ESTIMATION OF POWER SPECTRAL DENSITY 
OF SEISMIC DATA 
USING WELCH METHOD 

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Abstract 

Seismic data is processed to explore the details of geological structures of the 
Earth’s crust. In this paper raw seismic data is generated by the explosion of dynamite 
shot which is placed in the Earth’s crust. Raw seismic data is corrupted by unwanted 
signals from different sources i.e., the signal to noise ratio will be low. So, there is a 
requirement to obtain ideally characterized signal after processing but practically it is 
not possible. In order to get a signal with approximate ideal characteristics there should 
be increase in SNR to increase the signal strength. By estimating the power spectrum 
of input signal using Welch method there will be increase in SNR.
Keywords: Stochastic signal processing, Adaptive signal processing, Seismology, Applied statistics, Precise Estimation methodology.

1. Introduction

1.1. Seismic waves

Seismic waves are generated by earthquake, man-made explosions and volcanic eruptions. Seismic data is processed to explore the details of geological structures of the Earth’s crust. Due to some factors, seismic data is corrupted by unwanted signals like surface waves. The unwanted signal is said to be noise and the corrupted signal is said to be seismic data with noise. Seismic signal is random in nature with some unwanted signals. Seismic signals are produced by a source (an explosion). The signals which are producing from an explosion act as a transmitter and then transfer through earth layers. A few signals of these will be reflected, some will be refracted and some will be lost due to attenuation. The reflected signals will be recorded by a receiver called as geophone. The seismic analysis means collecting data by more number of geophones, transmission through a narrow band channel, storing of data for processing and interpretation [1].

The signal radiated from a seismic source which is generated from an explosion or man-made actions is generally a higher or lesser complicated displacement step function. Seismic data $s(t)$ is produced from the explosion of a dynamite shot. Fourier transform is used to convert any arbitrary function $s(t)$ which is in the time domain to its equivalent function $S(\omega)$ which is in the frequency domain. The following are the relations:

$$s(t) = 2(\pi) S(\omega) e^{i\omega t} d\omega$$  \hspace{1cm} (1)

$$\int_{-\infty}^{\infty} S(\omega) = s(t)e^{-i\omega t} dt = S(\omega) e^{i\varphi(\omega)}$$  \hspace{1cm} (2)

Sign conventions are used in the above equations to get wave vector which is positive in the direction of wave propagation eg: $e^{-i\omega t}$ is in eq(1) and is in $e^{i\omega t}$ eq(2)
1.2. Previous Study

Using two shaping processes the simultaneous de-noising and increasing resolution of synthetic model and post-stack seismic signal is enhanced for the first time. The two processes are shaping the estimated wavelet to ideal wavelet and shaping the inverted to more desirable model during repetitions [2]. An improved wavelet threshold de-noising algorithm is used to increase the signal quality and SNR of seismic signal. The designed system results high stability and improves SNR ratio [3]. It is difficult to get ideal characteristics by using traditional theoretical methods of spectrum analysis and wave filtering. So polarization filtering method is implemented to separate the noisy signal from the useful signal [4]. By interpolating the fractional order time differential equation with a parabolic and hyperbolic equation we will get a noise free seismic data. The adaptive variable time fractional-order anisotropic diffusion equation is used to remove noise present in the seismic data and for structure enhancement [5].

1.3. Welch Method

This paper presents a welch algorithm to calculate the power spectral density of seismic data produced from an explosion. Welch method is non-parametric method which includes periodogram. The possible implementation of Fast Fourier transform is possible by using periodogram. If data length samples are optimally selected then the periodogram technique using welch method is suitable to provide good resolution. Compared to Bartlett, Hanning, Blackman window Rectangular and Hamming give better results. The admirable resolution characteristics of sinusoids comparable strength will be good for rectangular window. The nonparametric welch method is the estimation of input power at different frequencies. A method to estimate the autocorrelation of a finite length signal is a periodogram. Welch method is the improvement of periodogram. Periodogram reduces unwanted signals in the estimated power spectrum of reducing the frequency resolution where SNR ratio is high [6].

Section 2 deals with mathematical modeling, section 3 deals with simulation and results, section 4 deals with conclusion.
2. Mathematical Modeling

There are two modifications in welch method. These modifications are proposed to Bartlett’s method. Assume $x_i(n)$ is a sequence and $w(n)$ is a window data. The first is allowing the sequences $x_i(n)$ to overlap and the second modification is applying the window data $w(n)$ to each sequence. By this a set of modified periodograms are produced and those are averaged. Here $i^{th}$ sequence is given when adjacent sequences are offset by $D$ points and length of each sequence is $L$.

$$x_i(n) = x(n + iD); \ n = 0, 1, ..., L - 1$$ (3)

If $K$ sequences cover $N$ data points then the amount of overlap between $x_i(n)$ and $x_{i+1}(n)$ is $L - D$ points.

$$N = L + D(K - 1)$$ (4)

If there is no overlap ($D = L$) then we get $K = N/L$ in Bartlett’s method as sections of length $L$ and sequences are allowed to overlap by 50%.

$$K = 2(N/L) - 1$$ (5)

Here, number of modified periodograms are doubled that are averaged then it maintains same resolution as Bartlett’s method. Hence it reduces the variance. However, by 50% overlap results in

$$K = (N/L) - 1$$ (6)

If we want to evaluate the performance of Welch method then the following expression estimate the performance of Welch method explicitly in terms of $x(n)$

$$P_w(e^{j\omega}) = \frac{1}{KLU} \sum_{i=0}^{K-1} \sum_{l=0}^{L-1} |\sum_{n=0}^{n-1} \omega(n) x(n + iD) e^{-jn\omega}|^2$$ (7)

The following expression is in terms of modified periodogram

$$P_w(e^{j\omega}) = \frac{1}{K} \sum_{i=0}^{K-1} p_i \ (i) \ (e^{j\omega})$$ (8)
Finally the expected value of Welch method

\[ \mathbb{E}\{P_w(e^{jw})\} = \mathbb{E}\{P_w(e^{jw})\} = \frac{1}{2\pi L U} P_x(e^{jw}) * |W(e^{jw})|^2 \]  

(9)

3. Simulation and Results

Step 1: Seismic data is taken from the book_seismic_data.mat [1]. Synthetic signal is generated by taking the sample number on x-axis and magnitude on y-axis as shown in Fig.1.

Step 2: Synthetic signal is contaminated with noise and power spectral density is calculated by taking normalized frequency on x-axis and PSD on y-axis. The normalized frequency is at 0.5 dB as shown in Fig.2.

Step 3: The raw seismic signal with trace 7 is loaded. It is as shown in Fig.3.

Step 4: The actual signal contains bias and the bias is removed. This removed mean and bias is said to be de-trended signal. The de-trended raw signal is as shown in Fig.4.

Step 5: Power spectrum of raw seismic signal is plotted using Welch’s algorithm as shown in Fig.5. This is before passing the BPF.

Step 6: Band pass filter spectrum is as shown in Fig.6.

Step 7: De-trended signal is convoluted with band pass filter spectrum. So the noise is removed using band pass filter of 8th order. FIR band pass filtered signal is plotted by taking sample no on x-axis and magnitude on y-axis. It is as shown in Fig.7.

Step 8: FFT spectrum is plotted after BPF by taking normalized frequency on x-axis and amplitude which represents in dB on y-axis as shown in Fig.8.

Step 9: Power spectral density is estimated and plotted as shown in Fig.9 after passing BPF using Welch algorithm.

\[ w = \frac{2\pi f}{f_s} = 0.1094\pi \quad \frac{2\pi f}{500} = 0.1094\pi \]

Tonal frequency \( f = 27.35 \text{ Hz} \)
Fig. 1. Synthetic signal using Welch.

Fig. 2. Power spectral density method for synthetic signal.

Fig. 3. Raw seismic signal.

Fig. 4. De-trended raw seismic signal.

Fig. 5. Raw signal spectrum.

Fig. 6. FIR band pass filter spectrum analysis using Welch method.
Fig. 7. FIR band pass filtered signal  
Fig. 8. FFT spectrum seismic after BPF  
Fig. 9. After BPF signal spectrum analysis using Welch method.

4. Conclusion

The power spectral density of seismic data is estimated using Welch algorithm. The SNR of raw seismic data and resolution is increased after calculating the power spectrum of seismic data by using Welch algorithm. Spectral leakage is reduced through the side lobes of data window. Welch method gives better results compared to some non-parametric methods.
5. References


