. The governing equations were solved by the Laplace transform technique. Vijaya, N. Ramana

Reddy, G.V[11] have analyzed the thermal diffusion effect on MHD free convection and mass transfer flow. V. Rajesh and S.V.K. Varma [9] have studied the importance of the effects of thermal-diffusion (mass diffusion due to temperature gradient). M.S. Alam, M.M. Rahman and M.A. Samad [2] studied the thermal-diffusion effect on unsteady MHD free convection and mass transfer flow past an impulsively started vertical porous plate. Recently, M. S. Alam, M.M. Rahman and M.A. Maleque [3], studied combined free convection and mass transfer flow past a vertical plate with heat generation and thermal-diffusion through porous medium. The aim of the present study is to the influence of flow past an parabolic started infinite vertical plate with variable temperature and variable mass diffusion in the presence of transverse applied magnetic field. The dimensionless governing equations are solved using Laplace-transform technique. The solutions for velocity, temperature concentration fields, Sherwood number and Nusslet number are derived in terms of exponential and complementary error functions.

2. Mathematical Formulation

The unsteady flow of a viscous incompressible fluids past an infinite vertical plate with variable temperature and variable mass diffusion in the presence of magnetic field has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration c'_∞. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are the same temperature T_∞ and concentration c'_∞. At time $t' > 0$,the plate is parabolic stared with the velocity $u = u_0 t'^2$ in its own plane against gravitational field and the temperature from the plate raised to T_w and the concentration level near the plate are also raised to $c'_w$with time $t$and the mass is diffused from the plate to the fluid. Since the plate is infinite in length all the terms in the governing equations will be independent of x and there is no flow along y-direction. The infinite vertical plate is also subjected to a uniform magnetic field of strength B_0 is assumed to be applied normal to the plate. Then the unsteady flow is governed by free-convective flow of an electrically conducting fluid in a parabolic started motion under usual Boussinesq's and boundary layer approximation is governed by the following dimensionless form of equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C" - C"_\infty) + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2{}_0}{\rho}u \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C"}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions:

$$\left.\begin{array}{l} u = 0, \quad T = T_\infty, \quad C' = C'_\infty \text{ for all } y, t' \leq 0 \\ t' > 0: u = u_0 t'^2,\ T = T_\infty + (T_w - T_\infty)At', C' = C'_\infty + (C'_w - C'_\infty)At' \text{ at } y = 0 \quad (4) \\ u \to 0, \quad T \to T_\infty,\ C' \to C'_\infty \quad \text{as } y \to \infty \end{array}\right\}$$

Where $A = \left(\frac{u^2{}_0}{v}\right)^{1/3}$,On introducing the following non-dimensional quantities:

$$\left.\begin{array}{l} U = u\left(\frac{u^2{}_0}{v}\right)^{1/3}, \quad t = \left(\frac{u^2{}_0}{v}\right)^{1/3} t', \quad Y = y\left(\frac{u^2{}_0}{v}\right)^{1/3} \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\beta\,(T_w - T_\infty)}{(vu_0)^{1/3}} \\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gc = \frac{g\beta(C'_w - C'_\infty)}{(vu_0)^{1/3}} \\ M = \frac{\sigma B^2{}_0}{\rho}\left(\frac{v}{u^2{}_0}\right)^{1/3}, \quad Pr = \frac{\mu C_p}{K}, \quad Sc = \frac{v}{D} \end{array}\right\} \quad (5)$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - MU \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (8)$$

The initial and boundary conditions in non-dimensional quantities are

$U = 0\,, \quad \theta = 0\,, \quad C = 0$ for all $Y, t \leq 0$

$t > 0$: $U = t^2\,, \quad \theta = t\,, \quad C \to t$ at $Y = 0$(9)

$U \to 0\,, \ \theta = 0\,, \quad C \to 0$ as $Y \to \infty$

The non-dimensional quantities are defined in the Nomenclature.

3. Method of Solution

The dimensionless governing equations (6) to (8), subject to the corresponding initial and boundary conditions (9) are tackled using Laplace technique and the solutions are derived as follows:

$$\theta = t\left[(1 + 2\eta^2 Pr)erfc(\eta\sqrt{Pr}) - \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}}\exp(-\eta^2 \mathrm{Pr})\right] \quad (10)$$

$$C = t\left[(1 + 2\eta^2 Sc)erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 \mathrm{Sc})\right] \quad (11)$$

$$\begin{aligned} U = & \left[2\,\frac{(\eta^2 + Mt)t}{4M}\left[\exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt}) + exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt})\right]\right. \\ & \left. + c\left[\exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt}) - exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt})\right] - \frac{\eta t}{2M\sqrt{\pi}}exp(-(\eta^2 + Mt))\right] \\ & + a\left[\frac{(\eta^2 + Mt)t}{4M}\left[\exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt}) + exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt})\right]\right. \\ & \left. + c\left[\exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt}) - exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt})\right] - \frac{\eta t}{2M\sqrt{\pi}}exp(-(\eta^2 + Mt))\right] \\ & + b\left[\frac{(\eta^2 + Mt)t}{4M}\left[\exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt}) + exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt})\right]\right. \\ & \left. + c\left[\exp(-2\eta\sqrt{Mt})erfc(\eta - \sqrt{Mt}) - exp(2\eta\sqrt{Mt})erfc(\eta + \sqrt{Mt})\right] - \frac{\eta t}{2M\sqrt{\pi}}exp(-(\eta^2 + Mt))\right] \\ & + a\left[d\left[(3 + 12\eta^2 Pr + 4\eta^4 Pr^2)erfc(\eta) - \frac{\eta\sqrt{Pr}}{\sqrt{\pi}}(10 + 4\eta^2 Pr)e^{-\eta^2 Pr}\right]\right] \\ & + b\left[d\left[(3 + 12\eta^2 Sc + 4\eta^4 Sc^2)erfc(\eta) - \frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10 + 4\eta^2 Sc)e^{-\eta^2 Sc}\right]\right] \end{aligned}$$

Where $a = \frac{Gr}{(1-Pr)+M}$, $b = \frac{Gc}{(1-Sc)+M}$, $c = \frac{\eta\sqrt{t}(1-4Mt)}{8M^{3/2}}$, $d = \frac{t^2}{6}$and $\eta = \frac{Y}{2\sqrt{t}}$(12)

4. Nusselt Number

From temperature field, TheNusselt number (rate of change of heat transfer) which is given in non- dimensional form as

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} \quad (13)$$

From equations (10) and (13), We get a Nusslet number as follows,

$$Nu = \left[-\frac{1}{2\sqrt{t}}\left[t\left(erfc(\eta\sqrt{Pr}) + \exp(-\eta^2 \mathrm{Pr})\left(\frac{\sqrt{Pr}}{\sqrt{\pi}}\right)\right)\right]\right]$$

5. Sherwood Number

From concentration field, The Sherwood number (rate of change of mass transfer) which is given in non- dimensional form as

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (14)$$

From equations (11) and (14), We get a Nusslet number as follows,

$$Sh = \left[-\frac{1}{2\sqrt{t}}\left[t\left(erfc(\eta\sqrt{Sc}) + \exp(-\eta^2 \mathrm{Sc})\left(\frac{\sqrt{Sc}}{\sqrt{\pi}}\right)\right)\right]\right]$$

6. Skin-Friction

The study skin-friction from velocity field. Is given in non-dimensional form as

$$\tau = -\left(\frac{\vartheta U}{\vartheta y}\right)_{y=0} \quad (15)$$

From equations (12) and (15), we get skin-friction as follows:

$$\begin{aligned}\tau = \Bigg[&-\frac{1}{2\sqrt{t}}\left[\frac{t^2}{2}\left(exp(2\eta\sqrt{Mt})erfc(\sqrt{Mt})\right) + c\left(exp(-2\eta\sqrt{Mt})erfc(\sqrt{Mt})\right)\right] \\ &+ a\left[\frac{t^2}{2}\left(exp(2\eta\sqrt{Mt})erfc(\sqrt{Mt})\right) + c\left(exp(-2\eta\sqrt{Mt})erfc(\sqrt{Mt})\right)\right] \\ &+ b\left[\frac{t^2}{2}\left(exp(2\eta\sqrt{Mt})erfc(\sqrt{Mt})\right) + c\left(exp(-s2\eta\sqrt{Mt})erfc(\sqrt{Mt})\right)\right] + a\left[d\left(\left(3\,erfc(\eta\sqrt{Pr})\right) - \frac{5\sqrt{Pr}}{\sqrt{\pi}}\right)\right] \\ &+ b\left[d\left(\left(3\,erfc(\eta\sqrt{Sc})\right) - \frac{5\sqrt{Sc}}{\sqrt{\pi}}\right)\right]\Bigg]\end{aligned}$$

Where $a = \frac{Gr}{M+(1-Pr)}$, $b = \frac{Gc}{M+(1-Sc)}$,$c = \frac{\sqrt{t}-4Mt^{3/2}}{8M^{3/2}}$, $d = \frac{t^2}{6}$ and $\eta = \frac{Y}{2\sqrt{t}}$

7. Results and Discussion

For physical understanding of the problem numerical computations are carried out for different parameters depends upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.21which corresponds to water vapor. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 7).We have plotted velocity profiles for different values of the physical parameters M(Magnetic field parameter), Pr(Prandtl number), Sc(Schmidt number) and t(time) and concentration(C) in Figure.1 to Figure.11 for the case of heating of the plate (Gr < 0, Gc< 0). The heating take place by setting up free convection current due to concentration gradient and temperature gradient.

Figure.1 shows that the different values of time t (t=0.2 ,0.4, 0.6) on the concentration profile for Sc=0.16. it is observed that the concentration of the fluid enhances with the increasing values of t.

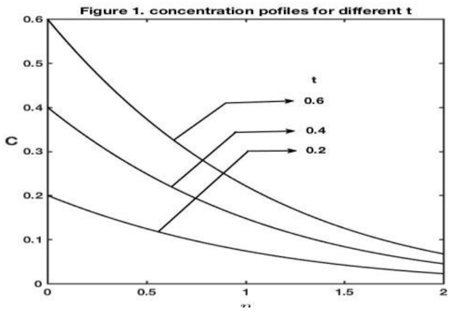

Figure .2 depicts the influence of Schmidt number Sc (Sc=0.16, 0.6, 2.01) on the concentration profiles at t=0.3. As the Schmidt number increases, the concentration decreases gradually.

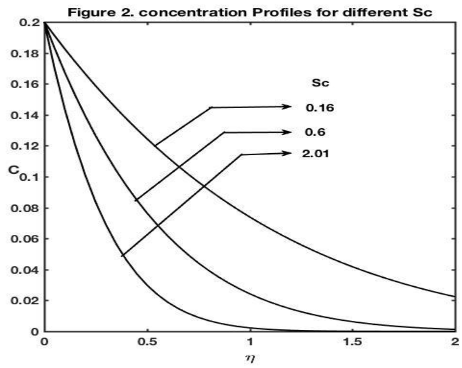

Figure.3 displays that the effects of time t(t=0.2 ,0.4 ,0.6) on the temperature profile for Pr=7. it is noticed that in time t results an increases in the temperature field.

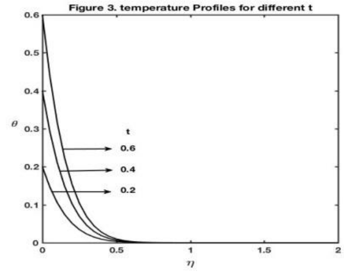

Figure.4 shows that the behavior of the temperature for different values of Prandtl number Pr(Pr=0.71, 2.0, 7.0) at t=0.2. It is perceived that an increases in the Prandtl number results decreases.

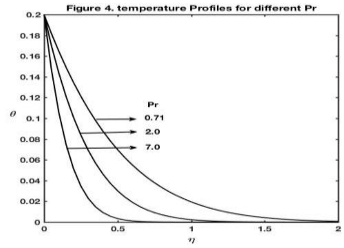

Figure.5 illustrate the influences of M(Magnetic field parameter) on the velocity field in cases of heating of the plate when (M=2,6,10),Gr=Gc=5,Pr=7 and t=0.3.It is observed that the velocity increases with decreasing values of magnetic field parameter. This shows that the increasing in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

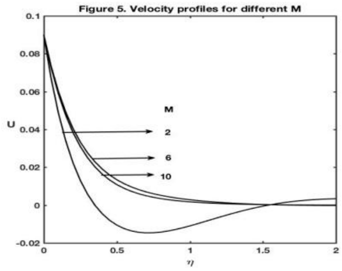

Figure.6 is a graphical representation which depicts the velocity profiles for different values of Pr [0.17,7.0,2.0]. It is clear that the wall concentration lowers with heightened value of Pr (Prandtl number).

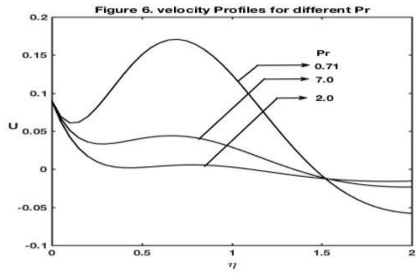

Figure.7 depicts that the flow behavior of the velocity profiles for different t (t=0.2,0.4.0.6,0.8), M=2, Gr=Gc=5, Pr=7 are discussed and implemented. Indeed, It maximizes the effects of magnetic flow field that the velocity increases with increasing values of time t.

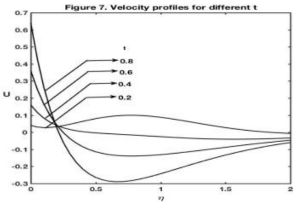

Figure 7. Velocity profiles for different t

Figure.8 shows that the velocity profile for different Schmidth number. The effect of velocity field for different value of time(t=0.2,0.4,0.6,0.8) in the presence of air. It is observed that the plate velocity increases with increasing value of time t.

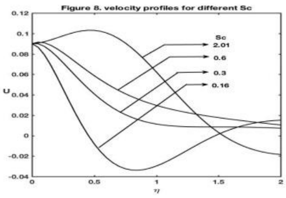

Figure 8. velocity profiles for different Sc

Figure.9 appears that the flow effects of different thermal Grashof number (Gr=2,5), mass Grashof number (Gc=5,10) with the corresponding Prandtl number(Pr)=7 and M=4 on the velocity at time t=0.3. It can be observed that the velocity increases with increasing values of the internal Grashof number or mass Grashof number.

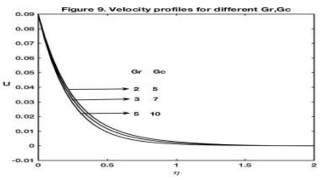

Figure 9. Velocity profiles for different Gr,Gc

Figure.10 explains that the Nusselt number with different value of time t (t=0.2 ,0.4,0.6,0.8),M=2,Gr=Gc=5,Pr=0.71 are graphically presented. Figure .11 shows that Sherwood number increases as increasing value of Sc(Schmidt number).

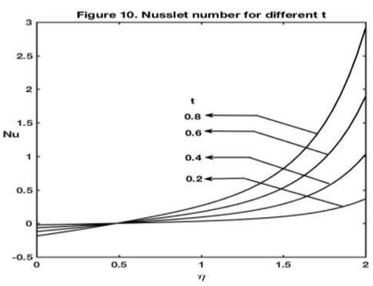

Figure 10. Nusslet number for different t

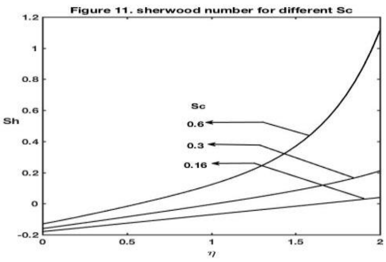

Figure 11. sherwood number for different Sc

Table 1 and Table 2 displays the effect of skin friction for the different values of thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, and time t. Talble 1 shows that the effect of skin friction in the presence of air (Pr=0.71) and table 2 depicts the effect of skin friction in the presence of water Pr(7.0). In both tables,we observe the skin friction increases with the increasing values of mass Grashof number and skin friction enhances with the increasing values of Sc. When time t increases, the value of Skin friction decreases.

Table 1: Skin friction profiles for air

t	Gr	Gc	Sc	τ
0.2	2	5	0.6	-0.3874
0.2	5	5	0.16	-0.5125
0.2	5	5	0.6	-0.5912
0.2	5	5	2.01	-0.2442
0.2	5	10	0.6	-0.8220
0.4	5	5	0.6	-0.2720
0.6	5	5	0.6	-0.4675

Table 2: Skin friction profiles for water

t	Gr	Gc	Sc	τ
0.2	2	5	0.6	-0.2437
0.2	5	5	0.16	-0.1532
0.2	5	5	0.6	-0.2319
0.2	5	5	2.01	0.1151
0.2	5	10	0.6	-0.4627
0.4	5	5	0.6	-0.1947
0.6	5	5	0.6	-0.5566

8. Conclusion

Skin Friction analysis of parabolic started infinite vertical plate with variable temperature and mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique, which can be obtained from the graphical results are:

- Velocity increases with increasing thermal Grashof number or mass Grashof number.
- Velocity increases with increasing values of the time t, but the trend is just reversed with respect to the Schmidt number.
- Nusselt number is greater with decreasing Prandtl number.
- Sherwood number is increases with increasing value of Sc.

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Nomenclature

A constants

B_0 external magnetic field

C dimensionless concentration

C_p specific heat at constant pressure $J.kg^{-1}.k$

$C^{'}$ species concentration in the fluid kgm^{-3}

C'_w wall concentration in the field
C'_∞ concentration in the fluid far away from the plate
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.k^{-1}$
Pr prandtl number
Sc Schmidt number
Sh Sherwood number
Nu Nusslet number
T temperature of the fluid near the plate
T_w temperature of the plate
T_∞ temperature of the fluid far away from the plate
t dimensionless time
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
U dimensionless velocity
y co-ordinate axis normal to the plate m
Y dimensionless co-ordinate axis normal to the plate
M Magnetic field parameter

Greek Symbols

β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $m^2.s^{-1}$
σ electrical conductivity
ρ density of the fluid $kg.m^{-1}.s^{-2}$
τ dimensionless skin-friction $kg\ .m^{-1}.s^{-2}$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions