SINGLE PRODUCT MULTIPLE MANUFACTURES SUPPLY CHAIN MODEL FOR FIXED LIFETIME PRODUCT

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Abstract: In this paper production inventory model is developed for single product with multiple manufacturers. The developed model considered two part coordination scheme (non coordination and coordination). In non coordination scheme Manufacture1 and Manufacture2 produces the same product and shortages are allowed only for Manufacture1. In coordination scheme Manufacture1 stop their production, purchase products from Manufacture2. In order to stimulate sales and reduce inventory Manufacturer2 frequently offers quantity discount to Manufacture1 in coordination scheme. The aim of the study is to determining the optimum multiples of orders which will minimize the total inventory cost. A numerical example is given to illustrate the solution procedure of the model. Finally, based on this example, we conduct a sensitivity analysis of the model.

AMS Subject Classification: 90B05, 65K10

Key Words: production, inventory, quantity discount, coordination, fixed life time products
1. Introduction

Products with limited shelf period like medicines, food and beverage etc., are manufactured by a large and small scale manufacturers. Sometimes the same product may be manufactured by a large and small scale manufacturer. In this situation, the small manufacturer (Manufacturer1) may decide to purchase the same product from the large scale manufacturer (Manufacturer2) instead of manufacturing by him. At that time, Manufacturer2 frequently offers discounted price to Manufacturer1 if a large-enough quantity of an item is purchased. The model proposed in this paper deals with this case.


Yongrui Duan et al. [14] analyzed buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product. In this model the situation is entirely varied and is applicable in case of inventory decision by two manufacturers. The remainder of this paper is organized as follows: Model Formulation, notation, assumption and are presented in Section 2. In Section 3,
the numerical example is provided to illustrate the solution procedure. Section 4 is concluded with remarks.

2. Model Development

The developed model deals, with and without coordination strategy for Manufacturer1 and Manufacturer2. Quantity discount is offered by the Manufacturer2 in the model with coordination.

2.1. Assumptions and Notations

Assumptions

1. Demand is constant and production rate is greater than the demand of an item.

2. Without coordination shortages are permitted and with coordination shortages are not permitted to the Manufacturer1 i.e., There is no shortage for coordination and there is shortage for absence of coordination.

3. During the production run the production of the item is continuous and at a constant rate until production of quantity $Q$ is complete and Manufacturer1 and Manufacturer2 produces a same product.

4. Under coordination strategy Manufacturer1 stop their produce, purchase the product with Manufacturer2.

Notations

$D_1, D_2$ Annual demand of Manufacturer1 and Manufacturer2
$P_1, P_2$ Annual production rate for Manufacturer1 and Manufacturer2
$L$ Life time of product
$k_1, k_2$ Manufacturer2 and Manufacturer1 setup costs per order, respectively
$h_1, h_2$ Manufacturer2 and Manufacturer1 holding costs, respectively
$p_1, p_2$ Delivered unit price paid by the Manufacturer2 and the Manufacturer1 respectively
$Q$ Manufacturer1 order quantity, $Q = Q_1 + Q_2$
$Q_1$ Amount remains in the inventory after satisfying
the shortage demand
\( Q_2 \)  Amount which is immediately taken to satisfy unfilled demand (Shortage period)
\( Q_0 \)  Manufacturer2 order quantity
\( m \)  Manufacturer2 order multiple in the absence of coordination
\( n \)  Manufacturer2 order multiple under coordination
\( K \)  Manufacturer1 order multiple under coordination.
\( KQ_0 \) Manufacturer1’s new order quantity
\( d(K) \)  Denotes the per unit dollar discount to the Manufacturer1 if he orders \( KQ_0 \) every time
\( TC_{M1} \)  Total cost of the Manufacturer1 without coordination
\( TC'_{M1} \)  Total cost of the Manufacturer1 with coordination
\( TC_{M2}(m) \)  Total cost of Manufacturer2 without coordination
\( TC_{M2}(n) \)  Total cost of Manufacturer2 with coordination.

**Case i: Development of model without coordination**

In this case Manufacturer1 and Manufacturer2 produce the same product and shortages are allowed for Manufacturer1. The model is formulated as follows:

The total annual cost for the Manufacturer1 is given by

\[
TC_{M1} = \frac{D_1k_2}{Q} + \frac{1}{2Q} \left( \frac{P_1}{P_1 - D_1} \right) \left[ h_2 \left( Q \left( 1 - \frac{D_1}{P_1} \right) - Q_2 \right)^2 + Q_2^2s_2 \right].
\]

Now \( \frac{\partial TC_{M1}}{\partial Q_2} = 0 \) and \( \frac{\partial TC_{M1}}{\partial Q} = 0 \), we get, \( Q_2 = Q \left( 1 - \frac{D_1}{P_1} \right) \frac{h_2}{h_2 + s_2} \). Without coordination strategy, the Manufacturer1 order quantity is

\[
Q = \sqrt{\frac{2D_1k_2(h_2+s_2)}{h_2s_2} \left( \frac{P_1}{P_1-D_1} \right)}
\]

and optimum total cost

\[
TC_{M1} = \sqrt{\frac{2D_1k_2h_2s_2}{h_2+s_2} \left( 1 - \frac{D_1}{P_1} \right)}.
\]

The Manufacturer2 order is equal to some integer multiple of

\[
Q_0 = \sqrt{\frac{2D_2k_1}{h_1} \left( \frac{P_2}{P_2-D_2} \right)},
\]

Particulars of Specific Case

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Development of model without coordination</td>
</tr>
<tr>
<td>2</td>
<td>Development of model with coordination</td>
</tr>
<tr>
<td>3</td>
<td>Development of model with coordination and order quantity adjustment</td>
</tr>
</tbody>
</table>

**Case ii: Development of model with coordination**

In this case Manufacturer1 and Manufacturer2 produce the same product and shortages are allowed for Manufacturer2. The model is formulated as follows:

The total annual cost for the Manufacturer2 is given by

\[
TC_{M2}(m) = \frac{D_2k_1}{Q} + \frac{1}{2Q} \left( \frac{P_2}{P_2 - D_2} \right) \left[ h_1 \left( Q \left( 1 - \frac{D_2}{P_2} \right) - Q_0 \right)^2 + Q_0^2s_1 \right].
\]

Now \( \frac{\partial TC_{M2}(m)}{\partial Q_0} = 0 \) and \( \frac{\partial TC_{M2}(m)}{\partial Q} = 0 \), we get, \( Q_0 = Q \left( 1 - \frac{D_2}{P_2} \right) \frac{h_1}{h_1 + s_1} \). Without coordination strategy, the Manufacturer2 order quantity is

\[
Q_0 = \sqrt{\frac{2D_2k_1}{h_1} \left( \frac{P_2}{P_2-D_2} \right)}
\]

and optimum total cost

\[
TC_{M2}(n) = \sqrt{\frac{2D_2k_2h_1s_1}{h_1+s_1} \left( 1 - \frac{D_2}{P_2} \right)}.
\]
i.e., order size = $mQ_0$ with the fixed intervals

$$t_0 = \sqrt{\frac{2k_1}{D_2h_1} \left( \frac{P_2}{P_2-D_2} \right)}.$$  

Here, average inventory held up per year of Manufacturer2 is given by

$$= \frac{(m-1)Q_0 + (m-2)Q_0 + \ldots + Q_0 + 0Q_0}{m} = \frac{(m-1)Q_0}{2}$$

Now the total annual cost for the Manufacturer2 is given by

$$TC_{M2}(m) = \frac{D_2k_1}{mQ_0} + \frac{(m-1)h_1Q_0}{2} \left( 1 - \frac{D_2}{P_2} \right)$$

$$= \frac{k_1}{m} \sqrt{\frac{D_2h_1}{2k_1} \left( \frac{P_2-D_2}{P_2} \right)} + (m-1)h_1 \sqrt{\frac{D_2k_1}{2h_1} \left( \frac{P_2-D_2}{P_2} \right)}$$

So without coordination, the Manufacturer2 model can be developed as follows

$$\min TC_{M2}(m) \quad \text{s.t} \quad \begin{cases} \frac{mt_0}{L} \leq 1, \\ m \geq 1 \end{cases}$$

here $mt_0 = L$ shows that the product is not overdue before they are sold up by the Manufacturer1.

**Theorem 1.** Consider $m^*$ be the optimum of (1), if $L^2 = \frac{2k_1}{D_2h_1} \left( \frac{P_2}{P_2-D_2} \right)$, then

$$m^* = \min \left\{ \sqrt{\frac{k_1h_2}{k_2h_1} + 1}, \frac{L}{\sqrt{2k_1D_2h_1}} \right\},$$

(2)

here $[x]$ is the least integer greater than or equal to $x$, $L^2 \geq \frac{2k_1}{D_2h_1} \left( \frac{P_2}{P_2-D_2} \right)$ is to ensure that $m^* \geq 1$.

**Proof.** Obviously

$$\frac{d^2TC_{M2}(m)}{dm^2} = \frac{k_1}{m^3} \sqrt{\frac{2D_2h_1}{k_1} \left( \frac{P_2-D_2}{P_2} \right)} > 0,$$
\( TC_{M2}(m) \) is strictly convex in \( m \). Considered \( m^*_1 \) be the optimum of \( TC_{M2}(m) \), then

\[
m^*_1 = \max \{ \min \{ m / TC_{M2}(m) = TC_{M2}(m+1) \} , 1 \} \\
= \max \{ \min \{ m / m(m+1) = \frac{2D_2 k_1}{Q_0^2 \left( 1 - \frac{D_2}{P_2} \right) h_1} \} , 1 \} \\
= \left[ \sqrt{\frac{h_2 k_1}{h_1 k_2} + \frac{1}{4} - \frac{1}{2}} \right] = 1.
\]

Put the value of \( t_0 \) into the constraints in (1), then we have \( m \sqrt{2D_2 k_1 \left( \frac{P_1}{P_2 - D_2} \right)} = L \). Take \( m^*_2 = \sqrt{\frac{2k_1}{D_2 h_1} \left( \frac{P_1}{P_2 - D_2} \right)} = 1 \), is true since \( L^2 = \frac{2k_1}{D_2 h_1} \left( \frac{P_1}{P_2 - D_2} \right) \). \( m^* = m^*_1 \) where \( m^*_1 = m^*_2 \), otherwise \( m^* = m^*_2 \). Therefore \( m^* = \min \{ m^*_1 , m^*_2 \} \), if \( L^2 = \frac{2k_1}{D_2 h_1} \left( \frac{P_1}{P_2 - D_2} \right) \).

**Remark 1.** Without coordination, the Manufacturer1 optimum total cost is \( TC_{M1} \), order size is \( \sqrt{\frac{2D_1 k_2 (h_2 + s_2)}{h_2 s_2} \left( \frac{P_1}{P_2 - D_1} \right)} \) and the Manufacturer2 optimum total cost is \( TC_{M2}(m^*) \), order size is \( m^* \sqrt{\frac{2D_2 k_1}{h_1} \left( \frac{P_2}{P_2 - D_2} \right)} \).

**Case ii: Development of model with coordination.** In coordination scheme, Manufacturer1 stop their production and purchase the product of Manufacturer2 and no shortages for Manufacturer1. In this strategy, Manufacturer2 given quantity discount with the discount factor \( d(K) \), if Manufacturer1 change his lot size by \( KQ_0, K > 0 \). Now the Manufacturer2 lot size is \( nKQ_0 \), where \( n \) is a positive integer and \( KQ_0 \)is the Manufacturer1 new order quantity. Now, the Manufacturer1 order quantity \( Q_0 = \sqrt{\frac{2D_1 k_2}{h_2} \left( \frac{P_1}{P_1 - D_1} \right)} \) and optimum total cost \( TC'_{M1} = \sqrt{2D_1 k_2 h_2 \left( 1 - \frac{D_2}{P_1} \right)} \). Therefore, Manufacturer2 total cost

\[
TC_{M2}(n) = \frac{D_2 k_1}{nKQ_0} + \frac{(n-1) \left( 1 - \frac{D_2}{P_2} \right) h_1 KQ_0}{2} + p_2 D_2 d(K).
\]

In coordination discount strategy, the problem can be developed as follows

\[
\min TC_{M2}(n)
\]
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subject to \[
\begin{align*}
& nKt_0 \leq L, \\
& \frac{D_2k_2}{KQ_0} + \frac{KQ_0(1 - \frac{D_2}{P_2})h_2}{2} - \sqrt{2D_2k_2h_2 \left(1 - \frac{D_2}{P_2}\right)} \leq p_2D_2d(K), \\
& n \geq 1,
\end{align*}
\] (4)

Now \(Kt_0 = L\) shows's that the product is not overdue before they are sold up by the Manufacturer1. The second constraint shows that the Manufacturer1 cost under coordination cannot exceed that without coordination.

**Theorem 2.** \(TC_{M2}(n^*) = TC_{M2}(m^*)\) is true, if \(m^*\) is optimum of (1) and \(n^*\) be the optimum of (4).

**Proof.** If the second constraint must be an equation, then \(p_2D_2d(K)\) takes smallest value and \(TC_{M2}(n)\) is optimized.

\[
d(K) = \frac{\frac{D_2k_2}{KQ_0} + \frac{KQ_0(1 - \frac{D_2}{P_2})h_2}{2} - \sqrt{2D_2k_2h_2 \left(1 - \frac{D_2}{P_2}\right)}}{p_2D_2}
\] (5)

If \(K = 1\), then \(d(1) = \frac{\sqrt{2D_2k_2h_1(1 - \frac{D_2}{P_2})} - \sqrt{2D_2k_2h_1(1 - \frac{D_2}{P_2})}}{p_2D_2} = 0\). So if \(K = 1\), then (4) is equivalent to (1). Therefore, \(TC_{M2}(n^*) \leq TC_{M2}(m^*)\) is true. \(\square\)

**Remark 2.** Theorem (2), ensures that Manufacturer2 will get more benefit to compare with Manufacturer1 if the manufacturer1 order size is \(KQ_0, K > 0\) because optimum total cost under coordination is less than without coordination.

Put equation (5) into equation (3), we have

\[
TC_{M2}(n) = \frac{D_2k_1}{nKQ_0} + \frac{(n-1)(1 - \frac{D_2}{P_2})h_1KQ_0}{2} + p_2D_2 \left(\frac{\frac{D_2k_2}{KQ_0} + \frac{KQ_0(1 - \frac{D_2}{P_2})h_2}{2} - \sqrt{2D_2k_2h_2 \left(1 - \frac{D_2}{P_2}\right)}}{p_2D_2}\right).
\] (6)
Let $K^*$ be the optimum of $T_{C_M^2}(n)$, we have

$$K^*(n) = \frac{1}{Q_0} \sqrt{\frac{2D_2\left(\frac{k_1}{n}+k_2\right)}{\left(1-D_2^2\right)[(n-1)h_1+h_2]}}$$

(7)

From first constraint of (4), we have

$$\left(\frac{k_1}{n}+k_2\right)n^2 = \frac{L^2Q_0^2h_2}{4k_2}\left(1-D_2\right)^2((n-1)h_1+h_2).$$

Take

$$g(n) = -k_2n^2 + \left(\frac{D_2L^2}{2}\left(\frac{P_2-D_2}{P_2}\right)h_1 - k_1\right)n$$

$$+ \frac{D_2L^2}{2}\left(\frac{P_2-D_2}{P_2}\right)(h_2-h_1).$$

(8)

Substituting (7) and $t_0 = \sqrt{\frac{2k_2}{D_2h_2}\left(\frac{P_3}{P_2-D_2}\right)}$ into (3), we have

$$T_{C_M^2}(n) = 2D_2\left[k_1\left(1-\frac{D_2}{P_2}\right)h_1 + \frac{k_1}{n}\left(1-\frac{D_2}{P_2}\right)[h_2-h_1]\right]$$

$$+ nk_2\left(1-\frac{D_2}{P_2}\right)h_1 + \left(1-\frac{D_2}{P_2}\right)k_2[h_2-h_1]\right] - 2Dh_2k_2\left(1-\frac{D_2}{P_2}\right)^{1/2}. \quad (9)$$

Therefore, (4) becomes

$$\text{min} T_{C_M^2}(n)$$

subject to

$$\begin{cases}
    g(n) = 0, \\
    n = 1,
\end{cases} \quad (10)$$

for $x \geq 0$, $\sqrt{x}$ is a strictly increasing so the above equation is equivalent to

$$\text{min} \; \hat{T}_{C_M^2}(n) = D_2\left[k_1\left(1-\frac{D_2}{P_2}\right)h_1 + \frac{k_1}{n}\left(1-\frac{D_2}{P_2}\right)[h_2-h_1]\right]$$

$$+ nk_2\left(1-\frac{D_2}{P_2}\right)h_1 + \left(1-\frac{D_2}{P_2}\right)k_2[h_2-h_1]\right] \quad \text{for } x \geq 0, \sqrt{x} \text{ is a strictly increasing so the above equation is equivalent to}$$
subject to \[
\begin{cases} 
g(n) \geq 0, \\
 n \geq 1
\end{cases}
\] (11)

Here, $\overline{TC}_M(n)$ is convex when $h_2 = h_1$, since

$$
\overline{TC}_M''(n) = \frac{2D_2k_1 \left(1 - \frac{D_2}{P_2}\right) [h_2 - h_1]}{n^3} > 0,
$$

otherwise it is concave. $g(n)$ is strictly concave because $g''(n) = -2k_2 < 0$.

**Lemma 1.** Let $n^*_1$ be the optimum of $\overline{TC}_M(n)$ for $n = 1$, then

$$n^*_1 = \begin{cases} 
\left\lceil \sqrt{\frac{k_1[h_2-h_1]}{k_2h_1}} + \frac{1}{4} - \frac{1}{2} \right\rceil, & \frac{k_1[h_2-h_1]}{k_2h_1} = 2 \\
1, & \text{otherwise}
\end{cases}$$

(12)

**Proof.** $\overline{TC}_M(n^*_1) = \min\{\overline{TC}_M(n^*_1-1), \overline{TC}_M(n^*_1+1)\}$ because $n^*_1$ is the minimum of $\overline{TC}_M(n), n \geq 1$. Now

$$\overline{TC}_M(n^*_1) - \overline{TC}_M(n^*_1-1) = \frac{-D_2k_1 \left(1 - \frac{D_2}{P_2}\right) [h_2 - h_1]}{n^*_1(n^*_1-1)} + D_2k_2 \left(1 - \frac{D_2}{P_2}\right) h_1 = 0,$$

$$\left(n^*_1 - \frac{1}{2}\right)^2 = \frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4}. \quad (13)$$

Similarly, by $\overline{TC}_M(n^*_1) - \overline{TC}_M(n^*_1+1) = 0$, we have

$$\left(n^*_1 + \frac{1}{2}\right)^2 = \frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4} \quad (14)$$

Hence, if $\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4} < 0$, $\overline{TC}_M(n^*_1) = \overline{TC}_M(n^*_1+1)$ for any given $n$, then $n^*_1 = 1$. If $\frac{k_1[h_2-h_1]}{k_2h_1} + \frac{1}{4} \geq 0$ by (13) & (14), $\sqrt{\frac{k_1[h_2-h_1]}{k_2h_1}} + \frac{1}{4} - \frac{1}{2} \leq n^*_1 \leq \sqrt{\frac{k_1[h_2-h_1]}{k_2h_1}} + \frac{1}{4} + \frac{1}{2}$. So $n^*_1 = \left\lceil \sqrt{\frac{k_1[h_2-h_1]}{k_2h_1}} + \frac{1}{4} - \frac{1}{2} \right\rceil$. Also note that, if $0 < \frac{k_1[h_2-h_1]}{k_2h_1} < 2$ then $n^*_1 = 1$, so (12) holds. \qed
3. Numerical Example

In this section, numerical examples are presented to illustrate the performance of developed model.

**Example 1.** Given $k_1 = 300$ per order, $k_2 = 100$ per order, $h_1 = 10$ per year, $h_2 = 5$ per year, $s_2 = 25$ per year, $P_1 = 8000$ units per year, $D_1 = 5000$ units per year, $P_2 = 15000$ units per year, $D_2 = 10000$ units per year, $p_2 = 30$ per unit, $\alpha = 0.5$, $L = 0.5$ year.

The computational result shows the following optimal values $k^* = 3.0000$, $d(k^*) = 0.0015$, $TC_{M2}(m^*) = 2236.1$, $TC_{M2}(n^*) = 1825.7$, $TC_{M1} = 1250$, $TC'_{M1} = 1369.3$.

**Example 2.** Given $k_1 = 400$ per order, $k_2 = 200$ per order, $h_1 = 15$ per year, $h_2 = 10$ per year, $s_2 = 55$ per year, $P_1 = 12000$ units per year, $D_1 = 10000$ units per year, $P_2 = 20000$ units per year, $D_2 = 15000$ units per year, $p_2 = 30$ per unit, $\alpha = 0.5$, $L = 0.5$ year. The computational result shows the following optimal values $k^* = 1.7321$, $d(k^*) = 0.0013$, $TC_{M2}(m^*) = 3354.1$, $TC_{M2}(n^*) = 2835.2$, $TC_{M1} = 2375$, $TC'_{M1} = 2582$.

3.1. Sensitivity Analysis

We now study the effects of changes in the value of system parameters $h_1$, $h_2$, $k_1$, $k_2$, $s_2$ on the Manufacturer1 and Manufacturer2 minimum total relevant cost per unit time $TC^*_{M1}$, $TC'_M$, $TC^*_{M2}(m)$ and $TC^*_{M2}(n)$ of the Example 1. The sensitivity analysis is performed by taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 1.

Computational result indicates that

1. The optimum total cost of Manufacturer2 under coordination is less than that without coordination. i.e., Manufacturer2 is comparatively highly benefited than Manufacturer1 in spite of giving quantity discount.

2. An increase in holding cost for Manufacturer2, the optimal total cost of Manufacturer1 and Manufacturer2 remain same or increase.

3. A reduce value of holding cost for Manufacturer1 tends to reduce in total cost for Manufacturer1 and Manufacturer2.

4. The set up cost for Manufacturer1 and Manufacturer2 decrease, automatically the total cost of Manufacturer1 and Manufacturer2 gets decreased.
Table 1: Effect of changes in the parameters of the inventory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k^*$</th>
<th>$d(k^*)$</th>
<th>$TC_{M2}(m^*)$</th>
<th>$TC_{M2}(n^*)$</th>
<th>$TC_{M1}$</th>
<th>$TC'_{M1}$</th>
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</thead>
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<td>$k_1$</td>
<td>400</td>
<td>3.3541</td>
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<td></td>
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</tr>
<tr>
<td>$k_2$</td>
<td>50</td>
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<td>2236.1</td>
<td>2124.7</td>
<td>885</td>
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<tr>
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<td>100</td>
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<td>1825.7</td>
<td>125</td>
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<td></td>
<td>150</td>
<td>2.5981</td>
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<td>7</td>
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4. Conclusion

This study develops single product and multiple manufacturers for fixed life time products. This model assumes coordination and non coordination scheme. In absence of coordination, Manufacture1 and Manufacture2 produce the same product and shortages allowed for Manufacture1. Manufacture1 more benefited under absence of coordination. Under coordination scheme Manufacturer1 stop their produce and purchase items from Manufacture2 with quantity discount. This paper concludes coordination scheme is more benefited to Manufacture 2 even though he offers quantity discount to Manufacture1. It is proved that the quantity discount is the best strategy to achieve system optimization and win – win outcome of Manufacture2. The goal of this paper is to minimize the total relevant cost to determined optimal decision variables. Numerical examples are also provided to illustrate the proposed model. Sensitivity analysis on the parameter changes is also performed. The proposed model can be extended by considering factors like multiple products, random discount offering, credit periods etc,
References


