VENDOR-BUYER COORDINATION MODEL WITH SHORTAGES AND SCREENING PROCESS

S. Ganesh¹, M.K. Vediappan², K. Srinivasan³

¹Department of Mathematics
Sathyabama University
Chennai, 600119, Tamil Nadu, INDIA

²,³Department of Mathematics
Vels University
Chennai, 600117, Tamil Nadu, INDIA

Abstract: The paper specifies deterministic inventory model for vendor-buyer with coordination and non coordination situations. In coordinate situation, the vendor provides quantity discount to the buyer for bulk purchase. In both situations total cost is developed for both buyer and vendor and order quantity is determined by analytically tractable solutions. This study is made to determine the optimal order quantity to lesser the total inventory costs. Numerical examples are provided to revels the developed model.

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Key Words: inventory, coordination, order quantity, screening cost

1. Introduction

Minimize the total inventory costs (setup cost, ordering cost, holding cost and shortage cost) is the main objective of traditional inventory models. Inventory models have lot of real time applications since we can implement these models easily in the organization. In inventory management some mechanism are
implemented to coordinate buyer and vendor. The mechanisms are quantity discount, sales rebate, trade credit, revenue sharing etc. Quantity discount is the best mechanism when it compared with the others.


2. Assumptions and Notations

The model use the following notations and assumptions.

Notations:

- $D$ Demand rate per time unit
- $R_1$ Buyer’s unit ordering cost per order
- $R_2$ Vendor’s unit setup cost per order
- $s$ Buyer’s unit shortage cost per order
- $p$ Buyer’s unit purchase cost per order
- $Q$ Economic Order quantity
- $Q_1$ Backorders level
- $H_b$ Buyer’s unit holding cost per order per unit
$H_v$ Vendor’s unit holding cost per order per unit

$s_c$ Vendor’s unit screening cost per order per unit

$n$ Vendor’s multiples of order without coordination

$m$ Vendor’s multiples of order with coordination

$k$ Buyer’s multiples of order with coordination

$d(k)$ Discount factor

**Assumptions**

(i) Demand rate is constant.

(ii) Without coordination scheme buyer having shortages and with coordination scheme no shortages are occurring.

(iii) Without coordination vendor screened the damaged items and with coordination buyer screened the damaged items for resale.

**3. Model Formulation**

In this section, model formulation for system is developed for with and without coordination. Without coordination scheme, buyer has shortages and vendor screened the damaged products. In coordination scheme, the buyer order quantity is large than regular quantity because the vendor offers quantity discount to the buyer. Hence, the buyer has no shortage and he himself screened the damaged products for resale.

**Model Formulation: Without coordination**

The total cost for buyer contains, ordering cost $\frac{R_1D}{Q}$, holding cost $\frac{H_vQ^2}{2Q}$ and shortage cost $\frac{s(Q - Q_1)^2}{2Q}$.

Thus, total cost for buyer can be written as

$$TC_b = \text{Ordering cost} + \text{Holding cost} + \text{Shortage Cost}$$

$$= \frac{R_1D}{Q} + H_vQ^2 + \frac{s(Q - Q_1)^2}{2Q}$$

The total cost for vendor contains the setup cost $\frac{R_2D}{nQ}$, the holding cost $\frac{H_vnQ}{2}$ and the screening cost $\frac{s_cQ}{2}$. 
Thus, total cost for vendor can be written as

\[
TC_v = \text{Setup cost} + \text{Holding cost} + \text{Screening Cost} \tag{3.3}
\]

\[
= \frac{R_2 D}{nQ} + \frac{H_v nQ}{2} + \frac{s_c nQ}{2} \tag{3.4}
\]

For optimality \( \frac{\partial TC_b}{\partial Q_1} = 0 \) and \( \frac{\partial^2 TC_b}{\partial Q_1^2} > 0 \) we get,

\[
Q_1^* = \frac{sQ}{H_b + s} \tag{3.5}
\]

For optimality \( \frac{\partial TC_b}{\partial Q} = 0 \) and \( \frac{\partial^2 TC_b}{\partial Q^2} > 0 \) we get,

\[
Q^* = \sqrt{\frac{2R_1 D}{sH_b}} \tag{3.6}
\]

**Model Formulation: With coordination**

The total cost for buyer contains, ordering cost \( \frac{R_1 D}{Q_c} \), holding cost \( \frac{H_b Q_c}{2} \) and screening cost \( \frac{s_c Q_c}{2} \).

Thus, total cost for buyer can be written as

\[
TC_{b1} = \text{Ordering cost} + \text{Holding cost} + \text{Shortage Cost} \tag{3.7}
\]

\[
= \frac{R_1 D}{Q_c} + \frac{H_b Q_c}{2} + \frac{s_c Q_c}{2} \tag{3.8}
\]

The total cost for vendor contains, setup cost \( \frac{R_2 D}{knQ_c} \), holding cost \( \frac{H_v knQ_c}{2} \) and buyer’s quantity discount \( pDd(k) \). Thus, total cost for vendor can be written as

\[
TC_{v1} = \text{Setup cost} + \text{Holding cost} + \text{Buyer’s discount factor} \tag{3.9}
\]

\[
= \frac{R_2 D}{knQ_c} + \frac{H_v knQ_c}{2} + pDd(k) \tag{3.10}
\]

For optimality \( \frac{\partial TC_{b1}}{\partial Q_c} = 0 \) and \( \frac{\partial^2 TC_{b1}}{\partial Q_c^2} > 0 \) we get,

\[
Q^*_c = \sqrt{\frac{2R_1 D}{H_b + s_c}} \tag{3.11}
\]
4. Numerical Examples

**Example 1.** Let $R_1 = 300$ per order, $R_2 = 100$ per order, $D = 1000$ units per year, $H_v = 2$, $H_b = 20$, $s_c = 2$, $s = 25$, $p = 5$, $n = 2$, $m = 3$, $k = 2$, $d(k) = 5\%$.

The optimal solutions are $Q^* = 34.64$, $Q_1^* = 19.25$, $TC_b(Q^*, Q_1^*) = 8.8527 \times 10^3$, $TC_v(Q^*, Q_1^*) = 1.5819 \times 10^3 Q_c^* = 115.47$, $TC_{b1}(Q_c^*) = 3.8682 \times 10^3$, $TC_{v1}(Q_c^*) = 1.0872 \times 10^3$.

**Example 2.** Let $R_1 = 400$ per order, $R_2 = 200$ per order, $D = 1500$ units per year, $H_v = 4$, $H_b = 25$, $s_c = 25$, $s = 25$, $p = 5$, $n = 3$, $m = 4$, $k = 2$, $d(k) = 10\%$.

The optimal solutions are $Q^* = 43.82$, $Q_1^* = 21.91$, $TC_b(Q^*, Q_1^*) = 1.3967 \times 10^4$, $TC_v(Q^*, Q_1^*) = 3.8176 \times 10^3 Q_c^* = 154.92$, $TC_{b1}(Q_c^*) = 5.9644 \times 10^3$, $TC_{v1}(Q_c^*) = 3.5918 \times 10^3$.

5. Conclusion

In this paper, vendor-buyer inventory model for deteriorating items is developed under screening process. Shortages are permitted in this inventory system for buyer only under non-coordination situation. The damaged items are screened for resale by the vendor for non-coordination situation and buyer for coordination situation. To compare with non-coordination, quantity discount coordination situation proves equal benefits of buyer and vendor. Our objective is to find the optimal order quantity to minimize the total inventory cost. Numerical examples are also provided to illustrate the proposed model. For the further researches, our proposed model can be extended in considering time, price or stock-dependent demand rate, credit period, quantity discounts, temporary discounts etc.,

References


