Spectral analysis of seismic signals using Burg algorithm

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Abstract: Seismic signal has high noise and it must be filtered to extract the original seismic signal from the seismogram. In this paper FIR band pass filter is used to reduce the noise and Burg algorithm is used to process seismic signal for the estimation of seismic signal spectrum where burg minimizes the forward and backward prediction errors.

Keywords: Applied Statistics, Adaptive signal processing, Stochastic signal processing, Seismology.

1. INTRODUCTION:
Most of the natural signals are analog signals and they are converted into digital signals by amplitude quantizing and time discretization. Analog signal is converted in to digital to occupy less memory. Noise in signals is the major factor which degrades the system performance and it can be eliminated by filters in signal processing.

Seismic waves are classified into body waves and surface waves. Body waves are fast, which are first received and can pass through solids which are classified into two types, primary waves which are parallel to the direction of propagation and secondary waves which are perpendicular to the direction of propagation [1]. Surface waves can be of two types Rayleigh waves and love waves. Rayleigh waves are compressional in motion and love waves are not compressional. Surface waves cause destruction due to high amplitude and can pass through free surfaces only. Seismic signal have very less frequency and it consists of noise from the external conditions like temperature and other sources which can be attenuated by de-convolution and stacking. Noise generated by the source is called coherent noise and can be eliminated by filtering [2]. Band Pass
Filter (BPF) is used for seismic signal filtering to improve overall gain of the seismic shot and it enhances the SNR of the signal by eliminating the low and high frequency noise including ground roll noise.

Signal processing is used for analysis of spectrum in a signal. FFT is not applicable for the seismic signal processing, as the seismic waves are random in nature [4]. FFT produces the average frequency of a signal over the entire time the signal acquired. White noise have energy which is present in all the frequencies have flat broadband frequency spectra [8]. Seismic signal processing has 3 stages for good seismic resolution they are Deconvolution, Stacking and Migration.

Deconvolution is made for the seismic signal after the BPF. Vertical resolution is decreased as there is loss in original wider frequency band so as to enhance the vertical resolution deconvolution is used by compressing the source wavelet. Wiener filter is used to perform the deconvolution in least squares method and it eliminates the truncation error between desired output and actual output and transforms one wavelet to another wavelet [6]. Deconvolution decreases the multiple reflections and increases the resolution.

Maximum Entropy or Burg de-convolution uses an entropy norm to produce the foreseeable and random elements of the data and has a strong spectral balance. The Fourier transform of the Auto correlation is the power spectrum of the random signal, power spectral density is classified into two types-classical or non-parametric. The most commonly used models are autoregressive (AR), moving average (MR), autoregressive moving average (ARMA), and harmonic (complex exponentials in noise) [6].

By using the autocorrelation and power spectrum [3], the signal can be foreseen from the previous sample. Information about the signal can be conveyed by degree of randomness. Uncorrelated input is excited and the signals can be modelled as output of the signal. The filter model gives the expected structure of the signal whereas the unpredictable part of the signal is given by random input. The burg model relay [7] for the least amount of backward and forward prediction errors. The estimation of parameters in autoregressive method is more accurate than autocorrelation as the burg algorithm does not use window to the data.

Section 2 deals with the mathematical modeling and with Burg algorithm. Simulations and results are discussed in section 3 and concluded in section 4 which summarizes the burg technique for spectral analysis.

2. MATHEMATICAL MODELLING:

The second class in spectrum estimation is the non – classical or parametric method, let \( x(b) \) is the p-th order autoregressive

\[
\hat{p}_a(e^{jw}) = \frac{1}{|\sum_{k=0}^{p} \hat{a}_p(k)e^{-jkw}|^2} \tag{1}
\]

The model parameters are estimated after selection of the model and the power spectrum is estimated by in co-operating the parameters in the parametric form.

\[
\hat{p}_s(e^{jw}) = \frac{|\sum_{k=0}^{d} \hat{b}_q(k)e^{-jkw}|^2}{1+|\sum_{k=0}^{p} \hat{a}_p(k)e^{-jkw}|^2} \tag{2}
\]
2.1 Autoregressive power estimation:
Yield of the all pole channel that is driven by those unit difference white noise is called autoregressive power estimation. [9].
The power spectrum of auto regressive process of order P is given by
$$\hat{p}_x(e^{jw}) = \frac{|b(0)|^2}{1+\sum_{k=0}^{P} a_p(k)e^{-jkw}}$$
(3)
If b (0) and \(a_p \) are estimated then power spectrum can be estimated using
$$\hat{p}_{AR}(e^{jw}) = \frac{|\hat{b}(0)|^2}{1+\sum_{k=0}^{P} \hat{a}_p(k)e^{-jkw}}$$
(4)
where \(\hat{p}_x(e^{jw}) \) accuracy depends on how the model parameters are estimated.

2.2 Auto correlation method:
All pole model of auto correlation model is used to estimate the AR coefficient by solving the autocorrelation normal equations. The autocorrelation matrix is nothing but a Hermitian and Toeplitz matrix with autocorrelation function \(R_{zz}(j)\)

$$R_y = E(zz^H) = \begin{bmatrix}
R_{zz}(0) & * & * & \cdots & * \\
R_{zz}(1) & R_{zz}(0) & * & \cdots & * \\
R_{zz}(2) & R_{zz}(1) & R_{zz}(0) & \cdots & * \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
R_{zz}(N-1) & R_{zz}(N-2) & R_{zz}(N-3) & \cdots & R_{zz}(0)
\end{bmatrix}$$

2.3 Linear prediction:
A sample value \(x(b)\) i.e. signal at time ‘b’, linear predictor model uses the continuously weighted combination of P previous samples [\(x(b-1), x(b-2), x(b-3) \ldots, x(b-P)\)].
The \(x(b)\) is given by
$$\hat{x}(b) = \sum_{k=1}^{P} a_k x(b - k)$$
(5)
where \(a_k\) is the predictor coefficients, the difference between the predicted sample and actual sample gives the prediction error \(e(b)\)
$$e(b) = x(b) - \hat{x}(b)$$
(6)
$$e(b) = -\sum_{k=1}^{P} a_k x(b - k) + x(b)$$
(7)
$$x(b) = e(b) + \sum_{k=1}^{P} a_k x(-k + b)$$
(8)
The forward prediction \(e(b)\) predicts samples from the past P signals [\(x(b-1), x(b-2), x(b-3) \ldots, x(b-P)\)] whereas the backward prediction \(y(b)\) gives samples \(x(b-P)\) from the upcoming samples \(x(b-P+1), \ldots\) x(b).
$$\hat{x}(b - p) = \sum_{k=1}^{P} c_k x(b - k + 1)$$
(9)
We know that backward prediction \(y(b) = -\hat{x}(b - p) + x(b - P)\) then
$$y(b) = -\sum_{k=1}^{P} c_k x(b - k + 1) + x(b - P)$$
(10)
The coefficients $c_k$ and $a_k$ can be predicted by using the Levinson-Durbin matrix.

### 2.4 Burg Method:
The addition of the squares of the forward and backward prediction errors [6] and minimizing is referred as burg method and can be denoted as

$$
E_{fb}^{(i)} = \sum_{b=0}^{N-1} \{ e^{(i)}(b) \}^2 + [y^{(i)}(b)]^2
$$

where $E_{fb}^{(i)}$ is minimised according to the reflection coefficient $k_i$, to get the reflection coefficient $E_{fb}^{(i)}$ is derivated and it is equalled to zero.

$$
k_i = \frac{2 \sum_{b=0}^{N-1} \{ e^{(i-1)}(b)y^{(i-1)}(b-1) \}}{\sum_{b=0}^{N-1} \{ [e^{(i-1)}(b)]^2 + [y^{(i-1)}(b-1)]^2 \}}
$$

We recognize that forward predictor coefficient vector is those turned around versify of the retrograde predictor coefficient vector. Therefore the $P^{th}$ order prediction filter can be estimated by minimising the sum of backward and forward prediction errors[10].

$$
E_{fb}^{(i)} = \sum_{b=0}^{N-1} \{ [y^{(i)}(b)]^2 + e^{(i)}(b) \}^2
$$

$$
E_{fb}^{(i)} = \sum_{b=0}^{N-1} \{ -\sum_{k=1}^{P} a_k x(b-k) + x(b) \}^2 + [x(b-P) - \sum_{k=1}^{P} a_k x(b-P+k)]^2
$$

### 3. SIMULATION AND RESULTS:
Step 1: The reference signal Book_Seismic_Data.mat [5] is taken from the Texas, USA by using dynamite as a source which is kept at 80-100 feet depth holes. One Trace is extracted with a sampling interval of 0.002 with 1501 samples and it is used for the spectrum analysis.

Step 2: The performance of the burg algorithm is evaluated with known synthetic signal to measure the tonals of the seismic signal.

Step 3: Let the synthetic signal with $0.98\exp\pm j\pi/5$ and $0.98\exp\pm j0.3\pi$ as poles be Auto Regressive process. $0.2\pi$ and $0.3\pi$ are the normalized frequencies. The poles will be at $0.5760\pm j0.7928$ and $0.7928\pm j0.5760$. The signal generated is given in Fig.1.

Step 4: The PSD of the synthetic signal is given in Fig.2. The figure indicates that the peak normalized frequencies are at 0.2 and 0.3. So we can conclude that burg algorithm is working well.

Step 5: The raw seismic signal data is given in Fig.3.

Step 6: The mean is subtracted from the original signal and it is called detrended signal shown in Fig.4.
Step 7: Burg algorithm is applied for the detrended signal and the PSD of the signal is given in Fig.5

$$\omega = \frac{2\pi f_s}{f_s} = 0.0958\pi$$

$$\frac{2\pi f}{500} = 0.0958\pi$$

Tonal frequency = $$f = \frac{500}{2} \times 0.0958\pi$$

$$f = 23.95\text{HZ}$$

Step 8: Band Pass filter with range [15Hz to 60Hz] with finite impulse response [FIR] order 8 is realized and is given in Fig.6.

Step 9: FIR BPF is convolved with the detrended seismic signal and the result is shown in Fig.7.

Step 10: Another insignificant tonal is: $$\omega_2 = 0.08798\pi$$, therefore

$$f = 250 \times 0.08798$$

$$= 21.995 \text{HZ}$$

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Fig 1. Synthetic signal

Fig 2. PSD of Synthetic Signal

Fig 3. Raw seismic signal

Fig 4. Detrended Seismic Signal
Fig 5. Seismic signal analysis using Burg

Fig 6. FIR filter frequency spectrum

Fig 7. Seismic signal convolved with FIR Filter

Fig 8. Freq Vs Mag Representation

Fig 9. Spectrum analysis after BPF using Burg
4. CONCLUSION

The raw seismic signal is detrended where mean is subtracted from the original signal and the PSD of synthetic signal is estimated using the burg. The burg method decreases the forward and backward prediction errors and the synthetic seismic signal is processed using the Burg to get the PSD of the synthetic signal.

5. REFERENCES
